Measurement of the Bulk Flow and Transport Characteristics of Selected Fractured Rock Aquifer Systems in South Africa

Report to the
Water Research Commission

by

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Executive Summary

In this research document the need for an effective method for regional bulk flow properties is discussed. An estimation method for transmissivity values in South Africa is given in respect to fractured rock aquifers with dual porosity properties. The influence of bulk regional transmissivity values for dual porosity medium is discussed in regards to the differences between the largest transmissivity value and the smallest. If relatively small differences (less than one order) are obtained for the host rock formation compared to the fractured zone, a mean value can be calculated using arithmetic, geometric or harmonic mean calculation methods. Once the difference between the lowest and the largest transmissivity value for a region is great than one order, only the geometric and harmonic mean calculation methods are reasonable estimates of bulk flow transmissivity. Two orders or greater the harmonic mean is the most appropriate method to apply for an effective area estimate. This has been confirmed for both natural and forced gradient scenarios with the application of a ModFlow simulation system.

The influence of sampling method was also reported and the results showed that low yielding boreholes should also be entered into the National Groundwater Database to improve regional bulk flow estimates. The National Groundwater Database was interrogated to evaluate data stored in it and the estimated mean transmissivity values were far above that obtained from field experiments. Two test sites were also included in the investigation to highlight the influence that the matrix or host rock has on the calculated transmissivity value. The Krugersdrift test site results indicated that regional transmissivity values are affected by geographical features as well as water divides. The second test site located at the University of the Free State illustrated the effect that pump tests have on the observed transmissivity values. Both uncased and cased boreholes were tested to determine transmissivity value, in both instances the transmissivity value decreased until it represented a mean between the host formation and the bedding plan fracture. The harmonic mean value gave the most effective estimate of the bulk flow parameter in the area even though packers were installed encapsulating the fractured zone.

Table of Contents

Table of Figures

List of Tables

Chapter 1 Introduction

Africa is an ever-changing human landscape, with social and political issues driving the development of the continent. In this regard the availability of freshwater is of notable concern, since the stability of a government depends heavily on its ability to provide services for basic human needs (CSA, 1996). Water resources management in this regard is gaining importance as a critical development issue (UNWater, 2006). Through management programs the population's environment can be improved. These include poverty reduction, agricultural productivity, industrial growth and sustainable growth in downstream communities (Davis et al., 2003). In Africa the World Health Organization (WHO) estimated that if access to basic water and sanitation services were improved, the health sector would save more than US$11 billion in treatment costs. People would gain 5.5 billion productive days each year due to reduced diarrheal disease (Tobin, 2008).

1.1 Aquifer Systems in Africa

Regional aquifers in Africa can differ substantially from one area to another and exist as alluvial, lacustrine, basaltic and sedimentary aquifers in coastal zones. The development of aquifer systems in Africa relies primarily on two major factors, *i.e.* the tectonic and climatic environments. In terms of aquifer setting, two completely different geological domains exist. Firstly, the mobile belt of the Atlas range and secondly the African platform, which are separated by the South Atlas Fault Zone located between the southern Saharan Craton and smaller microplate mesetas to the north (Steyl and Dennis, 2010).

The folded zones of northwest and southern Africa only occupy about 3 percent of the continent but sustains ca. 10 percent of its population (Zektser and Everett, 2004). In South Africa the region between Durban and Cape Town consists of paleozoic limestones and paleozoic sandstones, quartzites and shales. In the Atlas fault zone fissured carbonate aquifers of mainly limestone and dolomite of the Jurassic and Cretaceous age contribute to coastal and interbedded clastic porous aquifers.

Major aquifers of the African platform are found over an extensive area, interacting with coastal regions. Six major aquifer systems can be identified on their respective lithology; continental sandstone, carbonate, sandstone-carbonate (variable), alluvial, basaltic and crystalline basement aquifers (Zektser and Everett, 2004).

The continental sandstone aquifer group contains the Nubian Sandstone Aquifer, the continental intercalaire of the Sahara, the Karoo and the Kalahari. The carbonate aquifer group consists of the Jabal Akhdar-Sirte (coastal basin) in northern Libya. The sandstone-carbonate variable composition is found in the North Sahara basin in Algeria and Tunisia (Margat, 1994). The sandstone-limestone hydrogeological complex occurs mainly in coastal basins in Mauritania, Senegal, Côte d'Ivoire, Cameroon, Gabon, Angola, Somalia and Mozambique (Zektser and Everett, 2004).

Alluvial aquifers of the Neogene and Quaternary age occur in the sedimentary and coastal basins and are commonly found in Tunisia and the Atlantic coast of Morocco. The alluvial aquifer group contains the Congo basin and the Nile River alluvial aquifer (Egypt and Sudan). The latter is often clayey with the thickness of the alluvial aquifer increasing northwards from a few meters at Cairo to *ca.* 1000 m at the Mediterranean Coast. The northern coastal section of the Egyptian Delta Aquifer is less productive and contains brackish or saline water (RIGW/IWACO, 1988, Hefny et al., 1991).

Sedimentary basins are an important resource for water in the narrow coastal zones which includes the coastal basins of Gabon, Congo, Zaire, Angola and Mozambique. The Gabon coastal basin covers an area of about 55 000 km^2 and is composed of a multi-layered aquifer system (Figure 1-1). The aquifers consist of continental sediments, evaporites interbedded with carbonate rocks and marine sediments. Salinity, expressed as TDS (total dissolved solids) varies from less than 500 mg/l to 5 g/l, the higher salt load increases with depth and usually occurs at depths greater than 400 m.

Parts of Democratic Republic of Congo, Congo and Angola are underlain by the Congo basin; since large amounts of surface water are available very little attention has been paid to the groundwater aquifer system. In the river systems of this basin a large number of dams have been constructed in the recent past. However, recently the availability of potable water has shifted the focus back to investigating groundwater sources as a bulk supply (Supply, 2008).

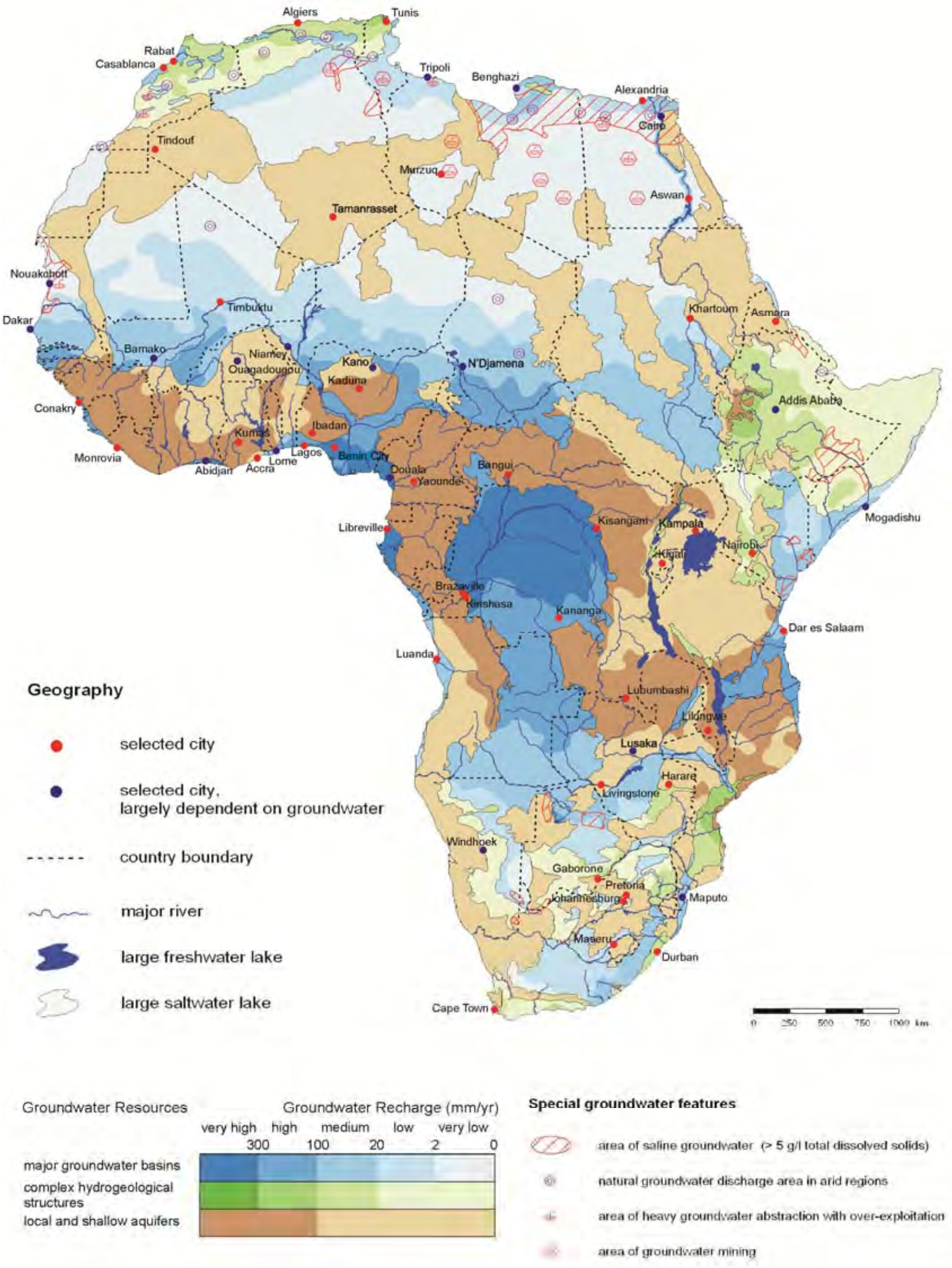

Figure 1-1 Groundwater regional map of Africa. Blue, green and brown represent respectively major, complex and shallow groundwater basins. Darker shading indicates higher recharge rates (WHYMAP, 2008, Steyl and Dennis, 2010).

1.2 The South African Perspective

South Africa can be classified as a developing country with an increasing water demand due to industrial and population growth (Robins et al., 2006). The country has a well-developed dam system and has expanded water supply systems to neighbouring countries such as Lesotho. Due to the ever expanding need for water and the arid climate of South Africa alternative water supply systems are required. One alternative is to focus on groundwater as a source of potable water for rural areas or small towns (DWAF, 2004). This is an attractive option, since reticulation networks are costly and would require construction over vast distances. In contrast local aquifers can be used and managed on a sustainable basis to provide potable water to a community (DWAF, 2004).

South Africa's groundwater resources are generally underutilised and less effectively managed than its surface waters (Robins et al., 2007). Characteristics that make groundwater attractive as a water resource for local communities is that the water stored in the aquifer can be abstracted as required. To a certain extent the groundwater, if managed correctly, can sustain a community during extended drought periods which is critical from a South African perspective (Robins et al., 2006). Due to the relatively low cost of drilling and pump installation, a water resource can be allocated close to the supply point. In general groundwater is of an acceptable quantity and quality to be used without further treatment; this significantly reduces costs and increases the range of usage for local communities. One of the key issues in South Africa is that reliable water resources are not evenly distributed through the country side. Two primary factors affect the distribution of groundwater resources in South Africa is the complex geology (Figure 1-2 and Figure 1-3) and variable local climate. Furthermore South Africa shares some of its aquifers systems with all of the neighbouring countries, i.e. Botswana, Lesotho, Namibia, Mozambique and Zimbabwe (Cobbing et al., 2008). This increases the responsibility on the South African side to effectively manage these respective trans-boundary aquifers.

1.2.1 Geology

As noted previously the geohydrology of South Africa tends to be complex; dominated by fractured aquifers (including karst limestone aquifers). The potential of many South African aquifers has not been fully developed and this is partly due to the lack of groundwater information (Robins et al., 2006).

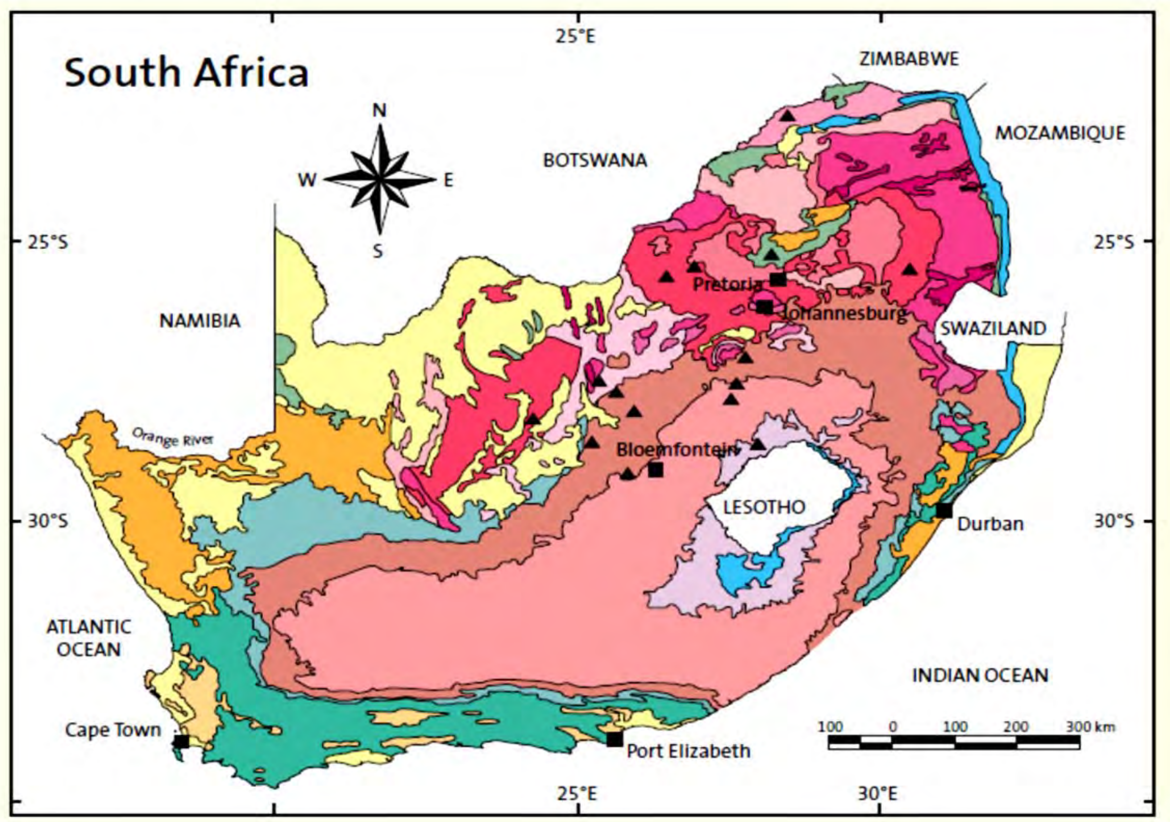

Figure 1-2 Generalised geological map of South Africa (legend key see Figure 1-3) (CGS, 2000, Johnson et al., 2006).

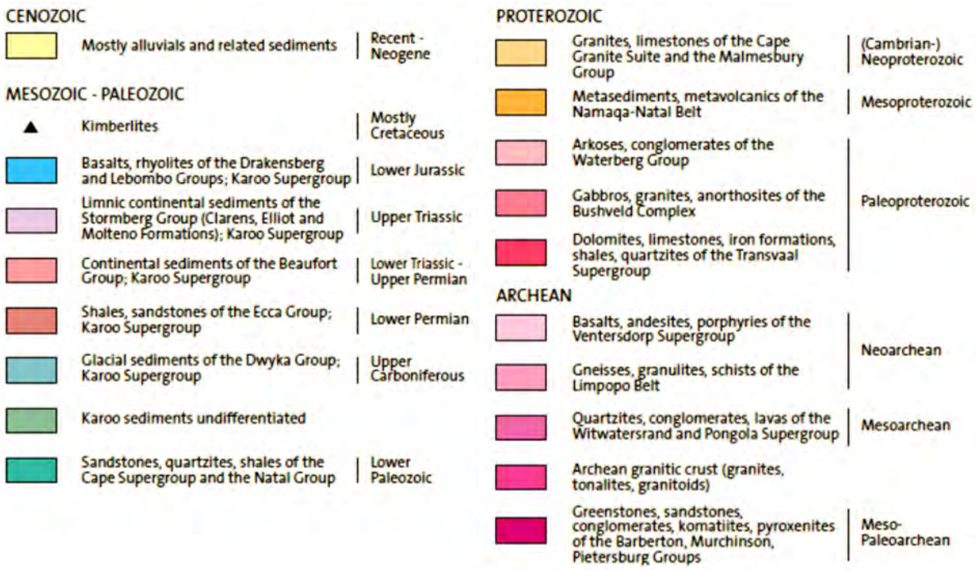

Figure 1-3 Legend key for Figure 1-2, showing the main geological features and groups (CGS, 2000, Johnson et al., 2006).

The investigation of aquifer systems is linked to the local geology of an area, and this regard South Africa consists mostly (> 80%) of the Karoo Supergroup (Figure 1-2). The Karoo Supergroup is dominated by hard, fractured rocks which might have dual porosity properties to a lesser or greater extent (Humphreys, 2000, Van der Linde and Van Biljon, 2000). The geometries of these aquifer systems is also complicated since intrusive structures and faulting commonly occur in these areas. In particular if drilling targets are required for water supply, it is advised to target the margins of dolerite intrusions. This is largely motivated by the baking affect that the dolerite intrusion would have had on the country rock. As the intrusive structure cooled, fracturing would have occurred and subsequently caused a preferred pathway to form along the dyke structure.

In respect to the crystalline basement (Granitic Plutons, Greenstone and Late Proterozoic sediments) specialized drilling techniques are typically required and the geohydrology of the aquifers are dominated by fractured rock systems.

Only four significant unconsolidated aquifers are present in South Africa, i.e. Kalahari, Atlantis, Langebaan and the Zululand/Mozambique aquifer underlying the St. Lucia world heritage site (Steyl and Dennis, 2010). The Cape Flats aquifer underlies *ca*. 630 km^2 and is an important source of water for Cape Town; however pollution has threatened its full development. The Mozambique/Zululand aquifer has a surface area of 7 000 km^2 and extends for 1 250 km along the coastline. It is in this area that the St. Lucia Wetland Park is situated; the existence of the nature reserve effectively protects this coastal region from exploitation. Due to forestry activities in the region, a reduction in water levels has been observed in the northern part of the Zululand aquifer. This has resulted in seawater intrusion along this area; however remedial action is currently underway to rectify this situation (DNWRP 2004).

1.2.2 Climatic factors

The influence of climatic conditions on recharge and water usage have been investigated recently in WRC reports (Dondo et al., 2010, Xu et al., 2007, Bredenkamp et al., 2007, Meyer, 2005), these reports all indicate that the process of groundwater recharge in South Africa is a highly complex problem. To illustrate this point a few maps were compiled showing different processes. A recent thesis by Dr. Van Wyk (Van Wyk, 2010) on groundwater recharge highlighted the effect of rainfall volume versus intensity when it comes to the volume of recharge. In Figure 1-4 the mean annual precipitation and rainfall concentration percentages is shown. The average annual precipitation in South Africa is ca. 500 mm, which is significantly less than that observed for the world with a value of 860 mm. Firstly, the mean annual precipitation in South Africa increases from the arid west to the tropical east. High precipitation values are observed along the eastern section extending into the north of South Africa. Comparing this mean annual rainfall pattern with the mean annual rainfall concentration it can be deduced that the northern section of South Africa has significantly more intense rainfall events (thunder storms). In contrast the southern half of South Africa and the eastern coastal section has a moderate continuous rainfall pattern.

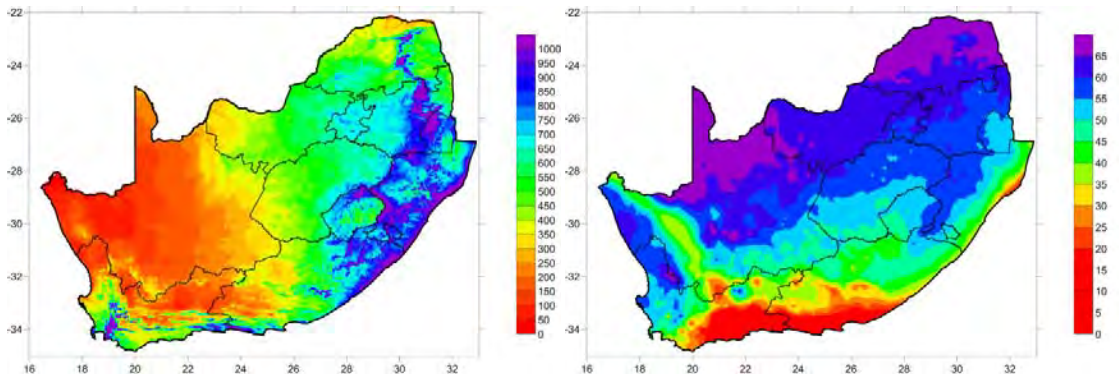

Figure 1-4 Mean annual precipitation (left hand, mm) and mean annual rainfall concentration as a percentage of total (right hand, %) of South Africa (Schulze et al., 1997).

It is expected that if episodic recharge does occur, it will generally be associated with high intensity rainfall events as noted previously (Van Wyk, 2010). Interestingly, the Karoo type aquifers are associated with high intensity rainfall areas which should increase recharge in these areas, however due to the low rainfall volumes it is expected that recharge would be in the order of a few millimetres per annum.

A second environmental factor that influences aquifer systems is the mean annual temperature and the associated mean annual evaporation potential (Figure 1-5). The average South African temperature range during the summer months is between 24-20^{o}C while during the winter time it ranges from 10-17^{o}C. The average temperature only decreases along the mountainous central belt and associated highlands, as indicated in Figure 1-5. This trend is not reflected in the evaporation potential with relatively low evaporation potentials along the eastern and coastal area (< 2 m) while inland and to the west the evaporation potentials is higher (> 2 m). This observation can be related to both the mean annual rainfall volume and mean annual temperature. It is expected that areas with high annual temperatures and evaporation potentials should be more reliant on groundwater sources, since aquifers are less vulnerable to these factors and thus represents a more constant source of water.

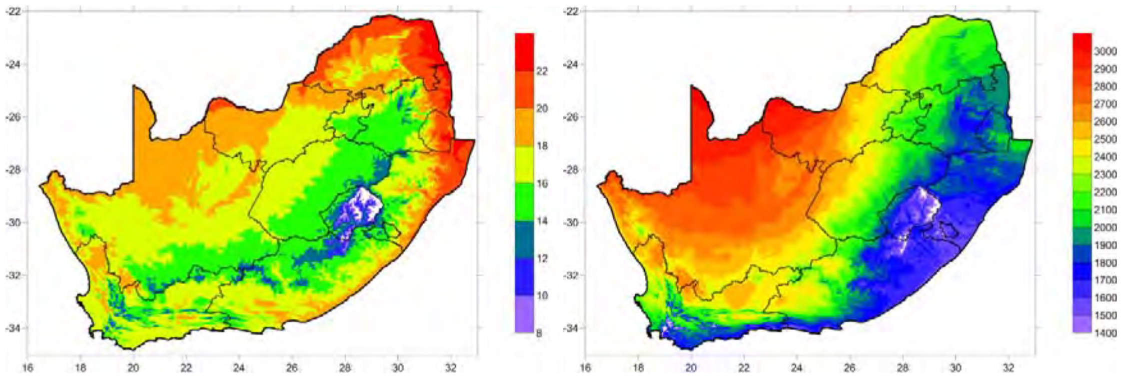

Figure 1-5 Mean annual temperature (left hand, ^{o}C) and mean annual evaporation potential (right hand, mm) of South Africa (Schulze et al., 1997).

Finally, the general rainfall patterns of South Africa are shown in Figure 1-6, illustrating the season progression and the major rainfall seasons. The seasonality of rainfall will also change the effective recharge observed in an aquifer, since lower evaporation potentials are dominant during the winter season allowing for pools to form which can recharge over a longer timeframe.

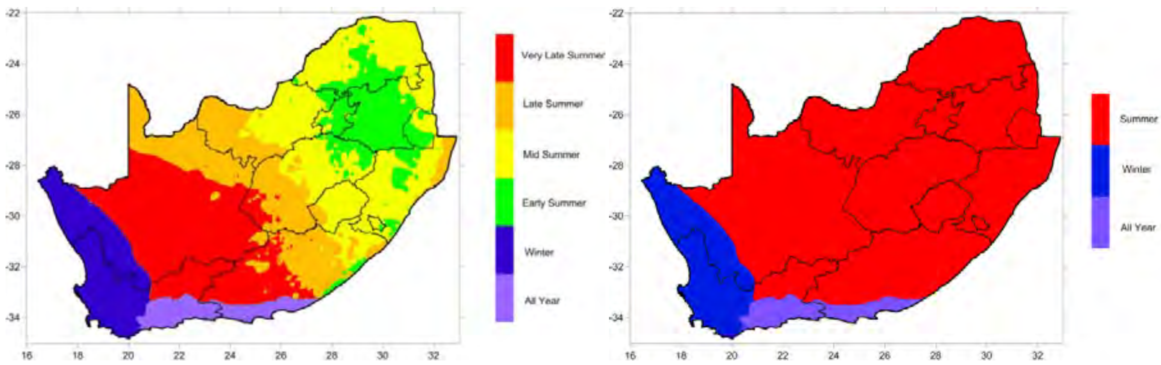

Figure 1-6 Generalised areas as a function of rain seasons in South Africa (Schulze et al., 1997).

In the next paragraph regional hydrogeological South African maps will be presented. The main focus is to supply a group of maps that will illustrate water resource directed management options and the impact it can have on South Africa’s water resources.

1.2.3 Hydrogeological factors

The majority of South African aquifers are found in hard rock geological formations, as noted previously the geological structure would have an impact on the transport properties of the aquifer. In regard to bulk flow parameters various estimations of transmissivity and storativity values have been attempted, however no satisfactory results have been obtained that could be applied in groundwater management (Murray et al., 2011). One major headache is the presence of dolerite dykes and sills, which infiltrated the Karoo Supergroup (Figure 1-2) during the early Jurassic period. The irregular distribution of high and low permeability zones complicates the effective estimation of regional groundwater flow parameters (Figure 1-7).

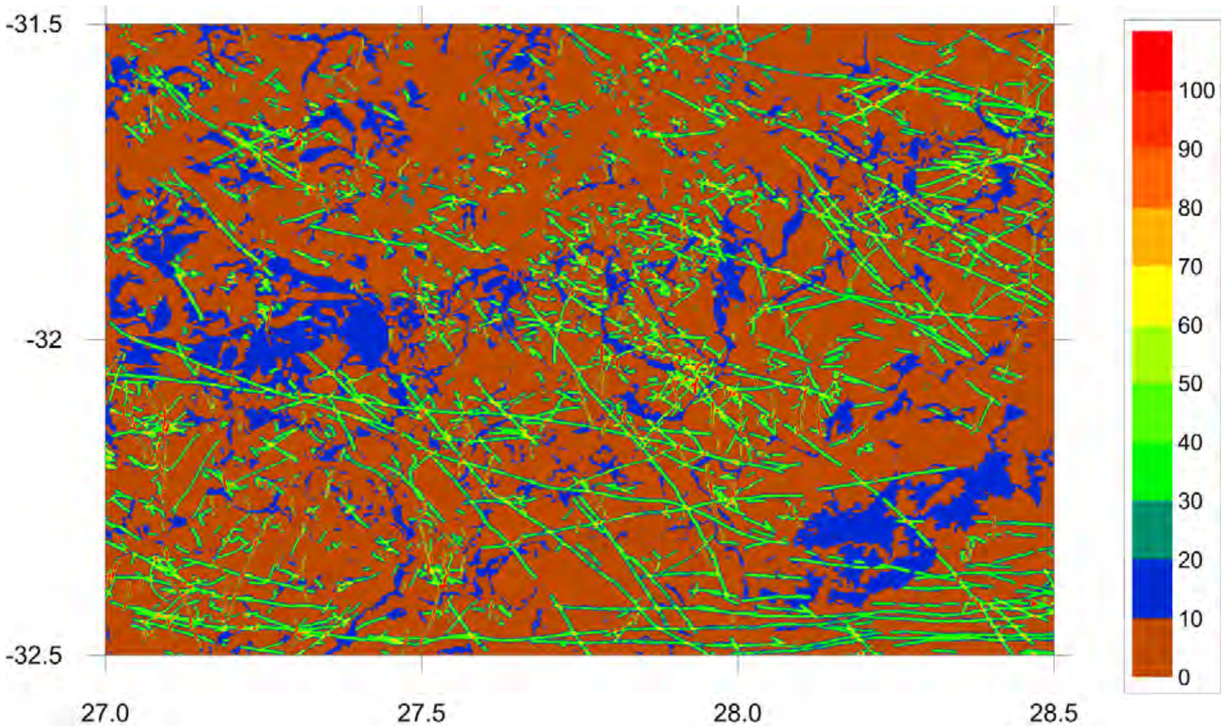

Figure 1-7 A regional map showing a subsection of the Karoo Supergroup with blue patterns indicating sills while green to red represents dykes in the area.

The estimation of transmissivity values have been done using various methods, i.e. conversion of borehole yield and pump test data (Figure 1-8). It is clear from the figures that different transmissivity values were obtained, this stems from the underlying observations used in determining the values. The first set of transmissivity values were calculated from borehole yields which would indicate a long-term average scenario. The maximum transmissivity that could be observed for this map was 44 m^2/d. In contrast if pump test data is used, maximum transmissivity values were observed in the range of 400-500 m^2/d. The discrepancy between the maximum values indicates that there might be a biasing factor in one of the methods or the methodology of obtaining the data itself.

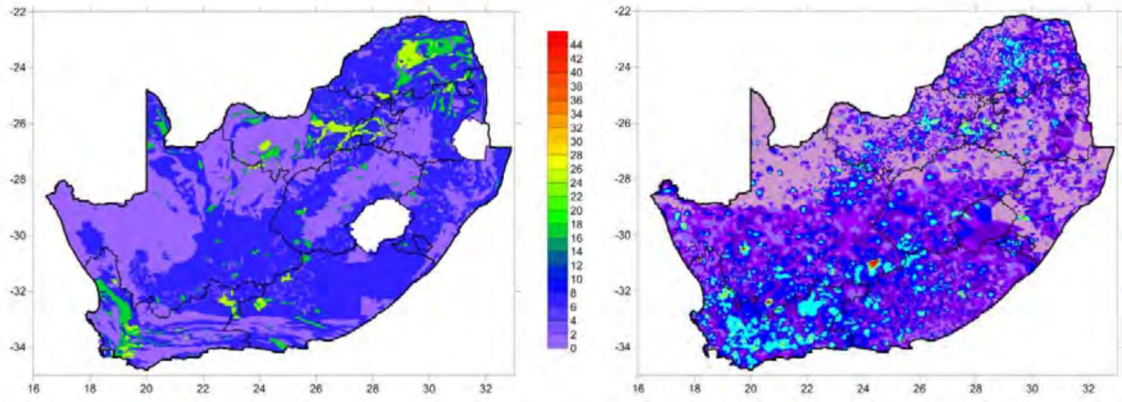

Figure 1-8 Estimated transmissivity values (m^2/d) for South Africa. Left-hand side constructed from reported borehole yield data points and right-hand side from the Groundwater Resource Directed Management (GRDM) Database.

Turning to storativity values for South Africa very little reported data exists which can be used for calculations. In general storativity values for the Karoo Supergroup is estimated to be in the range of 10^{-3}-10^{-5}. Considering the map (Figure 1-9) this assumption for South Africa holds, since 95% of South Africa's aquifers is located in the Karoo Supergroup formations.

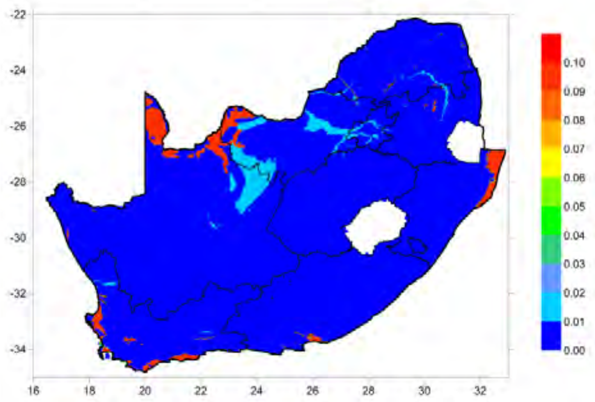

Figure 1-9 Estimated storativity values for South Africa (GRDM).

1.3 Research Statement

Groundwater resources in the past have played a vital role in the development and sustainability of rural South Africa communities. Due to reticulation costs, larger cities are investigating groundwater assets as an alternative water supply resource to assist in periods of drought or low water supply periods. One issue that hampers the development of a coherent strategy for developing this resource is the prospecting for adequate groundwater sources that is sustainable over the longer term.

Key factors that would assist in this endeavour are bulk flow parameters and the issue of scale. In general bulk flow parameters are heavily dependent on the geology of South Africa and the competency of the well test analyst in determining these parameters. A further complication is the synthesis of the obtained data into a management strategy as well as extrapolating the observed transmissivity values from a local to a regional scale.

In this work various case studies will be presented that range from small scale field investigations to large regional studies. The estimation of a representative transmissivity value for each of the study sites will be developed and eventually linked to a regional estimation value.

Thus the focal point of this study is to evaluate methodologies for use in converting bulk flow properties from a local perspective into parameters that could be applied on a regional level. In order to accomplish this objective, an investigation into estimation methods for bulk flow parameters was required.

A critical issue of this study was to determine methods that yielded representative transmissivity values in a heterogeneous aquifer setting. Furthermore, the conditions under which certain models could be applied that would add credence to the values obtained. A review of geostatistical methods will be presented and subsequently the calculation of an average value for a system.

1.4 Research Objectives

In order to obtain a balanced evaluation of methods to determine regional transmissivity values the following research objectives were set:

1. A literature review of current methods used in the groundwater industry to determine transmissivity values.
2. The construction of a conceptual model system to evaluate usage of different mean value calculation methods.
3. Investigate case studies to determine the effects of random sampling as compared to biased or directed sampling methods.
4. Discuss and evaluate different possible scenarios that could be encountered in the field as well as methods to estimate regional transmissivity values.
5. Present a possible methodology to indicate the way forward and requirements for geohydrological data in databases.

The following chapters will present each of the above objectives in a systematic manner. In every section an attempt will be made to comment on the applicability of each of the mean value calculation methods to estimate regional transmissivity or hydraulic conductivity values. The following chapter will present a short summary on the theory behind bulk flow parameters and the use of statistical methods in groundwater.

Chapter 2 Theory

2.1 Introduction

The basis of geohydrological investigations is that the core assumptions made by Darcy and subsequent researchers holds (de Marsily, 1986). Fluid flow through porous media is usually described with *Darcy's law*: The flux or the Darcy's velocity of the fluid v is proportional in magnitude to and coincidental with the negative gradient of the potential field or hydraulic head.

$$v = -k\frac{\rho g}{\mu}\nabla h$$

Equation 2-1

(ρ: density of the fluid, g: gravitational acceleration, ρg: specific weight of the fluid, k: permeability, μ: viscosity of fluid, h: fluid head)

Under the assumption that a fluid flows through a given cross section of a porous media without any obstruction by the sand grains, the *hydraulic conductivity* can be given as

$$K = k\,\frac{\rho g}{\mu}$$

Equation 2-2

Fluid properties in groundwater systems are usually independent of the head and may be assumed constant. Permeability is considered when discussing the transmissive capacity of the rock rather than hydraulic conductivity.

Transmissivity T, a concept related to permeability used in aquifer testing, is the vertically averaged product of the hydraulic conductivity and the saturated aquifer thickness, b, in a two-dimensional reservoir:

$$T = \int_{o}^{b} K\,(z)dz$$

Equation 2-3

z: elevation of a point above the datum

Storativity *S* is a measure of the ability of a reservoir to release or absorb fluid per unit surface area under a unit change in head In an aquifer testing storativity is considered as the vertically averaged product of the sum of the fluid compressibility, *c* and the pore compressibility, c_f , the porosity, φ, and aquifer thickness, *b*.

$$S = \int_{o}^{b} \phi(z)(c + c_f)dz$$

Equation 2-4

In general the hydraulic conductivity does not only depend on the fluid but also on the viscosity. It should be noted that the viscosity of water varies considerably with temperature. It is in this regard that one should be careful when dealing with water table aquifer systems in which temperatures and relative atmospheric pressures can change over a short time frame. Secondly, as noted in the introduction most of South African aquifers are characterised by fractured rock systems. There are two possible general methods for dealing with conducting fractured medium within an aquifer.

The first method relies on the idea of a continuous medium that is characterised by several conducting fractures. Thus each subset of fractures can be defined by a directional conductivity (hydraulic conductivity tensor) and successively the intensity and directions of flow can be combined to calculate the principal axis of anisotropy. The method of continuous medium is valid for a certain scale of observation since an average description of flow velocity and direction is used in the estimate. The estimation of the subset of fracture properties can be conducted using a statistical measure of an aperture, distance and dip or by in situ methods where the hydraulic conductivity of each fracture set is measured. Both estimation methods suffer from the same disadvantages in that it is assumed that the fracture network is infinite and are homogenous in the principal properties under investigation.

The second method of modelling flow in a fractured medium is done under the assumption that a discontinuous medium is present. Thus the elementary fractures or subset of fractures can be represented by an equivalent fracture of the same family (closed subgroup). The model is composed of nodal points where the hydraulic head is calculated and between the inter-nodal points a plane can be defined to compute the velocities. The most significant drawback of this method is that the property of each of the fractures should be known in space, this is however not possible on a regional scale.

2.1.1 Flow and Storage in Fractures and Porous Medium

In general when applying the Darcian method of flow in a porous medium, it is assumed that the water flow is steady and invariant with time. However if one considers transient flow, the fundamental properties of fractures appear such that it is clear that a dual or double porosity medium is present. In this instance both the hydraulic conductivity of the porous medium (K_m) and the hydraulic conductivity of the fracture (K_f) should be considered. In the steady state the dual porosity medium can be described by using an equivalent hydraulic conductivity and storativity values. However, under transient conditions fluid flow will be much greater in the fractures than in the porous medium ($K_f >> K_m$). In contrast the storativity of a fracture is much less than that for the porous medium that surrounds it ($S_m >> S_f$).

2.1.1.1 Validity Range of Darcy's Law

Darcy's Law is defined on a macroscopic scale which results in averaging effects occurring. However, at extreme values Darcy's Law is invalidated. The two extreme hydraulic gradients are represented at both low and high extremities.

Low gradients occur typically in fine cohesive material such as compacted clays (montmorillonite). If the gradient is too low the permeability in the medium is effectively zero (Figure 2-1), this can be represented by a threshold value (i_0). However, a second interval exists in which the relationship between the hydraulic gradient and the permeability is non-linear. The upper value of this non-linear boundary can be represented by a defined hydraulic gradient (i_1), see Figure 2-1. Hydraulic gradient values greater than i_1 corresponds proportionally to Darcy's Law ($i > i_1$).

In the instance where hydraulic gradients are too high, the proportionality between the gradient and the filtration velocity is negated. In order to make this transitional hydraulic gradient into a definable quantity, the Reynolds number in porous medium is used ($R_e = U\rho\sqrt{k}/\mu$) where U is the filtration velocity, k intrinsic permeability and μ viscosity. In practice Darcy's law is valid in a porous medium is in the range of 1-10, i.e. purely laminar flow.

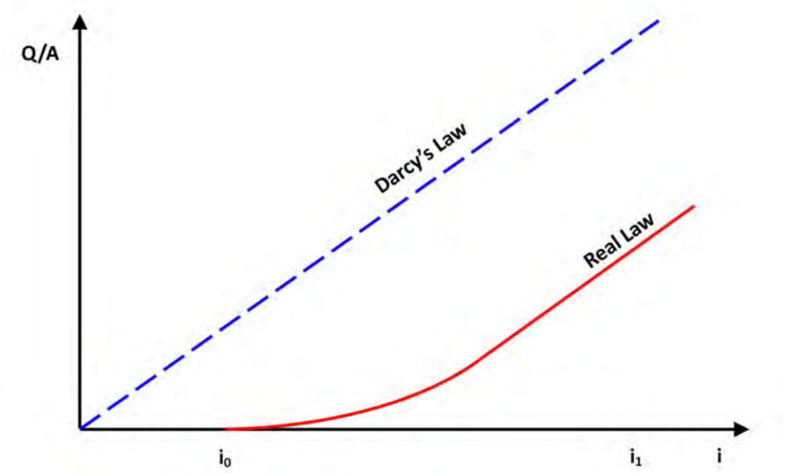

Figure 2-1 Effects on Darcy's Law at small gradients.

2.2 Difficulties Associated with Scientific Models in Practice

Although there are exceptions, the mathematical models associated with the conceptual models of physical systems are based on one or more partial differential equations. Take for example the one derived from Darcy's law in Equation 1.5, often used in groundwater flow investigations (Bear, 1972, Bear, 1979, Botha, 1996)

$$\boldsymbol{S_0}(\boldsymbol{x},\boldsymbol{t})\boldsymbol{D_t}\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{t}) = \boldsymbol{\nabla}\cdot[\boldsymbol{K}(\boldsymbol{x},\boldsymbol{t})\boldsymbol{\nabla}\boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{t})] + \boldsymbol{f}(\boldsymbol{x},\boldsymbol{t})$$

Equation 2-5

This equation contains in the terminology of Equation 2-5 two relational parameters S_0**(x,**t**)** (the storativity of the porous medium) and **K(x,t)**, the forcing function **f(x,**t**)** representing the strength of sources (+) and sinks (–) that may be present in the flow field and the observable piezometric head

φ(x,t**)**. Since Equation 2-5 is a partial differential equation one could use it to predict the behaviour of **φ(x,**t**)** at the positions **x** and time t, as is indeed done in practice. However, as is known from the theory of partial differential equations, this can only be done if the following conditions are met beforehand.

Step 1. The domain Ω spanned by **x** and its boundary $\partial\Omega$ must be known.

Step 2. The relational parameters, such as S_0**(x,**t**)** and **K(x,**t**)** in Equation 2-5, must be known at all points **x** in Ω and any time t for which the differential equation has to be solved.

Step 3. Any forcing functions that appear in the differential equation must be known.

Step 4. Boundary conditions, appropriate values of the dependent variable, e.g. **φ(x,**t**)** in Equation 2-5, must be known at all points along $\partial\Omega$ and again at any time t for which differential equation has to be solved.

Step 5. Initial conditions, appropriate values of dependent variable must be known across Ω for a suitable time, $t_0 = 0$ say.

These five conditions will henceforth be referred to collectively as constraining parameters. (Note that although these constraining parameters relate primarily to mathematical models based on differential equations, all mathematical models commonly used in science and technology today contain constraining parameters in one form or another.)

A method frequently used (especially historically) to satisfy the constraining parameters in a scientific model of a physical system, is to simplify the constraining parameters, geometrically or otherwise, in such a way that the underlying mathematical model reduces to an analytical model. The method is particular useful when a physical system either display a simple behaviour, or in a laboratory studies where the constraining parameters can be adjusted to satisfy a suitable analytic model.

There is little doubt that analytical models play a significant role in the development of the modern technological age. In fact, the question may be asked whether scientists do not sometimes unnecessarily disregard analytical models. Nevertheless, there exist physical systems, which cannot be modelled adequately with analytical models. This is especially the situation in what may be called the environmental and biological sciences. Take for example the scientific model of an aquifer with Equation 2-5 as a mathematical model. Although the application of remote sensing and similar techniques

(Hoffmann, 2005, Meijerink, 2007), may change the situation in the not too distant future, there do not exist methods today to determine the constraining parameters for this model. Results derived from this and similar mathematical models therefore will always be in doubt, unless values of the parameters can be derived in one or another way. This would not be too much of a problem, were it not for the fact that such mathematical models often represent the only viable approach to study the evolution of physical systems crucial to the environment on earth.

Three approaches are commonly used to solve the problem of unknown constraining parameters in scientific models: (a) observational-analytic modelling, (b) stochastically continuum modelling, and (c) inverse modelling, described separately below using with the mathematical model of groundwater flow in Equation 2-5 as basis. However, this should not be interpreted that the approaches only apply to models of groundwater flow or that the approaches can only be applied separately. On the contrary, the approaches can be applied separately or in combination to any scientific model subject to the problem of unknown constraining parameters.

2.2.1.1 Observational-analytic Modelling

Observational-analytic techniques refer to the use of suitable analytic models of the system under investigation to interpret observations on the system. The technique is fairly widely used in the interpretation of observations with existing theories and to derive values of the relational parameters in the mathematical model of a system. Witness, for example, the use of the Theis equation to derive values for the relational parameters S_0 and $\mathbf{K}$ in Equation 1.6 of a uniform infinite aquifer from hydraulic or pump tests (Kruseman and De Ridder, 1991) and the interpretation of geophysical surveys (Kirsch, 2006).

A glance at (Kruseman and De Ridder, 1991) and related books creates the impression that the observational-analytic technique can yield accurate values of the relational parameters in a large number of mathematical groundwater flow models. The conclusion thus quickly arose that the technique can be applied with confidence to the modelling of flow in the subsurface of the earth—a view supported by the following arguments (Black, 1993, Raghavan, 2004).

(a) The relational parameters derived from a hydraulic test tend to constant values if the test is run long enough.
(b) Engineers and modellers often twist the pressure head distribution derived from a model to observed values through the use of concepts such as skin factor and well loss, even though these concepts when initially introduced were intended to account for specific physical phenomena.
(c) There is a need in both the oil industry and groundwater investigations for information to be evaluated and acted upon in real time particularly because of cost, safety and necessity considerations.
(d) The earliest investigations of groundwater flow phenomena centred on aquifers in relative uniform sedimentary deposits whose internal geometry very much resembles the simple and well known geometry of a porous medium and therefore simply neglected.

Modellers in the oil industry and geohydrology consequently became very complacent with the use of observational-analytic technique. Nevertheless, there are a number of disadvantages associated with this approach, the first and most important of which is the neglect of the geometry of an aquifer or oil reservoir (Black, 1993, Botha et al., 1998). Models based on observational-analytic technique are consequently often restricted to sets of fixed relational parameters and horizontal flow, thereby neglecting properties such as anisotropy, deformation, preferential flow paths and flow in the vertical direction, which may lead to a significant underestimation of the extent—hence the economic value of the resource (Raghavan, 2004). The technique consequently adds little to a better understanding of the aquifer or reservoir and neglects the major advances that have been made in geological modelling over the past 20 years. Moreover, the technique essentially ignores any ‘messages’ in hydraulic tests, thus reducing the role these tests could and should play in the development of mathematical models for subsurface flow.

2.2.1.2 Stochastical Continuum Modelling

Observations show that natural rocks often are highly fractured and heterogeneous and hence exhibit pronounced spatial variability (Neuman, 2005). The result is that relational parameters of mathematical models derived with observational-analytic modelling for such rocks exhibit a similar variability. This caused the introduction of what is known today as the stochastic continuum concept over the years (Neuman, 2005, Raghavan, 2004). The basis of this assumption is to assume that a constraining parameter can be represented as a random field of given statistics that is spatially stationary. Although the concept could in principle be applied to any of the constraining parameters in a mathematical model, there is a tendency (at least in the oil industry and geohydrology) to restrict it to the permeabilities of boreholes. One reason for this is that the assumption of statistical stationarity implies, geologically speaking that the parameter can be described statistically with a distribution that do not depend on the position where the parameter is measured. One could therefore use a single distribution to generate suitable permeabilities for an aquifer, assuming that there exists at least one observed value.

There are at least three difficulties associated with the previous approach. The first is that it is not clear how one can extract a meaningful geological description of the medium borehole from a random field of permeabilities. The second is the question of scale dependence of the permeabilities (Hunt, 2006, Raghavan, 2004). For, as discussed by Hunt (Hunt, 2006) and Raghavan (Raghavan, 2004), statistical homogeneity implies that permeabilities must be scale-dependent, i.e. depend on the volume of the aquifer or oil reservoir used in the determination of permeability values. However, as pointed out by both Hunt (Hunt, 2006) and Raghavan (Raghavan, 2004) this dependence is more of an artefact related to the geometry of the aquifer or oil reservoir that can be removed by using an appropriate geometric model of the aquifer or oil reservoir. This means that there does not exist a so-called effective permeability or other effective parameters for that matter, as follows from the assumption of stationarity. However, this is not the place to discuss this controversy in detail. The discussion therefore concludes with the remark by (Raghavan, 2004), " … the subject is now a standard part of the vocabulary of reservoir modelling (Dagan, 2002), although it is not difficult to find people who will eschew this subject assiduously."

2.2.1.3 Inverse Modelling

The main advantage of physical theories is that they allow one to predict the future behaviour of a physical system, given a complete description of the physical system, especially the relational parameters—a procedure variously called the modelization problem, simulation problem, or the forward problem (Tarantola, 2005). However, the situation frequently arises in many branches of science and engineering that one have has some information on a physical phenomenon or physical system and want to determine values for the relational parameters in the mathematical model associated with the theory or scientific model. As one could expect this operation is commonly called the inverse problem.

A major difference between the forward problem and the inverse problem, from the mathematical point of view, is that the forward problem always has a unique solution (in deterministic physics), while the inverse problem has multiple solutions (in fact, an infinite number)—the so-called equifinality phenomenon (Beven, 2006). This means that one has to use special methods in handling the inverse problem. One approach to achieve this is to observe that the predicted values are generally not identical to the observed values, even in the case of the forward problem, for two reasons: measurement uncertainties and modelization imperfections. These two very different sources of error generally produce uncertainties with the same order of magnitude, because as soon as new experimental methods are capable of decreasing the experimental uncertainty, new theories and new models arise that allow one to account for the observations more accurately. For this reason, it is generally not possible to set inverse problems properly without a careful analysis of modelization uncertainties (Tarantola, 2005).

The way to describe experimental uncertainties is well understood and described in most textbooks on probability theory and statistics (Tarantola, 2005, Dekking et al., 2005). However, the proper way to put together measurements and physical predictions—each with its own uncertainties—is a matter in progress (Le and Zidek, 2006, Tarantola, 2005) and will not be discussed further here. A far more interesting aspect for the practical application of inverse problem is how to apply the inverse problem.

When solving inverse problems, scientists are often faced with two very different difficulties. The first is to find at least one model of the system that is consistent with the observations. The second difficulty arises in problems where finding at least one solution is doable, but how can one quantify the non-uniqueness of the result? Two different philosophies are in use today to try and solve this problem. The first carefully avoids using any a priori information on the model parameters that could 'bias' the inferences to be drawn from the data. The second philosophy, which is clearly Bayesian, asks the basic question: how does newly acquired data modify previous information? In other words, when starting with some a priori state of information on constraining parameters, what is the a posteriori state of information at which one arrive after 'assimilating' new data? Observations like these, lead (Tarantola, 2006) to introduce his concept of a Popperian-Bayesian approach that is use all available a priori information to sequentially create models of the system, potentially an infinite number of them. For each model, solve the forward modelling problem, compare the predictions to actual observations and use some criterion to decide if the fit is acceptable or unacceptable, given the uncertainties in the observations and the physical theory or model being used. The unacceptable models are the falsified models, and must be dropped. The rest represents the solution of the inverse problem that can be investigated further through the GLUE methodology of (Beven, 2006) for example and data assimilation techniques.

In the following sections the analysis techniques commonly applied in geohydrology will be presented and discussed. This is done in order to assist in the calculation of average transmissivity or hydraulic conductivity values for an area. Initially, the focus will be on geostatistical methods followed by unconfined aquifers, pump test interpretation and finally the effect of heterogeneous media on results obtained.

2.3 Geostatistical Methods

The term geostatistics is generally applied to a special branch of applied statistics. It was devised to treat problems that arise when conventional statistical theory is used to estimate changes in ore grade within a mine. Fundamental to geostatistics is the concept of regionalised variables, which has properties intermediate between a truly random variable and one that is completely deterministic. Regionalised variables are functions that describe natural phenomena that have geographic distributions, i.e. ground surface elevation, changes of grade within an ore body or water levels within an area. In contrast to random variables, regionalised variables are point wise continuous but cannot be determined or described by any definable deterministic function (Walder, 2008). Accordingly, a more formal definition can be given that describes the interchange between these parameters.

Definition 1: $\{Z(x)\}$, $x \in R^n$ is a random field if Z(x) is a random variable $\forall\, x \in R^n$. If $x = t \in R^n \rightarrow Z(t)$ is a random process.

Definition 2: A single realization $\{z(x)\}$, $x \in R^n$ of a random field $\{Z(x)\}$, $x \in R^n$ is regionalised variable.

Definition 3: The covariance function of the random field $\{Z(x)\}$, $x \in R^n$ is defined as

$$C(x,h) = Cov\big(Z(x), Z(h)\big) = E\{\big(Z(x) - EZ(x)\big)\big(Z(x+h) - EZ(x+h)\big)\}$$

Where E represents the mean of the corresponding random value or the product of the random differences.

The above definitions will only hold if the following two assumptions are made,

(i) First-order stationarity: $EZ(x) = EZ(x+h) = \mu = constant$. This result would specify that the mean of the random field is constant and that the mean value is the same at any point in the field.

(ii) Second-order stationarity: $C(x,h) = C(|h|)$. Indicating that the covariance between any pair of locations depends on the length of the distance vector (h), which would results in the second-order stationarity eliminating the directional effects.

Due to the inherent complexity of regionalised variables, it is generally impossible to determine all values within an area. More specifically the regionalised variables are presented as a sample of specific observations in an area, thus it is represented by a sub-set of the population. The size, shape, orientation and spatial arrangement of these observations constitute the support of the regionalised variable and if any of these observations change then an associated change in the characteristic of the regionalised variable will be observed.

In order to apply geostatistics in one, two or three dimensional space an estimate of the regionalised variable should be constructed. This is usually done by means of the semivariance (γ), which expresses the rate of change of a regionalised variable along a specific orientation. Approximating the semivariance requires an analysis technique similar to time-series analysis. Thus the semivariance is a measure of the degree of the spatial dependence between observations along a specific support (data points). If it is assumed that point measurements and equally spaced samples (γ) are gathered then the semivariance (γ_h) can be approximated as:

$$\gamma_h = \sum_{i}^{n-h} \frac{(x_i - x_{i+h})^2}{2n}$$

Equation 2-6

The x_i is a measurement of a regionalised variable, X, taken at location i and x_{i+h} at an interval of h. The number of points (n) can be measured, such that the number of comparison between points is n-h. The importance of the semivariance is its use as an accurate and precise statistic to quantify the dissimilarity of a variable between a chosen central point (X_i) and several other points (X_2, X_3, ..., X_n) with increasing distances from the central point. For each pair of points ((X_iX_2), (X_iX_3), ..., (X_iX_n)), the value of the semivariance is plotted on the y-axis versus the distance between the points h on the x-axis, to give a scatter plot called of the experimental semi-variogram. The relationship between the semivariance and distance from the central point will depend on the amount of regional dependence.

Once the experimental semivariogram has been plotted, a smoothed line of best fit called the theoretical semivariogram is fitted through the points with the restriction that it must start from a relatively low value at the central point, subsequently increase, but eventually plateau out at a constant value (Figure 2-2).

If there is some regional dependence, then the values of the variable at the central point and those nearby will be similar, thereby giving relatively small semivariances. As the distance from the central point increases, the amount of dependence reduces, so the semivariances will tend to increase but also become more scattered. At this distance (and beyond), the two points are equivalent to having been chosen at random from the population, so each of the widely scattered semivariances will estimate the population variance for samples of n = 2. Once the experimental semivariogram has been plotted, a smoothed line of best fit called the theoretical semivariogram is fitted through the points with the restriction that it must start from a relatively low value at the central point, subsequently increase, but eventually plateau out. The averaged value at the plateau gives a relatively good estimate of the population variance.

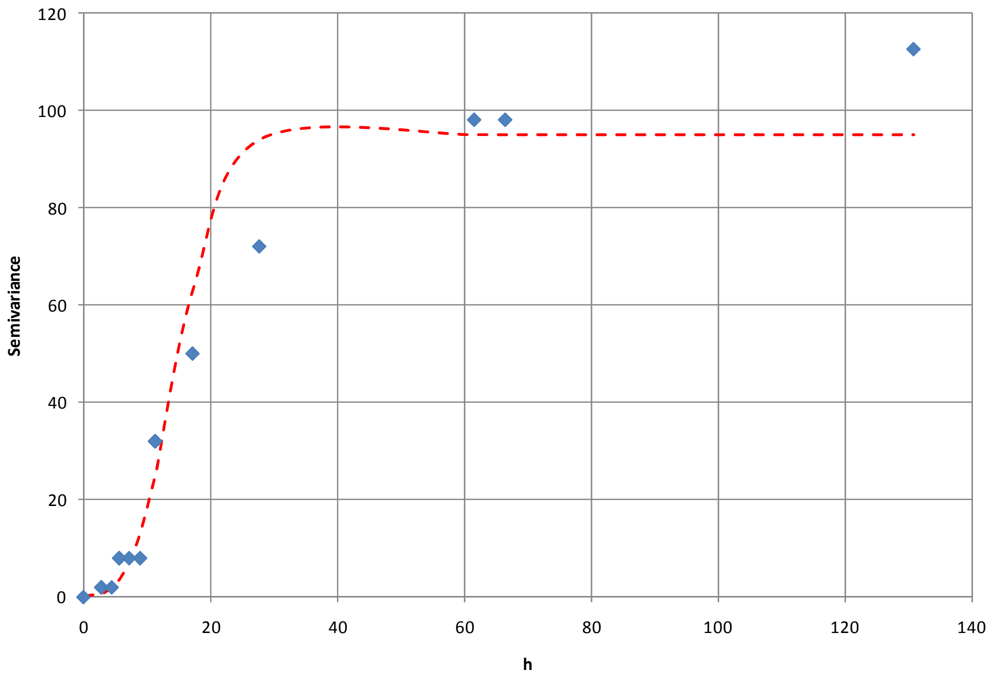

Figure 2-2 The experimental semivariogram is a scatter plot of the semivariance against distance between sampling points (blue diamonds). A variogram from the exponential family is fitted to the data, the nugget effect for parameters are excluded in this analysis.

The features of the theoretical semivariogram are shown in Figure 2-3. The semivariance at X = 0 is called the nugget or nugget effect. When the semivariance reaches its maximum height at the plateau, its value is called the sill. The outer limit of the region of influence surrounding the central point is defined as the value of X when the semivariance has reached 95% of the difference between the sill and the nugget.

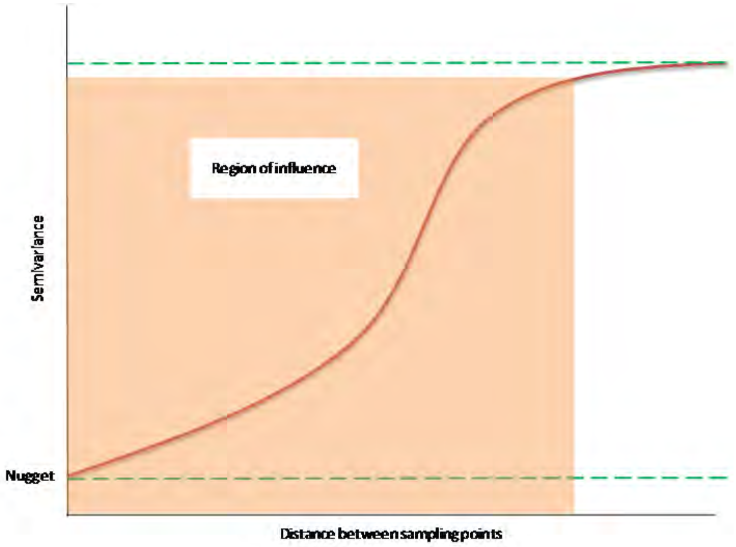

Figure 2-3 Description of theoretical semivariogram. If X = 0 and the semivariance does not equal zero then this is called the nugget. The maximum value, i.e. platue is refered to as the sill, with the region of influence representing the value of x for which the theoretical semivariance is 95% of the distance between sill and nugget.

One important application of the theoretical semivariogram is to predict the value of a variable at sites where it has not been measured. The width of the 95% confidence interval around the line of the theoretical semivariogram will depend on the amount of regional dependence. When there is no regional dependence, the line will rise rapidly and the 95% confidence interval around it will be relatively wide, because it is the smoothed average of many estimates made when n = 2. When there is regional dependence, the line will rise more slowly. Its 95% confidence interval will initially be very narrow because the regional dependence surrounding the central point will constrain the estimates of the semivariance to within a relatively small range. If a variable shows regional dependence and the point(s) at which you want to predict it lie within the regions of influence of known locations, it is possible to make quite precise estimates of its value. This is the basis of the method of interpolation called Kriging.

2.3.1 Variogram

Definition 4: Let $\{Z(x)\}$, $x \in R^n$ be a random field. A variogram of the random field is defined as

$$\gamma(x,h) = \frac{1}{2}E\{Z(x+h) - Z(x)\}^2.$$

Definition 5: If the variogram of a random field depends only on the length of the translation vector, the field is called intrinsic stationary, which means that $\gamma(x, h) = \gamma(|h|)$.

Some properties of the variogram of an intrinsic stationary field are $\gamma(0) = 0$ and $\gamma(h) \geq 0$. If the mean of a random field is known, i.e. $EZ(x) = \mu = constant$. Then the following relationships between the variogram and covariance functions holds $\gamma(h) = C(0) - C(h)$ and $C(h) = \gamma(\infty) - \gamma(h)$.

2.3.2 Parameter estimation

The variogram-based methods of estimation are widely used in classical geostatistics as a method to estimate unknown parameters. It starts with formulating a model for a particular application that involves both spatial and non-spatial exploratory analysis. Since $EZ(x) = \mu$ it can also be assumed that $\mu_i = \mu(x_i)$ and that mean value can be expressed in the general form of $\mu(x) = \beta_0 + \sum_{j=1}^{p} \beta_j d_j(x)$, where $d_j(x)$ are spatial explanatory variables. Considering it from a modelling perspective, the mean and covariance structure together define a linear Gaussian model for the data. There is however no implication that the data should follow a Gaussian distribution.

This in part describes the problem of estimating the transmissivity of hydraulic conductivity of an area. Given a domain such that only a few points are known implicitly in the domain (D), then it can be argued if a subdomain (S) is selected it can be unique (D ∉ S). This implies that a subdomain is not represented by samples in the larger domain. To counteract this effect the ergodicity hypothesis is adopted, i.e. the average of a process over time is equal to the average of that process over its statistical ensemble. This hypothesis strictly only applies for a homogeneous random stationary process, which in turn requires that the mean and covariance should not depend on time and space.

2.3.3 Analogy between hydrological and electrical flow

There is very often a strong erratic component in the transmissivity (spatially uncorrelated) values of an area, which might result in two boreholes located close to each other to have significantly different transmissivities. Due to spatial variability can one derive a representative average transmissivity value that could represent an area as a first order approximation? Using a deterministic approach, a set of uniform blocks can be placed in such a configuration that both parallel and serial flow could be simulated (Figure 2-4).

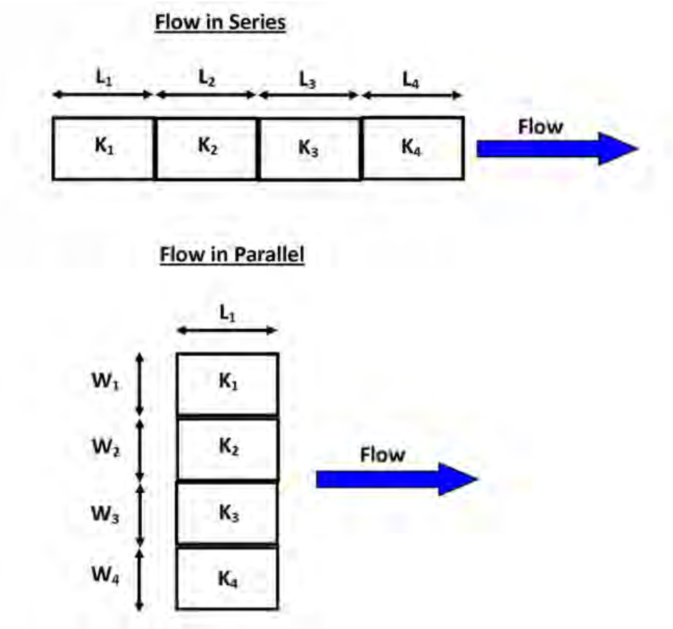

Figure 2-4 Illustration of blocks in series and parallel with flow direction indicated by blue arrow.

If blocks are placed in such a configuration that water flow is in series then the law of harmonic composition can be applied, i.e. $\sum L_i/K_{mean} = \sum L_i/K_i$. If flow is parallel to the blocks then the arithmetic composition can be used such that $K_{mean} \sum W_i = \sum W_i K_i$. Interestingly this is exactly the same as Ohm's law in electricity.

In the probabilistic approach the permeability can vary in all directions of space. Research in this field has led to the following observations (Matheron, 1967, Gelhar, 1976, Bakr et al., 1978, Gutjahr et al., 1978)

1. If the flow is uniform, the average permeability is independent of the spatial correlation of the permeability and the number of dimensions. The estimated values always range between the harmonic mean and the arithmetic mean of the local permeabilities.
2. If the probability distribution function of the permeability is lognormal and flow is two dimensional and uniform, then the average local permeability is equal to the geometric mean.
3. If flow is non-uniform and constant in time, there is no law of composition.

It is from point three that most of the pumping test analysis arrives. In general analytical solutions are derived for confined systems. However in the next section a closer inspection will be done for anisotropic unconfined aquifers.

2.3.4 Unconfined Aquifers

During a pumping test or water level fluctuation in an aquifer system the relative change in aquifer thickness can be as little as 1% or greater than 80%. If water levels drop significantly, a loss in transmissivity is observed since transmissivity is directly related to the product of the thickness (D) of the aquifer and the hydraulic conductivity (T = KD). Since most of the well field analysis methods are based on calculating the transmissivity of the aquifer, artefacts can presumably appear in analysed transmissivity values.

Neuman (1972, 1973b, 1974, 1975a,b) has investigated the phenomena of a fully or partially penetrating well pumping in an anisotropic unconfined aquifer. Furthermore, the delayed drainage of the unsaturated zone by gravity was included in the analysis. In the evaluation the anisotropy is defined is the interaction of the horizontal versus the vertical hydraulic conductivity. During the study it was assumed that the free surface or water table remains at a level (z = e) and that the boundary conditions at this surface is $K_z \frac{\partial s}{\partial z} = -\omega_d \frac{\partial s}{\partial t}, z = e, \forall r, t$. The proof of the analysis is similar to that of Streltsova (1975, 1976). If the well is fully penetrating and screened along its entire length, then the drawdown in a piezometer is given by:

$$s(r,t) = \frac{Q}{4\pi T} \int_0^\infty 4yJ_0\left(y\beta^{\frac{1}{2}}\right)\left[u_0(y) + \sum_{n=1}^{\infty} u_n(y)\right] dy$$

$$u_0(y) = \frac{\{1 - exp[-t_s\beta(y^2 - \gamma_0^2)]\}tanh(\gamma_0)}{[y^2 - (1+\sigma)\gamma_0^2 - (y^2 - \gamma_0^2)^2/\sigma]\gamma_0}$$

$$u_n(y) = \frac{\{1 - exp[-t_s\beta(y^2 - \gamma_n^2)]\}tanh(\gamma_n)}{[y^2 - (1+\sigma)\gamma_n^2 - (y^2 - \gamma_n^2)^2/\sigma]\gamma_n}$$

Equation 2-7

The values for γ_0 and γ_n can be obtained as the roots of

$$\sigma\gamma_0 \sinh(\gamma_0) - (y^2 - \gamma_0^2)\cosh(\gamma_0) = 0, \quad \gamma_0^2 < y^2$$
$$\sigma\gamma_n \sin(\gamma_n) + (y^2 + \gamma_0^2)\cos(\gamma_n) = 0$$
$$(2n-1)(\pi/2) < \gamma_n < n\pi, \quad n \geq 1$$

Equation 2-8

The general parameters are defined as r is the distance from the piezometer to the well, Q the constant discharge rate, T transmissivity, J_0 the Bessel function of the first kind and zero order, $t_s = Tt/Sr^2$.

The interpretation is done on log-log paper using a three step process. The solution unfortunately is only an approximation obtained by linearization. It does not take into account the reduction in the saturated thickness with time. Although the aquifer is assumed to be of uniform thickness, this condition is not met if the drawdown is large compared with the aquifer's original saturated thickness. A corrected value for the observed drawdown s then has to be applied. Jacob (1944) proposed the following correction factor **s'** = **s** - (s^2/2D), where **s'** = corrected drawdown, s = observed drawdown and D = original saturated aquifer thickness. According to Neuman (1975), Jacob's correction factor is strictly applicable only to the late-time drawdown data, which fall on the Theis curve.

The preceding paragraph highlights a specific problem that one has with unconfined aquifer systems. It is generally assumed that in fitting curve sets to data that the aquifer thickness is relatively constant. However in unconfined aquifers under aggressive pumping conditions this is clearly not true at which point the relation T = KD should be applied. Thus the observed transmissivity is related to the thickness of the aquifer or more specifically to the water level. A second observation that can be inferred from the above as that the hydraulic gradient should also play a role in the observed transmissivity although it is not clear what the relationship would be over an extended time period. This is typically observed during pumping in which the initial gradient close to the well is large and as time progresses the gradient reaches a constant value. The gradient change would thus also influence the calculated transmissivity of the well and/or area.

2.4 Review of Pump Tests and Possible Interpretations

It is generally assumed in a pump test that one is working with a single-phase flow of fluids to a well in a heterogeneous porous media (Raghavan, 2004). Wells play an important role in estimating aquifer properties in both hydrocarbon recovery and groundwater systems. Resolution of uncertainty in complex geological environments is essential for management of resources in both disciplines. The main of these investigations is to forecast an aquifer's performance over the long term and methods of improving the yield from the system.

In order to construct a sensible model for an aquifer the following points should be considered.

- A conceptual geological model should be determined for the local and regional scale;
- Geological model that is quantitative and that is based on interpretations of geology and geophysics and measurements based on core analysis, outcrops and petrophysics;
- Fine-scale computational model that is based on variables that are directly pertinent to fluid dynamics;
- Coarse scale version of a fine-scale computational model that is suitable for rapid computations over a regional scale.

Computational models are focused on the roles of wells, particularly the connectivity of wells to the aquifer/reservoir and also connectivity between the wells. The measured interconnectedness of the aquifer appears to be dependent on the number and location of the wells tapping the system. Thus in evaluating the pump test results a possible interpretation of the aquifer is made.

2.4.1 Support Volume

According to Raghavan, (Raghavan, 2004), support volume plays an important role in the matter of hydrogeologic scaling in the groundwater literature. The properties of the porous medium and its shape affect the volume of the aquifer investigated by pumping tests. Single well tests in the form of pressure build-up tests are used to evaluate reservoirs containing hydrocarbons in the petroleum system.

Measurements at observation wells located at a distance from a pumping well are considered to be more reliable for aquifer parameter estimations than measurements at a pumping well itself.

The differences between petroleum reservoirs and the groundwater systems are:

- The nature of the fluids in the porous medium (the compressibility of fluids in a hydrocarbon reservoir is much larger than that of water filled aquifer); and
- Well spacing (i.e. wells are usually at a considerable distance from each other in the petroleum reservoir) due to budget constraints.

Well spacing and compressibility of water enables the groundwater aquifer to be tested under steady flow conditions.

2.4.2 Further issues

Theis method imposes a single value for the properties of the aquifer volume that is sampled even if the geological formation may exhibit a continuous variation in properties. Thus a simplification of the real aquifer system is obtained. Similarly, methods such as Cooper-Jacob also produce an average estimation of transmissivity and storativity for a specified system.

Permeability or transmissivity as well as porosity and storativity generally depend on the volume of the porous medium sampled. Permeability cannot be derived without recognising the factors that influence its estimation. The concept of a single effective permeability for a single or group of well(s) should be avoided in the evaluation of pumping tests since it is a self-defeating proposition when complex geological formations are considered (Raghavan, 2004).

2.5 Testing and Heterogeneity – the Statistical Approach

Models which do not consider heterogeneity are only used due to a lack of tools and concepts to describe the aquifer accurately in three dimensions. These kinds of models should only be used as a mean of simplified parameter evaluation and for evaluation of short-term effects.

The issue of heterogeneity with respect to well testing and performance have been addressed by:

- Simulation of variations in properties of the porous medium applying deterministic (i.e. no randomness) methods and correlating the properties of the porous medium with properties derived by conventional means. and;
- Statistical methods.

Cardwell and Parsons (Cardwell and Parsons, 1945) method considers steady state, radial flow to a well in an aquifer in the form of a circle. The method shows that the equivalent transmissivity (T_e) is bounded by the weighted and harmonic means of the transmissivity of different aquifer regions.

$$\frac{\int_\Omega \frac{dx}{r^2(x)}}{\int_\Omega \frac{dx}{T(x)r^2(x)}} \leq T_e \leq \frac{\int_\Omega \frac{T(x)dx}{r^2(x)}}{\int_\Omega \frac{d(x)}{r^2(x)}}$$

Equation 2-9

Where Ω = region of aquifer bounded by the outer boundary and the well bore

$r(x)$ = distance from point **x** to the central well, and

$d(x) = r(x)drd\theta$ is an infinitesimal area

Hence they considered a deterministic approach to estimate the transmissivity of a porous medium.

However, deterministic methods are not sufficient to predict uncertainty in well performance because of lack of knowledge of hydrogeological properties, an insufficient numbers of observations and the high degree of spatial variability. Statistical methods are based on the concept that the porous medium is statically homogenous, i.e. the porous medium consists of a single facies.

In geological terms, the assumption of statistical homogeneity implies that the porous medium consists of single facies that is as explained by Anderson (Anderson, 1997), variability is independent of location in shape. The logarithm of the permeability, k (known as the log permeability), is defined by and presumed to be Gaussian.

$$Y = \ln(k)$$

Equation 2-10

In groundwater literature, the statistical properties of *Y* is described by its geometric mean,

$$k_G = exp\langle Y\rangle$$

Equation 2-11

Where angle brackets represent ensemble average and the two point correlation

$$C_y(x,y) = \sigma^2 \rho_Y\,(\mathrm{x},\mathrm{y})$$

Equation 2-12

σ^2: Variance of Y

$\rho_Y\,(\mathrm{x},\mathrm{y})$: Autocorrelation function of *Y*

The mean is usually assumed to be independent of location, that is, $\langle Y(\mathrm{x})\rangle \equiv \langle Y\rangle$, and the two point covariance depends only on the separation vector, $\mathbf{s} = |\mathrm{x} - \mathrm{y}|$, and not the actual values of **x** and **y**. The system is isotropic if the covariance depends only on the magnitude of **s,** and the covariance is characterised by a unique measure known as a range or integral scale.

Matheron (Matheron, 1967), illustrated that the effective permeability, k_{eq}, of a two dimensional system under the assumption that the porous body is a lognormal, subisotropic and ergodic medium is given by

$$\ln(\,k_{eq}) = E\ [\ln k_G\,]$$

Equation 2-13

k_G : geometric mean of the permeability of the individual elements of the heterogeneous medium.

2.5.1 Heterogeneous Media and the Theis Method

Warren and Price (Warren and Price, 1961) derived a statistical method which considers flow to a well and concluded that if the porous medium is assumed to consist of a quilt or patchwork of heterogeneous porous elements that are randomly distributed in three dimensions, then such a medium may be represented by a porous rock with a conductivity or permeability equal to the geometric mean of the permeability of the individual elements. They have predicted lower estimates of permeability for the 3-D flow, and this is attributed to the numerical bias results when finite difference models are used to analyse well response. It is furthermore difficult to deduce results for 3-D flow if the well penetrates the aquifer completely especially by computational methods.

Interference tests in heterogonous two-dimensional formations analysed with the Theis method by the Vandenberg (Vandenberg, 1977), appear to yield storativity estimates which vary over a much wider range than the transmissivity estimates. This indicates that transmissivity estimates, *T*, appear to be dependent upon the distance between the pumping and observation wells. The same theory was confirmed by Meier et al. (Meier et al., 1998) by interpreting drawdown curves at many observations wells with Jacob (Jacob, 1946) method. Whenever a heterogeneous system is approximated by a homogenous one, anisotropy should be considered and that the estimate of transmissivity reflects an estimate that reflects a tensor. If not considered then estimates of *S* may not be representative of the porous medium, for significant variations in *S* are obtained even in relatively homogeneous but anisotropic reservoirs when reservoir anisotropy is considered.

The problem with evaluating pumping test responses at the observation boreholes is that there are very few diagnostic features in the drawdown curve to reflect the underlying complex geology. Raghavan (Raghavan, 2004) suggest that drawdown responses be evaluated simultaneously in all observation boreholes with a realistic model that incorporates the underlying heterogeneity to some extent.

Toth (Toth, 1967) matched the different parts of the measured drawdown to different parts of the Theis curve and observed that the time rate of drawdown along the first limb is determined by the actual transmissivity of the aquifer from which water is pumped. Toth (Toth, 1967) concluded that calculated equivalent transmissibility's are expected to decrease as the length of the pumping test increases, approaching a value that is determined by the permeability and thickness of the whole rock complex contributing to the well. The observations of Toth (Toth, 1967), are illustrated by the Figure 2-5, below taken from Raghavan et al. (Raghavan, 2004). The derivative responses at active (pumping) wells for a number of fields are illustrated. The responses are from well producing all types of fluids from a variety of geological environments.

Common features from three well responses, in Figure 2-5 are:

- The small values of the derivative curve at an early time suggest that the wells appear to be located in a highly permeable region;
- the rapid rise in the derivative curve at intermediate times appear to suggest a loss in connectivity between the well and the reservoir as the fluid is produced; and
- Some of the curves appear to stabilize at later times, suggesting that the reservoir permeability is low at least when compared with the value corresponding to that of the near well.

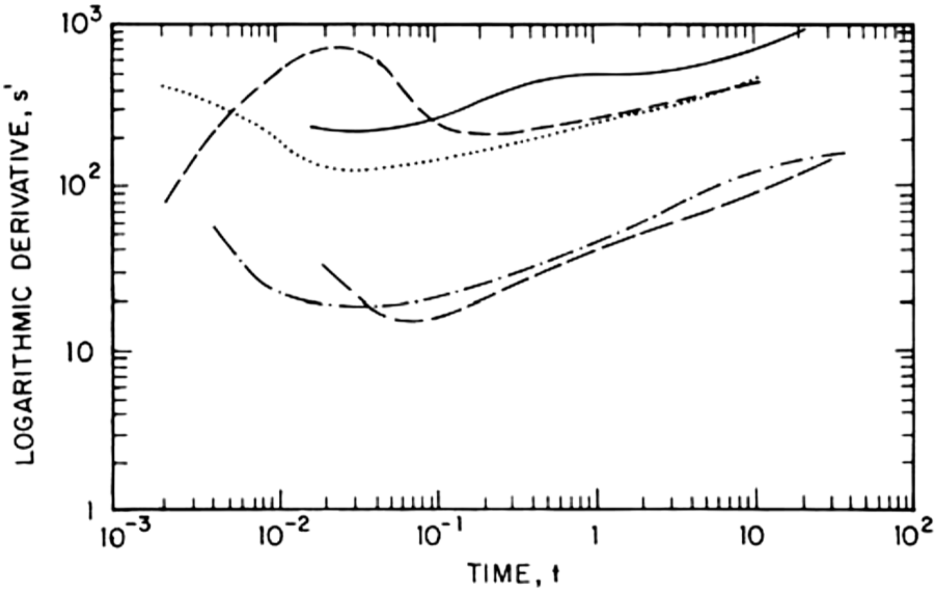

Figure 2-5: Derivates responses for a number of field test at active well (Raghavan, 2004).

2.5.2 Stochastic modelling

This method focuses on flow to wells in heterogeneous formations by assuming a random, but spatially stationary permeability field of given statistics. The method evaluates pumping test responses in heterogeneous formations assuming that the properties are a random function of the well coordinates.

Shvidler (Shvidler, 1966) examined the role of permeability by assigning discrete values of permeability to the location of the wells, resulting in obtaining a network of values of permeability within certain accuracy.

Permeability, *k(**x**)* at a point x is an intrinsic property of the rock which doesn't change with time and is independent of the well location. Over the years a stochastic approach to modelling flow and transport has been developed to account for the spatial variability of conductivity and also account for the uncertainty concerning such variations.

To reconcile permeability values on a small scale, obtained from core and other static measurements to conductivities obtained from pumping tests, a power-averaging scheme where the pumping permeability is of the form were applied in Eq. 2-14.

$$k = \left[\frac{1}{V}\int_{V}^{\prime} k^{\omega}\, dv\right]^{1/\omega}$$

Equation 2-14

V: volume of interest over which the point of values of permeability are available

2.5.3 Some other approaches

Raghavan (Raghavan, 2004), have focused on studies that assume the underlying heterogeneity may be handled by Gaussian models. The limitation of Gaussian models can be handled by multipoint geostatistics and fractals. Neuman et.al (Neuman et al., 1990) suggests that distinct geological units that are statistically homogeneous on a hierarchy of scales may be described by a fractal model. Raghavan

(Raghavan, 2004), states that the well responses (Figure 2-5) can be analysed by assuming a fractal model to represent the heterogeneity of the porous medium.
Oliver and Indelman (Oliver, 1990, Indelman, 2003) indicate that for a mildly heterogeneous condition, it should be possible to use Theis (Theis, 1935) solution; however the interpretation of the permeability needs to be different. The results mentioned above (Oliver, 1990, Indelman, 2003) do not help in interpreting test like those shown in Figure 2-5. A continuously increasing derivative indicates that the effective permeability decreases significantly with distance.

2.6 Hydrogeological Scaling

Freeze (Freeze, 1985) has suggested that heterogeneity may be incorporated into analyses of groundwater system by using stochastic methods. According to Raghavan, pressure test invariably yield permeability estimates that are much larger than estimate obtained from core analysis. Riva et al. (Riva et al., 2001) confirmed that for a steady flow to a well that discharges at a constant rate, the permeability increases with support volume. The problem is that the values obtained from field tests are well above the limit of k_G predicted by stochastic theories.

Most classical concepts of (pumping test) analysis suggest that hydraulic conductivity increases linearly with the test radius, i.e. the permeability increases with increasing support volume for steady flow to a well that discharges at a constant rate.

According to Raghavan (Raghavan, 2004), the effective value of *k* appears to change, and invariably appears to decrease with distance (i.e. losing of connectivity) in a single-well test (converging and diverging flow). He clarified this observation by considering the derivative responses shown in Figure 2-5. As the derivative curve increases with time for t> 10^{-1}, there is no possibility to conclude that permeability increases with support volume. Neuman and Di Federico (Neuman and Di Federico, 2003) concluded that statistical homogeneity often inferred from standard geostatistical analyses may not be a true characteristics of hydrogeologic medium properties but an artefact of the scale of investigation and method of inference.

2.6.1 *Geology and Testing*

There have been increases in the use of seismic and geological data in the interpretation of pumping test.

The work mentioned above falls into three types:

- The use of numerical model to predict the effect of the geological model upon the well test responses
- The conditioning of geostatistically generated geologic models to reflect the well response; and
- The history matching of the numerical model with observed well test data.

2.6.2 Geologic Setting

Care need to be given to the geological environment that we are attempting to model. it is critical to consider whether these models are developed in a geological context. Geological models that are based on the assumption that aquifer /reservoir may be described by stochastic methods beg the question as to whether such techniques are applicable to real-world conditions. Conceptual geologic models provides the information on the properties of regions of high conductivity (existence, distribution, connectivity, etc.) and provides boundary within which we need to work.

2.6.3 *Scale-Up*

Scale-up / up-scaling is the aggregation of fine scale cells around features that significantly influence fluid flow without increasing the total number of cells to excessively due to computational limitations. Scale-up is usually carried out by attempting to concentrate nodes around features that significantly influence fluid flow

Those involved in scale-up primarily concern themselves with two issues:

- Ensuring that the pressure-flow relationship is maintained at the fine-scale and coarse scale levels; and

- Ensuring that breakthrough characteristics of all reservoir fluids are preserved at both scales.

Scale-up algorithms consider steady state or quasi-steady conditions. Scale up process needs to be part of the optimization algorithm where pumping or well test are concerned. Pumping or pressure test should be analysed once the scale that will preserve the connectivity is determined. Scale-up requires that the differences in the scales over which the properties vary be quite distinct (large).

Scale-up techniques are of two kinds, local and global. Global techniques impose a global pressure drop across the model and evaluate patterns within the system to determine regions that may be aggregated. Permeability values in three principal directions at the fine scale yield a tensor representation at the coarse scale. Local techniques presume that we may impose pressure gradients across a small group of cells that are to be scaled. Local techniques do no yield permeability representations in the form of a tensor. The transmissivities that are obtained by the scale-up algorithm by global technique depend on boundary conditions that are used and also the well location.

Raghavan et al. (Raghavan, 2004) found out that using a scale up criteria based on steady state procedures appears to be inadequate for capturing responses of the kind of interest to those who examine transient behaviour.

Factors governing aggregation and scale-up appear to be different:

- Aggregation and scale up processes are affected by the location of boundaries, discontinuities and connectivity;
- Approaches to scale-up assume a priori steady flow and this option is unavailable if we wish to examine transient flow; and
- If the pressure-derivative curve is used as the measure to evaluate and ascertain the adequacy of the coarse-scale model, then this measure indicates that the degree of coarsening permitted is much less than that permitted by simply considering pressure drops.

Scale-up technique will usually eliminate the underlying correlation that governs porosity-permeability relationship and any small-scale feature that may be of interest. According to Raghavan (Raghavan, 2004), it is impossible to deduce geological characteristics, except for large-scale, structural features from a model that has been scaled.

2.7 Inversion Algorithm

Jacquard and Jain (Jacquard and Jain, 1965), were the earliest to attempt to adjust permeability and porosity of the reserve by perturbing the permeability and porosity fields in a systematic way. Raghavan concluded that a pumping test curve will not on the surface tell us how we should go about resolving small-scales values of conductivity or permeability. With a pumping test curve it is possible to incorporate these small-scale properties along with their complex arrangements of these effects in a rigorous and quantitative way.

According to Raghavan, it is useful to recognize that we are concerned with four different models, i.e. model of reality, the conceptual model which can be subdivided into the quantitative model and the computational or numerical model. Each of these models reflects a particular scale to describe aquifer properties. It is virtually impossible to comment on anything about the underlying geological model on a quantitative level based on adjustments made at the computational level.

Scale-up process, at least as far as pumping well tests are concerned and needs to be part of the optimization algorithm (Figure 2-6). Analysis of pumping or pressure test should only be done once the scale that will preserve the connectivity of the well to the reservoir is determined.

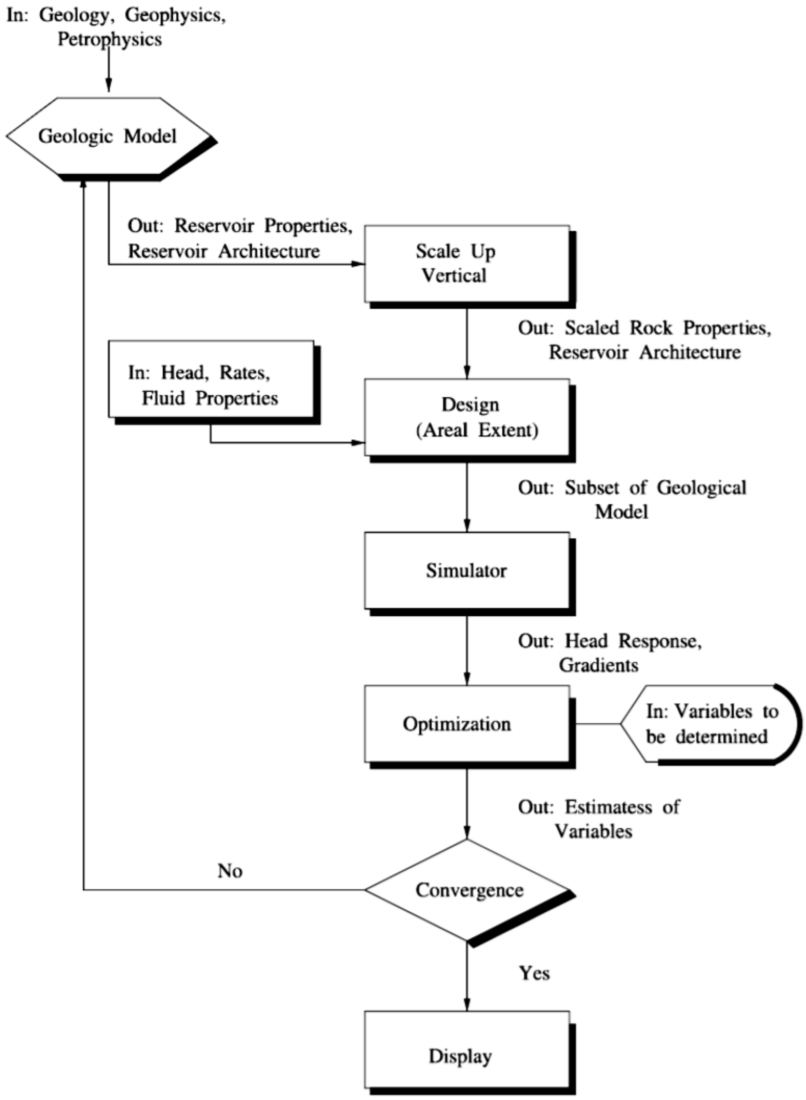

Figure 2-6: Outline to suggest scheme to estimate properties. (Raghavan, 2004)

2.8 Conclusion

In this chapter a review of methods applied in geohydrology were presented. The most important bulk flow parameter estimation procedures were discussed as well as the assumptions that are related to the systems. The methods of Theis and Cooper-Jacob permit geohydrologists to determine transmissivity values from pump tests. The major assumption in these methods is the concept of a homogeneous isotropic medium. Few diagnostic clues are directly available to deduce information on heterogeneity from pressure measurements unless a drastic change in permeability influences the test. Furthermore, the heterogeneity of the geology in an area greatly affects the determination of the transmissivity or hydraulic conductivity in that region. The use of multiple well tests in a study area can be used to construct an average hydraulic conductivity, although great care should be taken in evaluating the result.

In the following section a conceptual model of regional hydraulic conductivity values will be used. This will be done to highlight the effect that conductive zones have on estimated hydraulic conductivity values. A distinction between natural systems (no pumping) and forced gradient systems (pumping) will be made to evaluate further the effect on the estimation of a regional hydraulic conductivity value.

Chapter 3 Conceptual Models

3.1 Introduction

In this chapter various case studies will be reported to illustrate the effect of hydraulic tests on stationary and forced gradient systems. To evaluate these systems a random selection of points will be investigated that can be assigned a discrete value. In addition to illustrate the effect of ranges of difference, a set of values will be chosen such that a low variance between values is observed and conversely a set with high value differences. These values can then be applied to a numerical system in which the effect of the distribution of hydraulic conductivity values can be determined. Subsequently, a constructed natural system, which does not include any abstraction, should be investigated to determine if the random distribution of hydraulic conductivity and the inherent variance in these values effect mean value estimates. Finally, forced gradient conditions should be investigated to evaluate if the variance in hydraulic conductivity adversely affect the estimation of hydraulic conductivities from pump tests.

3.2 A Theoretical Investigation into Hydraulic Conductivity Distributions

Resource determination of an area is usually driven by the determination of the bulk flow parameters, such as hydraulic conductivity and storativity values. At this stage a decision usually is made on the basis of either maintaining the area under natural conditions (no pumping) or an abstraction (pumping) scenario is envisaged. In both instances water levels, hydraulic testing and distribution of the water resource (aquifer) is required. Since it is not possible to evaluate the total area for these parameters certain assumptions have to be made such that an average bulk flow parameter for an area can be determined. In wide-ranging situations a simple average of observation points is assumed to be sufficient.

It is to the above paragraph that the following sections are devoted too, i.e. what is in an average of an area. It is assumed that the storativity for the theoretical area is constant throughout and that the study site consists of a dual porosity medium. Two types of environments are possible for the system, firstly a natural system with no pumping in the area and secondly a pumped system.

3.2.1 Hydraulic Conductivity Distribution under Natural Conditions

In order to evaluate the effect of hydraulic conductivity distribution in a natural system a mathematical approximation of Figure 1-7 was constructed. A clear variance between the country rock and the dolerite intrusions can be observed. The simplest approach was to use a randomly distributed matrix (Figure 3-1) that contained discrete integer values that ranged from 1 to 3. These values were specifically chosen to mimic the system observed in Figure 1-7 and the hydraulic properties of the dyke structures ($K_f \rightarrow \infty$ m/d) are significantly larger than the country rock ($K_m = 10^{-3}$ m/d). A few transitional areas were also included since baking and sedimentation processes does occur in the vicinity of the dykes.

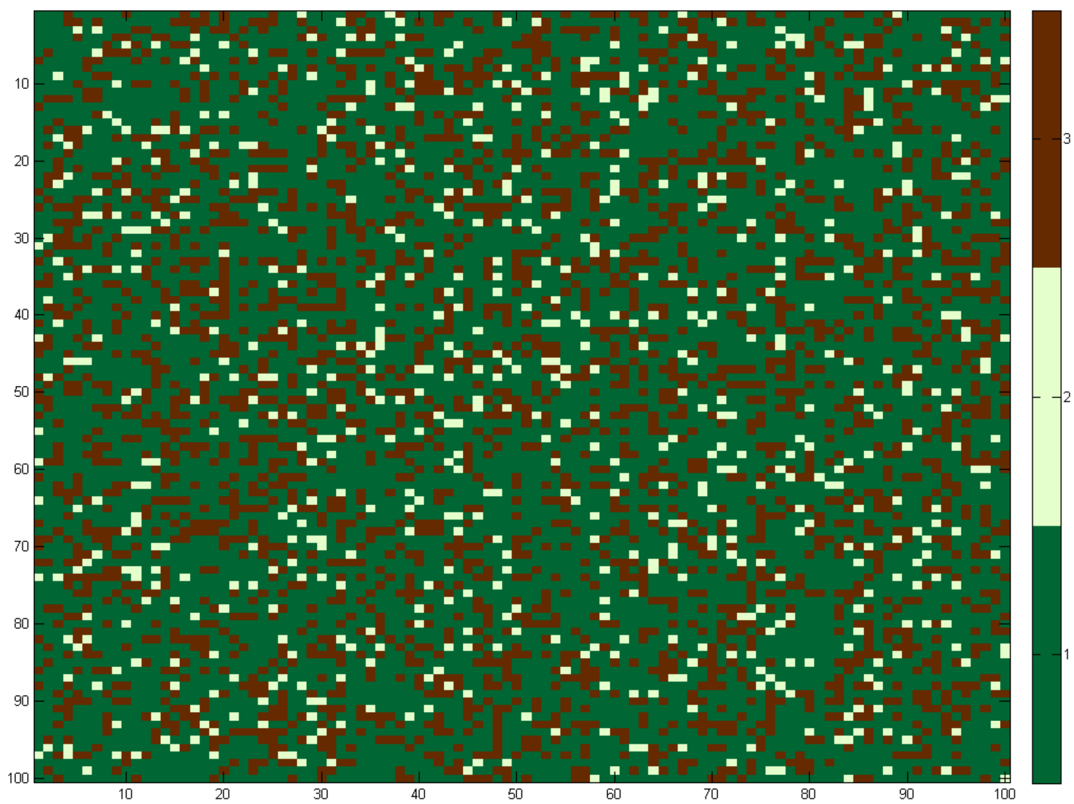

Figure 3-1 Randomly distributed transmissivity values over a two dimensional surface (100 x 100 elements).

Thus, because the hydraulic conductivity values of the whole area is known no uncertainty could exist in the analysis. In addition to the 100 x 100 randomly distributed matrix a second larger system (1000 x 1000 elements) was incorporated to evaluate the change in calculated averages of the system. In the current discussion the focus will be on the following mean values – arithmetic, geometric and harmonic mean. The standard deviation and minimum/maximum values were included to illustrate the changes observed compared to the average values obtained. A further interesting observation is that the absolute change in hydraulic conductivity between the country rock and the dolerite intrusions can be of different orders, i.e. differ by 1, 10 or 100. In Table 3-1 these values are listed and the associated histogram is shown in Figure 3-2. A graphical representation of the averages and associated minimum and maximum values are illustrated in Figure 3-3-Figure 3-5.

Table 3-1 Evaluation of mean values for differing ranges of hydraulic coductivity values in an area as depicted in Figure 3-1.

	100x100	1000x1000	100x100	1000x1000	100x100	1000x1000
Min	1	1	1	1	1	1
Max	3	3	100	100	10000	10000
Arithmetic Mean	1.599	1.304	27.362	15.389	2663.400	1441.674
Geometric Mean	1.405	1.184	3.628	2.014	17.007	4.065
Harmonic Mean	1.269	1.116	1.427	1.186	1.505	1.190
Standard Deviation	0.878	0.706	43.369	34.695	4407.400	3507.570

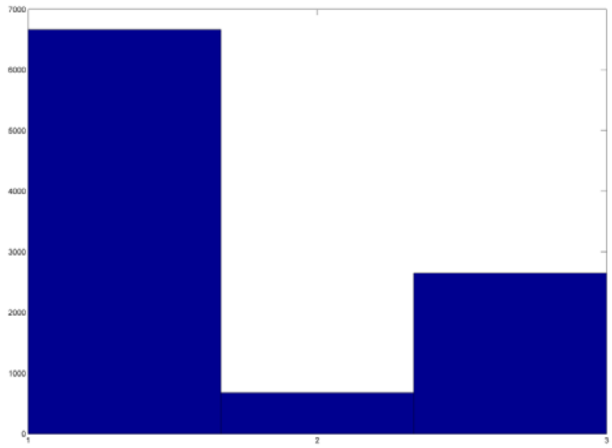

Figure 3-2 Histogram of transmissivity distribution values as represented for each random function in a 100 x 100 matrix (Figure 3-1).

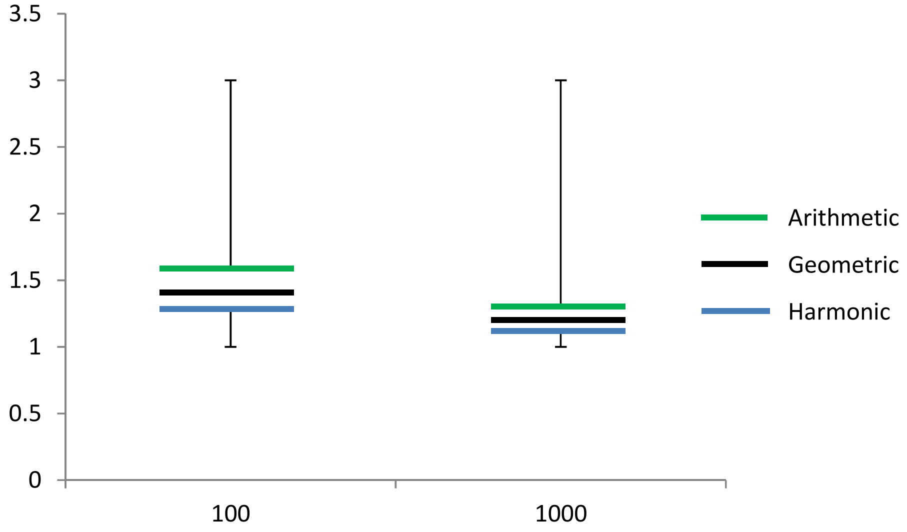

Figure 3-3 Basic statistical analysis of (1, 2, 3) map distribution in both 100x100 and 1000x1000 sample areas.

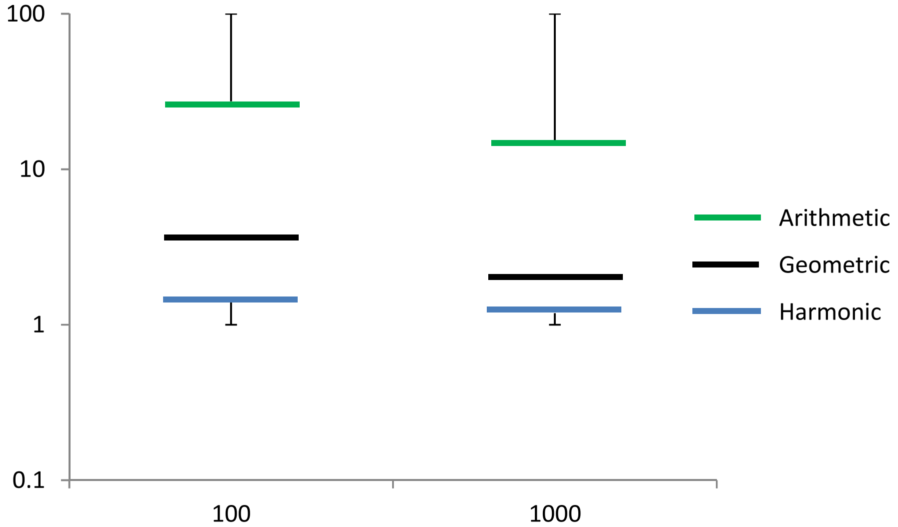

Figure 3-4 Statistical analysis of (1, 10, 100) map distribution in both 100x100 and 1000x1000 sample areas. Y-axis scale in logaritmic units.

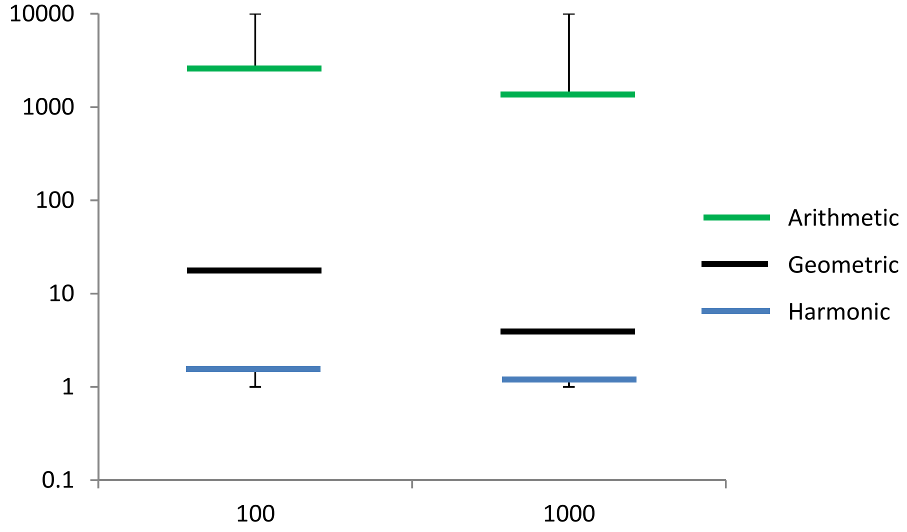

Figure 3-5 Statistical analysis of (1, 100, 10000) map distribution in both 100x100 and 1000x1000 sample areas. Y-axis scale in logaritmic units.

The following general observations can be made from Table 3-3, Figure 3-3-Figure 3-5.

1. The relative difference between the highest and lowest hydraulic conductivity value plays a significant role in the mean values calculated for each system. The arithmetic mean over estimates the hydraulic conductivity in an area, while the geometric and harmonic mean tends to give lower order estimates.
2. In each instance as compared to the order of the hydraulic conductivity difference the larger sample set gives a lower mean value. This is a result of the relative distribution of higher versus lower hydraulic conductivity zones as indicated in Figure 3-2 and it is directly reflected in the method of calculating these mean values.
3. The mean values as calculated for the change in order (left to right over Table 3-3) decreases in variance. With the harmonic mean being nearly constant, while the geometric mean has a steady increase as the order is increased in the system. The most aggressive growth in the mean value is observed for the arithmetic mean, which is adversely affected by higher hydraulic conductivity values.

As noted in Section 2.3 the natural flow system is controlled by zones of low hydraulic conductivity if parallel flow is considered over an area, thus it is reasonable to expect that the average hydraulic conductivity value for an area should be close to that of either the harmonic or geometric mean of the system. However, since natural flow is dependent on the local hydraulic conductivity and the hydraulic gradient (which can approach zero); it is expected that the harmonic mean would be a good approximation since it stays nearly constant throughout all of the order changes.

3.2.1.2 Estimating Hydraulic Conductivity as a Function of Discharge over an Area

In the above discussion the main focus was on the distribution of the hydraulic conductivities over an area and estimating the mean value. This section will attempt to illustrate a method of estimating the hydraulic conductivity by using Darcy's Law (Q = KiA). Due to memory limitations only a 500 by 500 element system (each element 10 m x 10 m) could be constructed with a total thickness of 100 m for the aquifer system. A random distribution for the hydraulic conductivity values were used and are illustrated in Figure 3-6. A constant head boundary was placed on the left and right most border of the theoretical area, secondly a hydraulic gradient was imposed on the system of 0.0001. Natural flow of the system was simulated and flow is from left to right in Figure 3-6. A water balance on either side and in the middle was setup which spanned the whole 5000 m of the model. The hydraulic conductivity values for the one order system is defined as the following set $\{0.01, 0.1, 1\}$, while the two order system is defined by $\{0.01, 1, 100\}$ for the lowest and highest values (Figure 3-6).

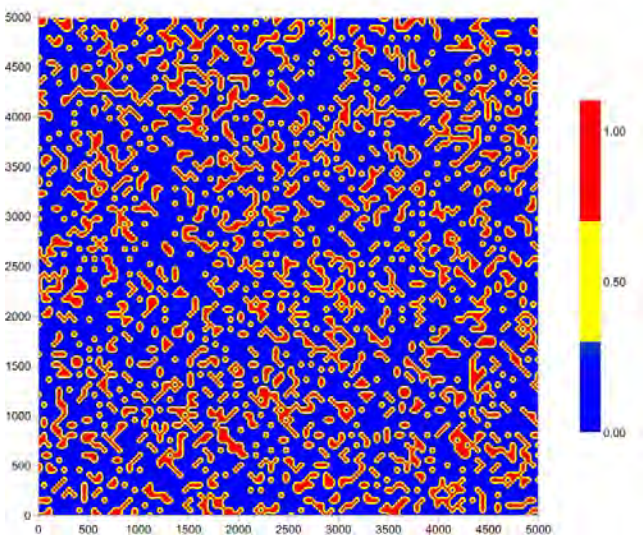

Figure 3-6 Random distribution of hydraulic conductivites for a specific test area.

With the above in mind an estimate water balance was derived for the right, left and central zones of Figure 3-6. The respective values were 0.9196, 0.9359 and 0.9231 m^3/d for the first order difference while for the second order difference the values are 0.9806, 1.0010 and 0.9838 m^3/d. It is interesting to note that although no restraints were placed on the model, except the order differences in hydraulic conductivities, the approximate discharge volume values are relatively similar. Converting these values to hydraulic conductivities results in the following values of 0.0184, 0.0187 and 0.0185 m/d for the first order difference and for the second order difference the values are 0.0196, 0.0200 and 0.0197 m/d, respectively. A change in hydraulic conductivity can be observed, however the change is less than 10% indicating that the low hydraulic conductivity value of the country rock dominates the natural flow regime. The arithmetic, geometric and harmonic mean of the hydraulic conductivity zones are defined for the one and two order systems as 0.2608, 0.0358, 0.0142 m/d and 24.8698, 0.1283, 0.0144 m/d. In both instances the harmonic mean gives the closest approximation of the hydraulic conductivity of the area.

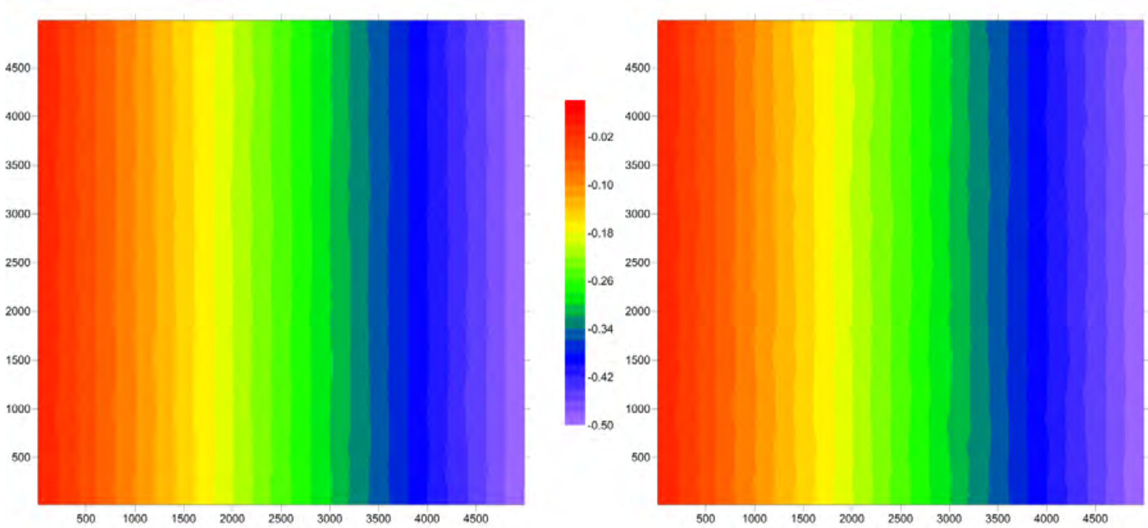

Figure 3-7 First order (left hand side) and second order (right hand side) hydraulic head value results after a 6 year simulation period.

In Figure 3-7 no clear difference can be observed for the respective hydraulic head values for the one and two order difference systems. However, if a difference image (Figure 3-8 left hand side) is constructed between the one and two order systems, a strong variance can be detected between zones. The random distribution of higher hydraulic conductivity areas can be observed, especially if the random distribution of hydraulic conductivity zones is overlain over the difference map (Figure 3-8 right hand side). The two order difference map was subtracted from the first order difference system, thus positive (yellow to red) values indicate an elevated hydraulic head in the one order system compared to the two order system. The inverse is also true in which the two order difference is greater than the one order systems resulting in a negative (green to blue) value. Thus, the blue regions indicate a build-up of water or hydraulic head since a large hydraulic conductivity difference between the country rock (0.01 m/d) and the simulated hydraulic conductivity of the dolerite intrusions (10000 m/d) is observed.

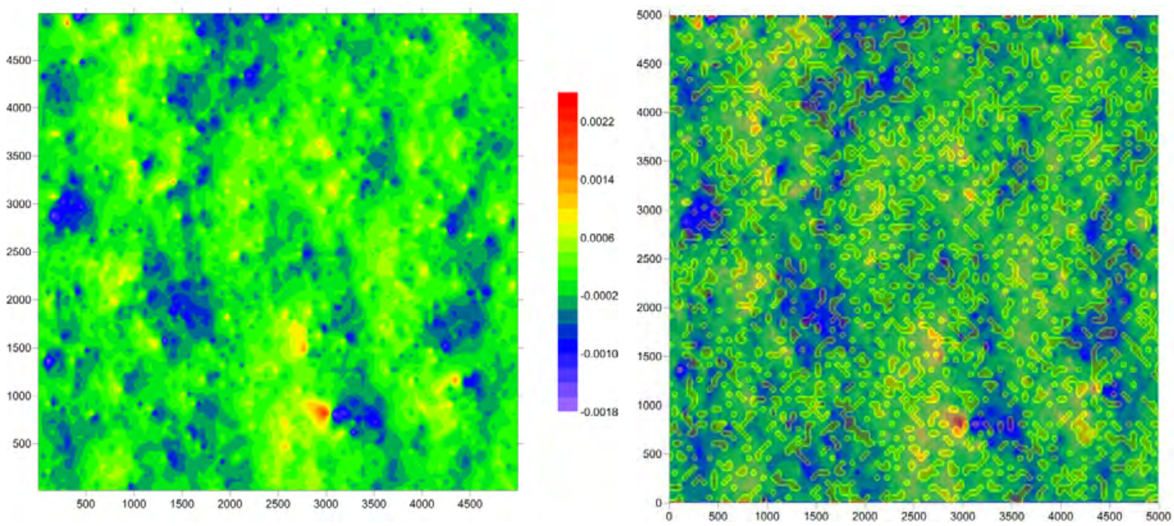

Figure 3-8 Difference map of first and second order hydraulic conductivity under simulated natural flow.

Furthermore, visually comparing the distribution of light blue and dark blue zones to that of yellow and red zones, a higher number of blue zones are observed. This illustrates that the higher two order hydraulic conductivity areas, has a larger effect on the transport parameters, thus affecting the movement of water. It is interesting to note that no simple linear correlation exist which would indicate where head build up would occur in the system (Figure 3-8).

A final evaluation of the natural system is to estimate what would occur if a sink is placed in the middle of the area. A rectangular area (1000 m x 1000 m) was placed in the centre of the model and the water level within this area was kept at 50 m below surface level (Figure 3-9). In nature this scenario can represent a sheer bank in which groundwater falls to a river level.

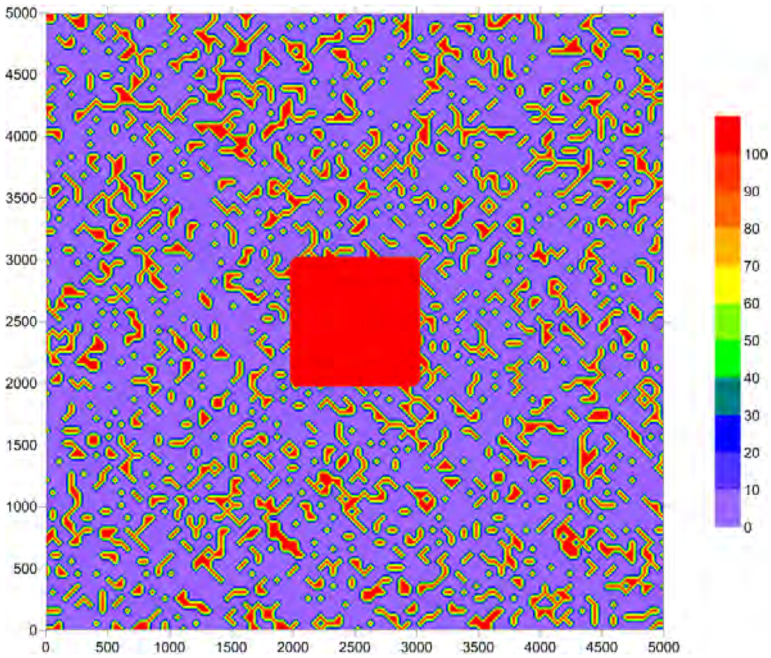

Figure 3-9 Hydraulic conductivity map of the study area with a cavity in the central region.

To assist in the evaluation the influx from one of the walls were measured over an extended time frame (6 years) and the results are reported in Table 3-2 and Figure 3-10. The initial discharge into the cavity for the one and two order systems is 2824 and 18970 m^3/d, respectively. It can be clearly observed from Figure 3-10 that both the one and two order solutions tend towards the same result (141-148 m^3/d); this would again indicate that the initial high hydraulic conductive zones are overshadowed by the influence of the country rock.

Table 3-2 Summary of values for the discharge rate of the one and two order systems for the cavity model.

Time (d)	Discharge 1 order (m^3/d)	Discharge 2 order (m^3/d)
2	2824	18970
12	1186	1222
112	437	454
360	267	277
720	204	212
1080	176	184
1440	160	168
1800	149	156
2160	141	148

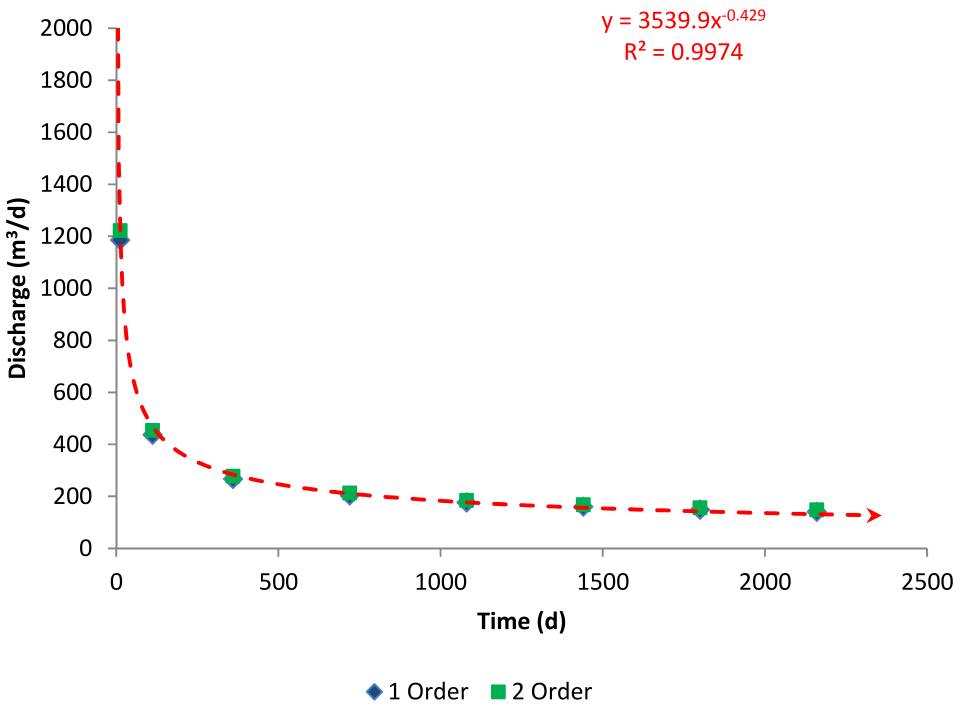

Figure 3-10 Discharge from seepage wall over an extended time period of 6 years. Both the one order difference and two order difference scenarios are given for the same model.

In this section the calculation of mean (arithmetic, geometric and harmonic) values were discussed in order to determine the most suitable average value to use if an average value is required for an area. The harmonic mean gave consistently the most stable results. It was shown that the geometric mean was susceptible to change if orders of magnitude were observed between the low and high hydraulic conductivities zones.

Subsequently, the natural calculated hydraulic conductivity of an area can be estimated by determining the harmonic mean. It can be reasoned that the hydraulic properties of the area is controlled by the lower hydraulic conductivity values instead of the geometric mean values as proposed by pure statistical approaches.

This concludes the discussion on the natural low hydraulic gradient systems and in the next section forced gradient results will be presented.

3.2.2 Hydraulic Conductivity Distribution under Forced Gradient Conditions

In this section a model was created that had 500 x 500 cells (1 cell = 10 m x 10 m) with a random hydraulic conductivity distribution as presented in Figure 3-11. A similar approach as that followed in the natural system was used, in that the influence of the change in the order of hydraulic conductivity values was of interest. The hydraulic conductivity values for the one order system is defined as the following value set $\{0.01, 0.1, 1\}$, while the two order system is defined by $\{0.01, 1, 100\}$ for the lowest and highest hydraulic conductivity values.

Furthermore, a second set of scenario's were of interest and that was the presence of a high or low hydraulic conductivity zone around the pumping well (Figure 3-12). This was done to determine the long-term effect that the final hydraulic properties of the system would have on the hydraulic conductivity estimate.

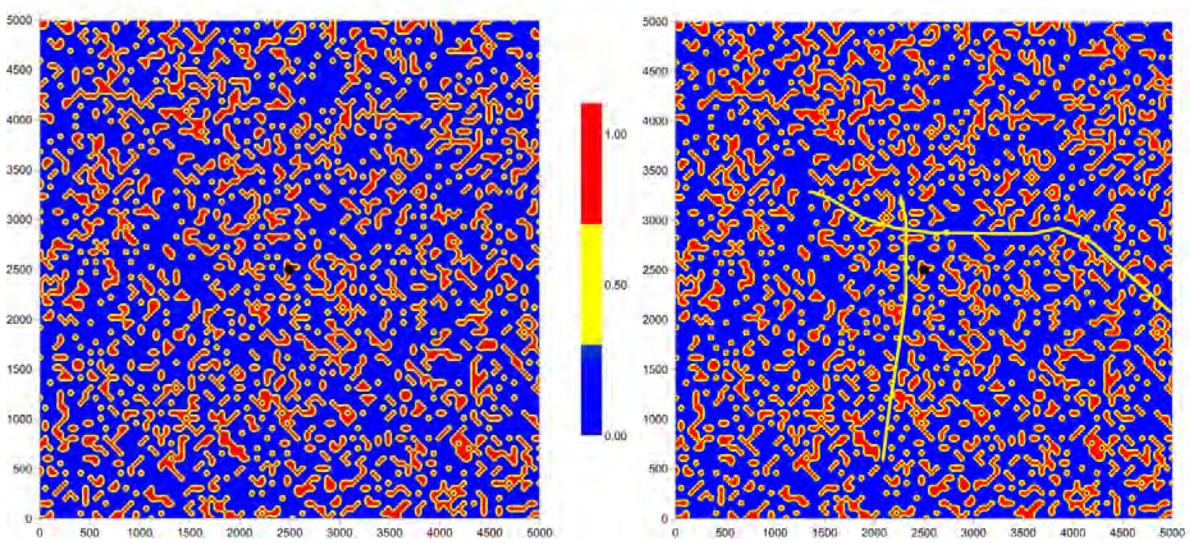

Figure 3-11 Random generated maps, on the left hand only a random distribution of hydraulic conductivity values are represented. On the right hand side lineaments or line segments have been included that could act as high conductivity zones.

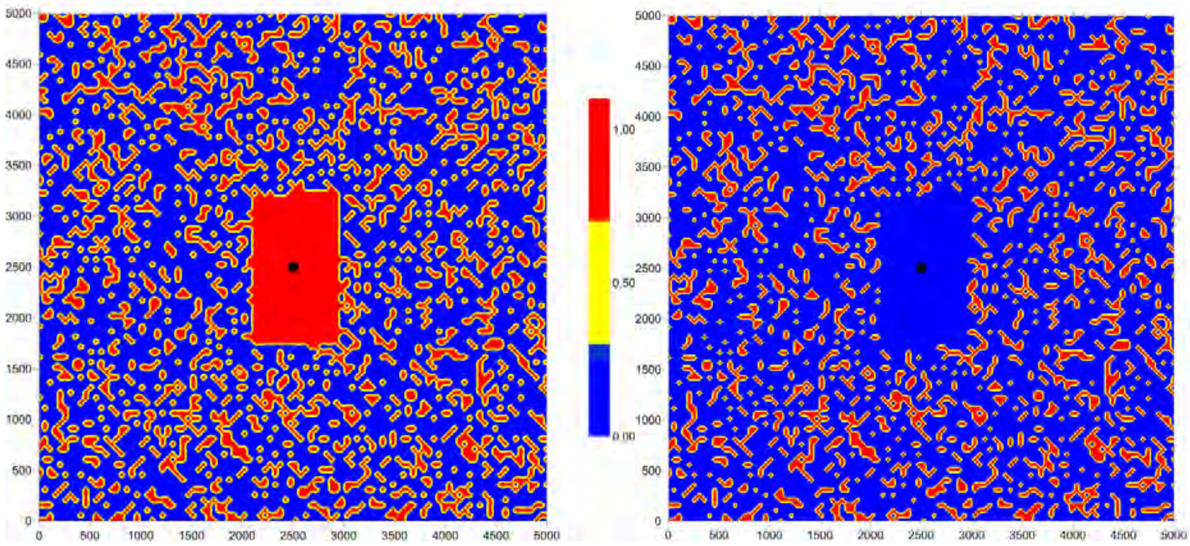

Figure 3-12 Hydraulic conductivity maps showing the presence of a high (left) and low (right) conductivity blocks in the middle of the randomly generated values.

In all four of the scenarios a pumping well was placed in the centre and set to pump at a rate of 100 m^3/d, over a six year time period. The final drawdown contours is presented in Figure 3-13-Figure 3-14. It is clear from Figure 3-13 that the inclusion of low conductivity zones had no real apparent effect on the cone of depression of the system. If however the blocked zones that have high and low hydraulic conductivity values are compared. The high hydraulic conductivity block has a significantly smaller drawdown in the centre as compared to the low hydraulic conductivity block scenario.

The drawdown values over a time period in the borehole was recorded and fitted using the Cooper-Jacob straight line method (Figure 3-15). The fits were done at late time to give an estimate of the hydraulic conductivity of the system. Since the Cooper-Jacob method gives an estimated value based on graphical fit, it was expected that large differences in the distribution of the hydraulic conductivity values would yield significant results. However, from Table 3-3 and Table 3-4, the method proofed itself relatively robust in estimating the hydraulic conductivity values.

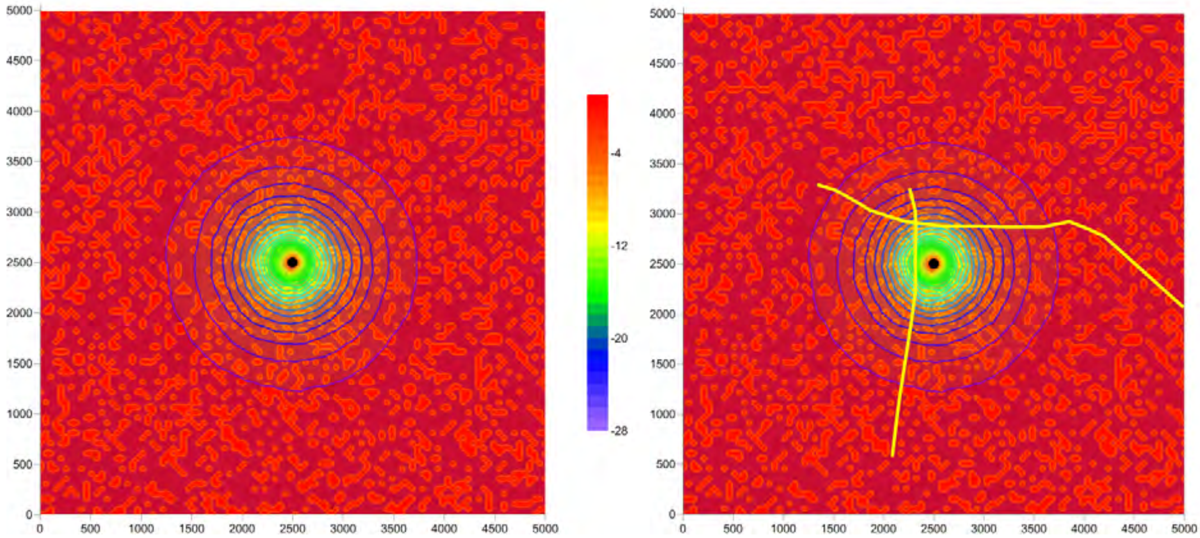

Figure 3-13 Contours maps of drawdown at the end of six years of constant pumping at 100 m^3/d. Left hand side illustrate the contours in a random field while the right hand side indicates the effect of two low conductive zones on the cone of depression contours.

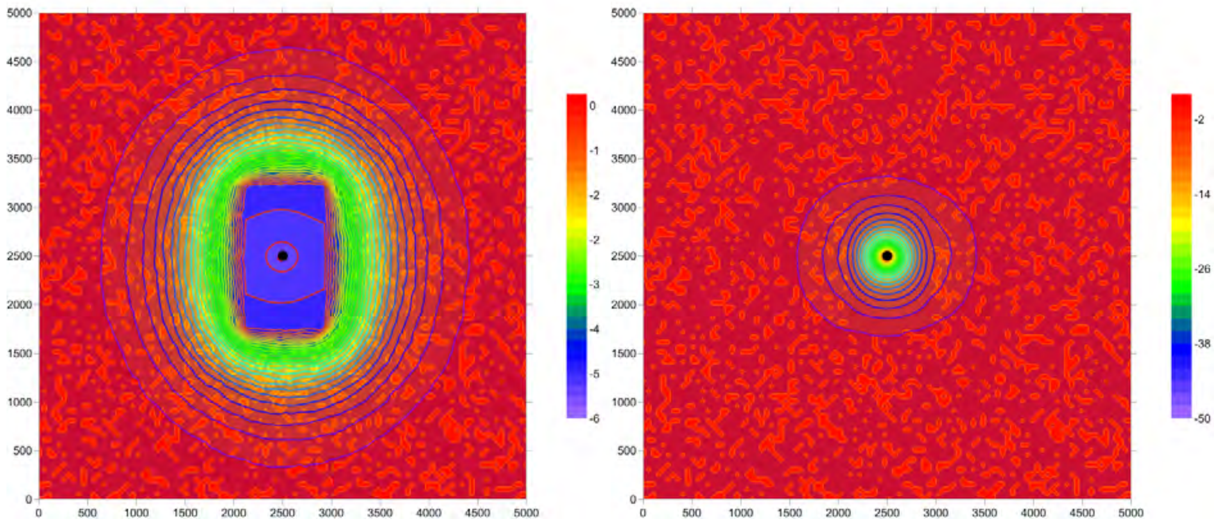

Figure 3-14 Contours maps of drawdown at the end of six years of pumping at a 100 m3/d. Left hand side illustrate the contours in a random field with a high hydraulic conductivity zone located in the middle. The right hand side indicates the effect of a low hydraulic conductivity zone on the cone of depression contours.

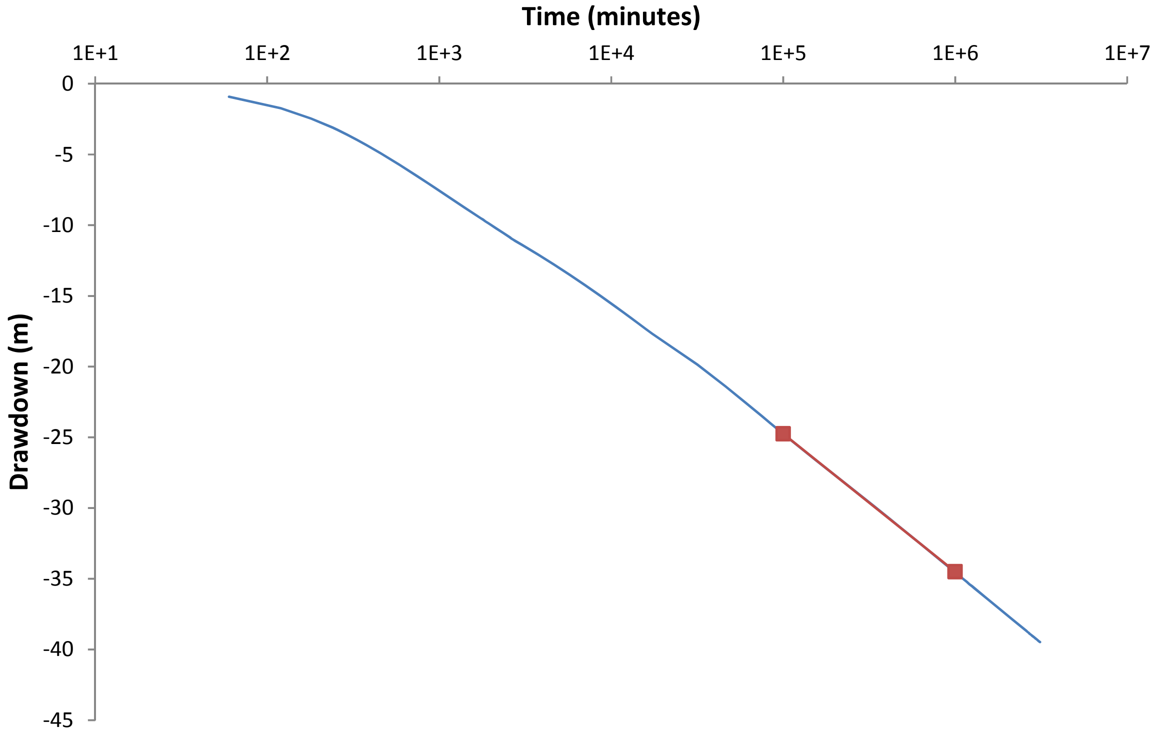

Figure 3-15 Cooper-Jacob fit of simulated pump test data for the random one order system. The estimated hydraulic conductivity for the pump test is 0.0183 m/d.

Table 3-3 The influence of conductive zones on the average hydraulic conductivity of different order systems. Randomly generated low conductivity zones (lines) were included see Figure 3-11.

Order – Mean	Arithmetic	Geometric	Harmonic	Observed
1 order				
Random	0.2608	0.0358	0.0142	0.0183
1 line	0.2592	0.0355	0.0142	0.0192
2 lines	0.2582	0.0353	0.0142	0.0181
3 lines	0.2565	0.0350	0.0141	0.0174
2 order				
Random	24.8698	0.1283	0.0144	0.0203
1 line	24.7098	0.1262	0.0144	0.0203
2 lines	24.6084	0.1249	0.0143	0.0190
3 lines	24.4448	0.1228	0.0143	0.0177

Table 3-4 The influence of high and low conductive block zones on the average hydraulic conductivity of different order systems.

Block – Mean	Arithmetic	Geometric	Harmonic	Observed
Low $\{0.01, 0.1, 1\}$				
1 order	0.2485	0.0336	0.01395	0.0100
2 order	23.6515	0.1133	0.01409	0.0199
High $\{0.01, 1, 100\}$				
1 order	0.2971	0.0422	0.01497	0.0325
2 order	28.5546	0.1779	0.01514	0.0411

The results from Table 3-3 are interesting since in all instances the harmonic mean underestimates the hydraulic conductivity of the area by 20-30%. In contrast the geometric mean overestimates the hydraulic conductivity by 80-100% for the one order system and 500-600% for the two order system. This clearly illustrates the effect of dual porosity systems on the estimation method for areal hydraulic conductivity mean values. It should be kept in mind that the difference in orders between the one and two system is $\{0.01, 0.1, 1\}$ and $\{0.01, 1, 100\}$, respectively. Thus in both instances the low hydraulic conductivity value for the area is 0.01. Compared to both the observed and the dominant background value, it is expected that the harmonic mean would give results which is more consistent with the actual regional hydraulic conductivity. Ironically, the arithmetic mean in all instances gives nonsensical values and will be discarded in the rest of this discussion.

In the instance of the high and low hydraulic conductivity block values (Table 3-4) the influence of large homogeneous structures on the bulk flow parameter were investigated. The harmonic mean gives relatively good estimates that range from 30-60% errors. The geometric mean overestimates by 200-400%, except in one instances (30% error) and that is when the high conductivity block is used with the one order system. This is most likely a fortuitous event and is not reflected in the general analysis; nevertheless the geometric mean overestimates the hydraulic conductivity in an area.

Considering all of the systems in Table 3-3 and Table 3-4 an average value was calculated and the difference between each of the mean value methods was calculated. The harmonic mean gave errors that ranged from 6-60%, with the best results obtained for the low hydraulic conductivity block (6%) and the randomly distributed (20-30%) systems. The high hydraulic conductivity block yielded the largest average underestimation difference of 60%. The geometric mean gave average errors that ranged from 90-550%, with the best results obtained for the randomly distributed one order systems. The remainder of the average geometric mean values were greater than 200%, rendering the estimated hydraulic conductivity results useless. Thus, if a mean value of an area needs to be estimated it is most likely best to use the harmonic mean, with the knowledge that at least 30% of the calculated values should be added to derive a realistic estimate of the hydraulic conductivity of the region.

3.3 Conclusion

In this section two distinct environments were investigated to determine what type of mean value should be used to determine a reasonable estimate of the hydraulic conductivity. It was assumed that the hydraulic conductivity in the whole area was known and that no uncertainty exists as to the location or variance of the distribution. Thus it was deemed necessary to use a numerical model system in which different possible scenarios could be tested. The scenarios tested included the idea that the relative difference in the hydraulic conductivity zones could dramatically affect the mean values calculated for an area. In addition the difference in hydraulic conductivities was set such that an order of difference could be observed, i.e. $\{0.01, 0.1, 1\}$ and $\{0.01, 1, 100\}$.

Firstly, a natural flow scenario (no pumping) was investigated in which a low gradient was used to determine the flow through a region. Since two sets of hydraulic conductivities were used the values obtained indicated that the harmonic mean, although it underestimated the hydraulic conductivity for the area yielded more accurate values. It was shown that the geometric mean was susceptible to change if orders of magnitude were observed between the low and high hydraulic conductivities zones. Thus it can be reasoned that the hydraulic properties of an area is controlled by the lower hydraulic conductivity values, instead of the geometric mean values as proposed by pure statistical approaches.

Secondly, a forced gradient scenario (pumping) was evaluated in which a pumping well was placed in the middle of the study area. Considering all of the systems investigated the harmonic mean gave errors that were far less than that observed for the other methods. Similar to the natural flow scenarios, the geometric mean was sensitive to large differences between hydraulic conductivity zones. It was determined that the harmonic mean underestimates the regional mean hydraulic conductivity by a third and should be adjusted to give a reasonable approximation.

Chapter 4 Case Studies

4.1 Introduction

In the following sections an investigation into reported hydraulic conductivity or transmissivity values will be presented and discussed. In order to evaluate these systems the previously constructed random map (Conceptual Model section) will be used to illustrate the effect on hydraulic conductivity values from a totally random selection of points and secondly the influence of a directed search method on regional hydraulic conductivity values. Subsequently, field data obtained from the WRC (Murray et al., 2011) and other regional studies conducted at the IGS will be included to demonstrate the observed effects of estimating regional transmissivity values.

4.2 The Effect of Random Selection on Regional Hydraulic Conductivity Estimates

As noted in the preceding section the constructed random map (Figure 4-1) will be used with the first order difference hydraulic conductivity values of $\{0.01, 0.1, 1\}$. A random selection of points in the map was taken in order to mimic the effect of site exploration. The number of sampled points was steadily increased from 10, 100, 1000 and 10000 over the possible 250000 data points in order to illustrate the effect of sample size on observed values.

The values obtained were tabulated in Table 4-1 with four different samples being taken to illustrate the random behaviour of the system. As noted in the previous chapter the expected regional hydraulic conductivity values should be in the order of 0.0184-0.0187 m/d. The arithmetic mean is an order larger than the expected regional hydraulic conductivity value; in addition the increase in sample size does not improve the result but instead worsens the approximation. Furthermore, if sample 4 in Table 4-1 is considered the initial estimate with 10 samples sites is significantly worse than with 100 sample sites. This would indicate that not only does the arithmetic mean suffer from over estimating the regional hydraulic conductivity but that it also gives highly variable results as a function of sampling.

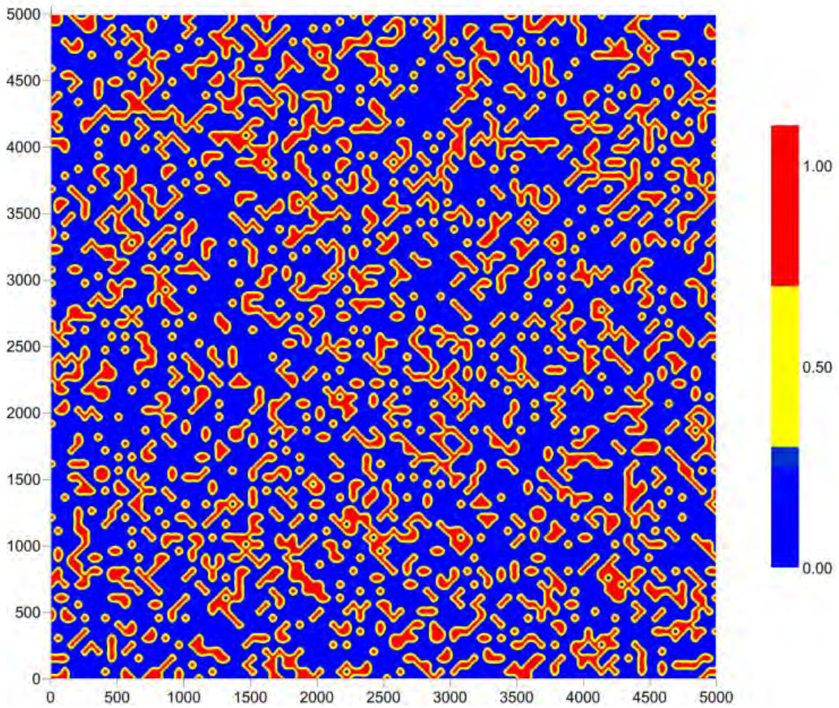

Figure 4-1 Random distribution of hydraulic conductivites for a specific test area.

Table 4-1 Mean values of randomly sampled points in a first order difference hydraulic conductivity map (0.01, 0.1, 1).

	1	2	3	4	Average
Arithmetic					
10	0.2080	0.2080	0.2170	0.5050	0.2845
100	0.2881	0.2404	0.2755	0.2161	0.2550
1000	0.2606	0.2542	0.2790	0.2837	0.2694
10000	0.2558	0.2532	0.2617	0.2685	0.2598
Average	0.2531	0.2389	0.2583	0.3183	0.2672
Geometric					
10	0.0251	0.0251	0.0316	0.1000	0.0455
100	0.0372	0.0309	0.0407	0.0309	0.0349
1000	0.0349	0.0343	0.0385	0.0408	0.0371
10000	0.0348	0.0345	0.0361	0.0372	0.0357
Average	0.0330	0.0312	0.0367	0.0522	0.0383
Harmonic					
10	0.0125	0.0125	0.0140	0.0198	0.0147
100	0.0140	0.0134	0.0151	0.0139	0.0141
1000	0.0140	0.0140	0.0145	0.0150	0.0144
10000	0.0141	0.0141	0.0143	0.0144	0.0142
Average	0.0137	0.0135	0.0145	0.0158	0.0143

The geometric mean has a far better performance in the estimation of the hydraulic conductivity values for the region. In general the values are overestimated by a hundred percent, however with an increase in sample sites a steady growth of the hydraulic conductivity can be observed. As in the previous section, sample 4 shows a significant deviation from the other samples. In this instance the geometric mean recovers as sample sites are increased. If the average value for different sampled sites is compared no real systematic improvement can be observed. The average of all the samples results in an overestimation of a hundred and five percent.

In the instance of the harmonic mean values, the values tend to be in the same range as the expected regional hydraulic conductivity (0.0184-0.0187 m/d). Interestingly, the harmonic mean underestimates the regional hydraulic conductivity by approximately thirty percent. However as the number of sample sites are increased, an increase in estimated hydraulic conductivity is observed, the underestimation is reduced to twenty percent. The harmonic mean has a far better analysis of sample 4, with an estimated value which is initially larger than the observed regional hydraulic conductivity but eventually reduces to the expected harmonic mean values for the other samples. If the average of the different samples is taken an average underestimation of twenty three percent is calculated. The harmonic mean values for the hydraulic conductivity values are more robust in estimating final values as compared to the other methods.

Subsequently, it is of interest to evaluate what would happen to a sample set that had two orders of difference in the hydraulic conductivity values $\{0.01, 1, 100\}$ as noted in the conceptual model chapter. The values obtained were tabulated in Table 4-2 with four different samples being taken to illustrate the random behaviour of the system. As previously noted in the conceptual model chapter the expected regional hydraulic conductivity values for the second order difference map should be in the range of 0.0196- 0.0200 m/d.

The arithmetic mean is still an order greater than the regional hydraulic conductivity value. The random samples taken in this section shows a greater variance in the samples, with sample 4 resulting in the best approximation. As the sample size increases an initial decrease in estimated hydraulic conductivity values is observed. However with large sample sets a steady increase in the arithmetic value is observed which results in excessive estimates.

Table 4-2 Mean values of randomly sampled points in a second order difference hydraulic conductivity map (0.01, 1, 100).

	1	2	3	4	Average
Arithmetic					
10	0.4060	0.3070	0.3070	0.2080	0.3070
100	0.1945	0.2836	0.2557	0.3268	0.2652
1000	0.2577	0.2675	0.2702	0.2713	0.2667
10000	0.2616	0.2638	0.2596	0.2675	0.2631
Average	0.2799	0.2805	0.2731	0.2684	0.2755
Geometric					
10	0.0631	0.0398	0.0398	0.0251	0.0420
100	0.0269	0.0407	0.0372	0.0437	0.0371
1000	0.0352	0.0368	0.0378	0.0366	0.0366
10000	0.0359	0.0361	0.0357	0.0369	0.0361
Average	0.0403	0.0384	0.0376	0.0356	0.0380
Harmonic					
10	0.0166	0.0142	0.0142	0.0125	0.0144
100	0.0132	0.0149	0.0147	0.0146	0.0144
1000	0.0142	0.0143	0.0145	0.0142	0.0143
10000	0.0142	0.0142	0.0142	0.0144	0.0143
Average	0.0145	0.0144	0.0144	0.0139	0.0143

The geometric mean has significantly smaller estimate values for the hydraulic conductivity of the system, with the overestimated estimated values ranging from 0.0631-0.0269 m/d (0.0196-0.0200 m/d). This results in an overestimation that is in the order of thirty five to two hundred percent. The variance in values between each sample is also great and no clear trend can be observed to evaluate possible methods of improving the outcome. If the average of the different samples is taken an average underestimation of ninety percent is calculated. Increasing sample size also does not improve the estimation of hydraulic conductivity values, with samples 2 and 3 showing a decreasing trend. However Samples 1 and 4 has the opposite trend.

The harmonic mean values tend to underestimate the regional hydraulic conductivity value by fifteen to thirty percent. If the average of the different samples is taken an average underestimation of thirty percent is calculated. In general all harmonic mean values for the system are in the same order and no significant variance is observed between samples.

The results obtained in this section indicate that random sampling of a region results in variation in the average observed hydraulic conductivity values. The arithmetic mean was shown to be sensitive to changes in both the first and second order difference systems. The geometric mean gave a better estimation of the region hydraulic conductivity values, although it overestimated

4.3 The Effect of Directed Selection on Regional Hydraulic Conductivity Estimates

Due to the method of borehole site selection a certain amount of bias is observed in regional zone. Firstly, the topography and local geology of the area is considered and if possible any geological structures could be observed. Secondly, geophysical methods such as magnetic, gravimetric, electromagnetic and electrical resistivity surveys are employed to determine the most favourable borehole site locations. Finally, local knowledge can also greatly improve the changes of siting a high yielding borehole. These methods are by no means fool proof and low yielding boreholes can be sited by these methods, however these methods do improve borehole siting procedures and generally give favourable results. In addition to the above, most borehole contractors and private land users are only interested in high yielding boreholes (> 1 l/s) and typically backfill lower yielding borehole sites.

To cover this possibility, a biasing procedure needs to be used to evaluate different yielding scenarios in both a one and two order difference systems. In order to assist in this a pseudo magnetic approach was implemented since South Africa's geology is riddled with dolerite intrusions (Figure 1-7). A smoothing algorithm was used on both the first and second order difference maps. Smoothing was done using a 3x3 matrix with a specified weighting scheme (Table 4-3) to normalise the values obtained. In the instance of the first order difference map it was a quarter while in for the second order difference map it was 1/400.

Table 4-3 Smoothing matrix format as applied to first order difference map.

0.25	0.5	0.25
0.5	1	0.5
0.25	0.5	0.25

The obtained value was then evaluated against a threshold value to determine a biasing matrix. The biasing matrix was then applied to the original hydraulic conductivity map and all values not included in the biasing map were set to zero. This would in effect mimic the use of a skilled geohydrologist, which would ignore all other areas except where anomalies were observed in a region. A typical biasing matrix is spatially represented in Figure 4-2 compared to the original map of Figure 4-1. Thus only these regions will be considered as possible borehole drilling sites.

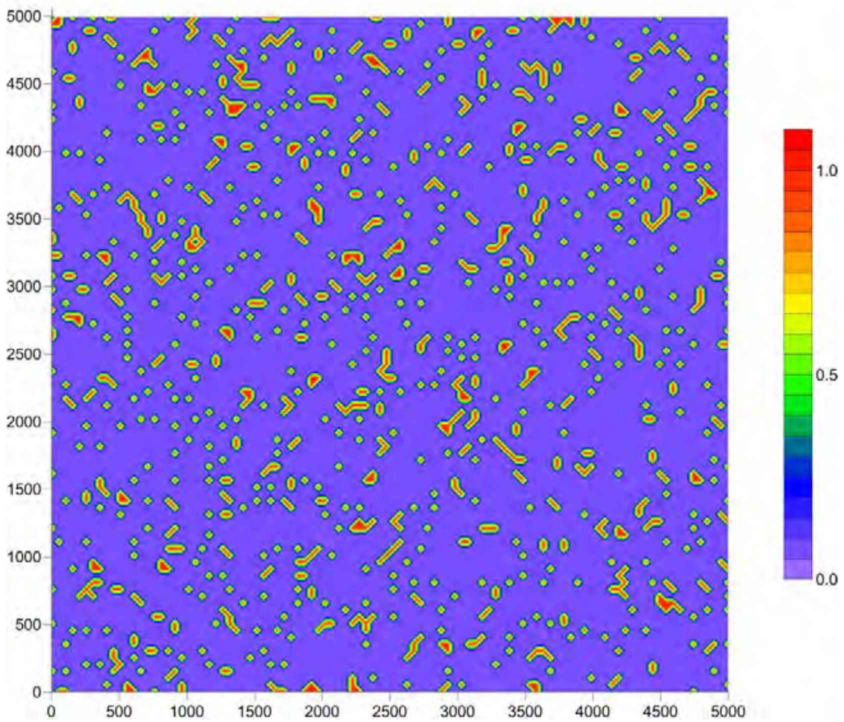

Figure 4-2 Effect of smoothing and filtering of original hydraulic conductivity map (filter set at 50% of highest peak).

Figure 4-2 is then used as a reference map to select the hydraulic conductivity values from Figure 4-1, which in turn is then randomly sampled to give values to be used in estimating mean hydraulic conductivity values. These values are tabulated in Table 4-4 for a first order difference map and Table 4-5 for a second order difference map, respectively.

As noted previously the expected regional hydraulic conductivity values for the first order difference map should be in the order of 0.0184-0.0187 m/d. Compared to the values reported in Table 4-4, it can be clearly observed that the arithmetic mean does not even come close to representing the expected regional value. Similarly, the geometric mean fails to represent the general value and is in this instance useless. The mean estimate error is in the order of thousands of percentages for both the arithmetic and geometric mean.

Table 4-4 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map (0.01, 0.1, 1).

	1	2	3	4	Average
Arithmetic					
10	1.0000	1.0000	0.9100	0.7120	0.9055
100	0.8920	0.8614	0.9028	0.7939	0.8625
1000	0.8830	0.8938	0.8853	0.8619	0.8810
10000	0.8828	0.8841	0.8798	0.8814	0.8820
Average	0.9144	0.9098	0.8945	0.8123	0.8828
Geometric					
10	1.0000	1.0000	0.7943	0.3162	0.7776
100	0.6166	0.5248	0.6607	0.3981	0.5501
1000	0.5902	0.6194	0.6012	0.5433	0.5885
10000	0.5905	0.5946	0.5832	0.5875	0.5889
Average	0.6993	0.6847	0.6598	0.4613	0.6263
Harmonic					
10	1.0000	1.0000	0.5263	0.0461	0.6431
100	0.0910	0.0673	0.1099	0.0500	0.0796
1000	0.0835	0.0911	0.0876	0.0745	0.0842
10000	0.0838	0.0850	0.0822	0.0832	0.0836
Average	0.3146	0.3109	0.2015	0.0635	0.2226

In contrast to the arithmetic and geometric mean the harmonic mean give results which are better than expected. The harmonic mean hydraulic conductivity values for the regional area does show a dependence on sample size, although sample 4 has the best mean value approximation. The mean estimate error is in the order of three hundred percent for the harmonic mean, this clearly illustrates the influence of bias in the data. The absence of lower hydraulic conductivity values have skewed the results and in this instance not even the harmonic mean can give a reasonable assessment.

Considering the second order difference map, a similar approach to that of the first order difference map was followed. The biasing in this instance gives more pronounced effects. As noted previously the expected regional hydraulic conductivity values for the second order difference map should be in the order of 0.0196-0.0200 m/d. Firstly, the arithmetic mean values for the regional hydraulic conductivity does not even resemble an approximation. Similarly the geometric mean fails to estimate the regional hydraulic conductivity. The lowest approximation for the hydraulic conductivity value is 15 m/d which is three orders greater than the actual value. Both these mean value calculation methods diverge from the regional hydraulic conductivity value, and thus is over sensitised to large values of hydraulic conductivity in the sample sets.

Table 4-5 Mean values of randomly sampled points in a second order difference biased hydraulic conductivity map (0.01, 1, 100).

	1	2	3	4	Average
Arithmetic					
10	80.0020	100.0000	90.0010	100.0000	92.5008
100	89.0011	89.0209	90.0109	89.0209	89.2635
1000	90.0069	87.4131	87.4171	89.0090	88.4615
10000	88.5908	87.8524	87.9128	88.2798	88.1590
Average	86.9002	91.0716	88.8354	91.5774	89.5962
Geometric					
10	15.8489	100.0000	39.8107	100.0000	63.9149
100	36.3078	39.8107	41.6869	39.8107	39.4040
1000	40.9261	33.1131	33.7287	37.6704	36.3596
10000	36.5426	34.3716	34.6258	35.3346	35.2187
Average	32.4064	51.8239	37.4631	53.2039	43.7243
Harmonic					
10	0.0500	100.0000	0.0999	100.0000	50.0375
100	0.0908	0.1108	0.1109	0.1108	0.1058
1000	0.1062	0.0876	0.0907	0.0979	0.0956
10000	0.0956	0.0905	0.0913	0.0919	0.0923
Average	0.0857	25.0722	0.0982	25.0751	12.5828

In comparison the harmonic mean does give results which are nonsensical, first entries for sample 2 and 4. However, in general the calculated harmonic mean values are significantly closer that that obtained from the other methods. The mean estimate error is in the order of three hundred percent for the harmonic mean, this clearly illustrates the influence of bias in the data. The absence of lower hydraulic conductivity values have again skewed the results and in this instance not even the harmonic mean can give a reasonable assessment of the regional hydraulic conductivity values.

To further investigate the influence of the bias of the system a stepwise increase from zero to fifty percent was performed on the first order system. It was done in for 0, 20, 40 and 50% filter values to determine if there is a cut off in which a reasonable estimate for the regional hydraulic conductivity could be estimated. The results are tabulated in Table 4-4 and the respective biasing maps are shown in Figure 4-3-Figure 4-4.

Table 4-6 Mean values of randomly sampled points in a first order difference biased hydraulic conductivity map.

Percentage	0	20	40	50
Arithmetic				
10	0.2080	0.4150	0.6040	1.0000
100	0.2881	0.4780	0.7174	0.8920
1000	0.2606	0.4309	0.7838	0.8830
10000	0.2558	0.4361	0.7895	0.8828
Average	0.2531	0.4400	0.7237	0.9144
Geometric				
10	0.0251	0.0794	0.1585	1.0000
100	0.0372	0.0933	0.2951	0.6166
1000	0.0349	0.0789	0.3945	0.5902
10000	0.0348	0.0804	0.4024	0.5905
Average	0.0330	0.0830	0.3126	0.6993
Harmonic				
10	0.0125	0.0195	0.0246	1.0000
100	0.0140	0.0197	0.0397	0.0910
1000	0.0140	0.0188	0.0516	0.0835
10000	0.0141	0.0189	0.0524	0.0838
Average	0.0137	0.0192	0.0421	0.3146

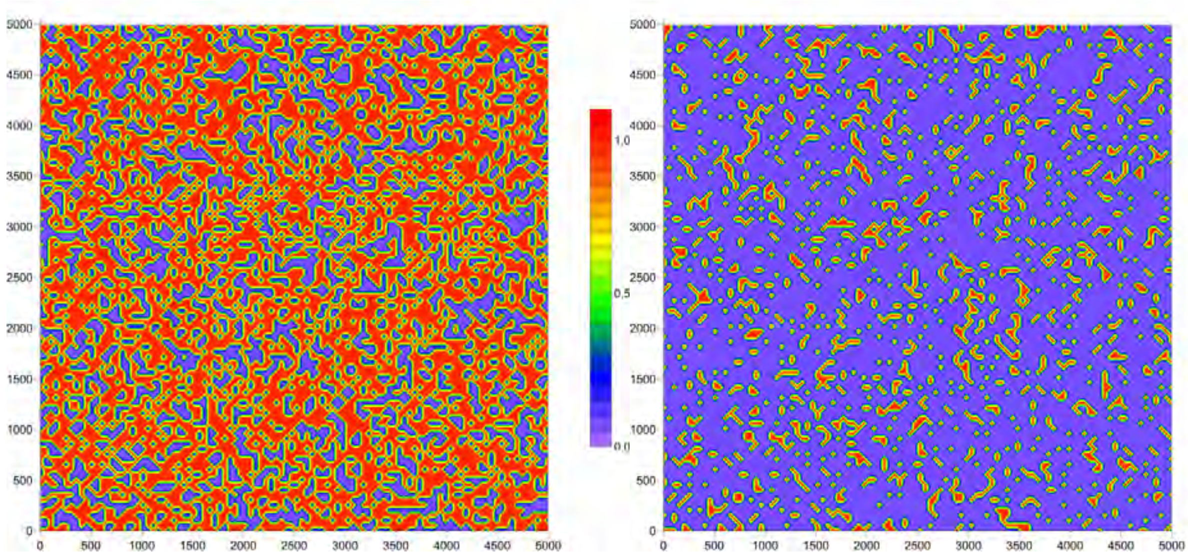

Figure 4-3 Effect of bias on the filter of hydraulic conductivity map (left filter set at 20% and right 40%).

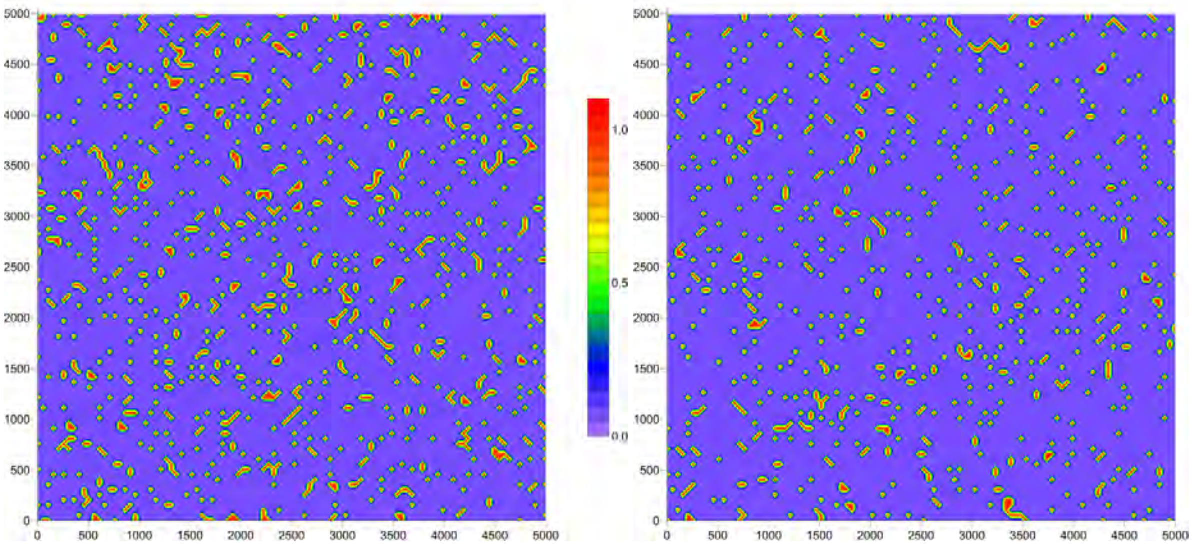

Figure 4-4 Effect of bias on the filter of hydraulic conductivity map (left filter set at 50%). Right hand indicates the change of filter from 40 to 50%.

In Figure 4-3 the difference between the 20 and 40% filter can be clearly observed, red indicates areas that are active sampling points. The 20% biased map leaves most of the sampling area unchanged, i.e. some of the lower conductivity zones are included in the mean value calculations. An approximate doubling in the arithmetic mean value is observed for the 0 to 20% filter values. The geometric mean is still below 0.1 m/d, but is roughly 4 times the expected value. The harmonic mean values are less than ten percent different from the expected hydraulic conductivity values. Although the estimation of the actual hydraulic conductivity for the region is obtained from the 20% filter values, it does give a rough estimate to determine possible field values. A possible approximation of the field values can be obtained by including one fifth of lower yielding boreholes in the high yielding data set, thus resulting in a better estimate.

The change from 20 to 40% is quite significant; a far lesser area in the original hydraulic conductivity map is left unchanged. This also results in a dramatic change in the mean values calculated for the system. The arithmetic and geometric mean is orders different from the actual regional hydraulic conductivity value. The hydraulic conductivity as estimated by the harmonic mean has also deteriorated but still is only a hundred percent greater than the expected value. This result indicates that although drastic changes in the sampling method were implemented, it is still possible to calculate a reasonable estimate of the regional hydraulic conductivity.

The difference between 40 and 50% filtering was not expected to have a great impact, however the change is significant. The difference map between 40 and 50% is shown in Figure 4-4 (right hand side); the filtering removed a significant part of the sample sites in the original hydraulic conductivity map. This in part is the reason behind the jump in mean values, since the values used are mostly on high conductivity zones.

It is clear from this section that as much bias as possible should be removed from the system before a mean hydraulic conductivity value estimate for a region is calculated. To remedy this data from low yielding boreholes should be included as well as conducting pump tests in areas where the country rock dominates the hydraulic parameters of the region. Furthermore for a rough estimate it is expected that approximately twenty percent of the hydraulic conductivity or transmissivity values should come from low yielding boreholes.

4.4 Estimating the Regional Hydraulic Conductivity from Field Data

The work presented here is based on data obtained from a WRC project on field measurements of transmissivity values (Murray et al., 2011). In this section geological formation data will be discussed and methods of obtaining a reasonable estimate of the hydraulic conductivity or transmissivity values. To further reduce the complexity of the system it will be assumed that in each value reported an aquifer thickness of 100 m is in effect, thus T = KD can be used to convert the transmissivity values (dependent on aquifer thickness) to hydraulic conductivity which is independent of aquifer thickness.

In addition to the above a short discussion on the conceptual model of dolerite dykes and sills will be included in this section. The rationale behind the discussion is the impact that these structures have on the transport of water in the Karoo aquifer systems. Firstly, dolerite dykes can act as conduits with high hydraulic conductivity zones (Figure 4-5). Secondly, the dyke structures can create an obstruction which can intercept or hinder the flow of water across the formation. Finally, it is rare that a dolerite dyke would not have weathered close to the surface and if the water level rises high enough a connection between the upstream and downstream aquifer systems is highly likely (Figure 4-6). The impact of this is that two aquifer systems can act in unison or can be totally disconnected from each other.

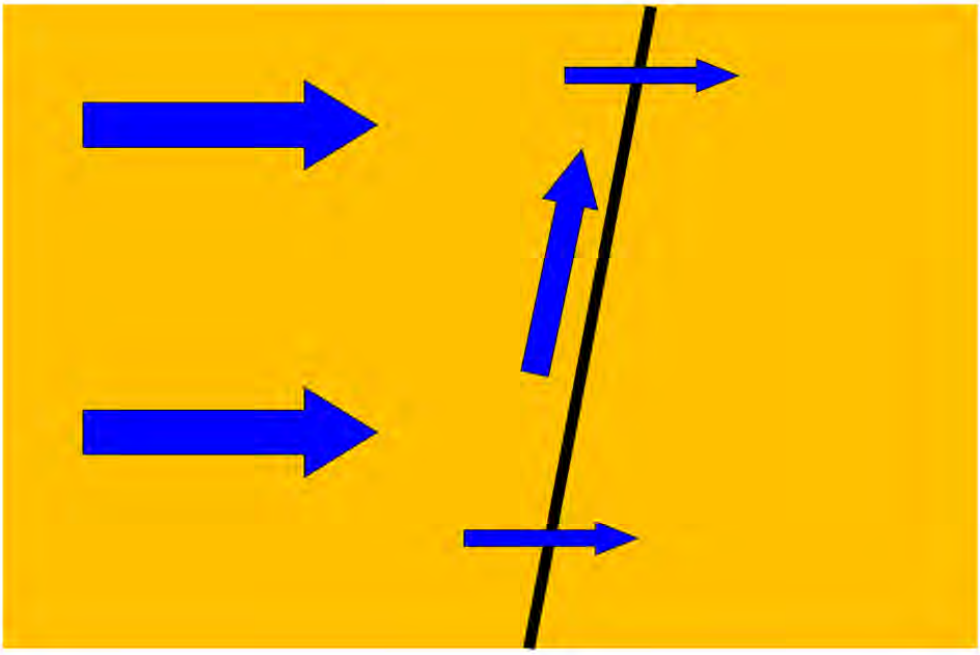

Figure 4-5 A plan view of the regional water flow direction in the presence of a dyke structure.

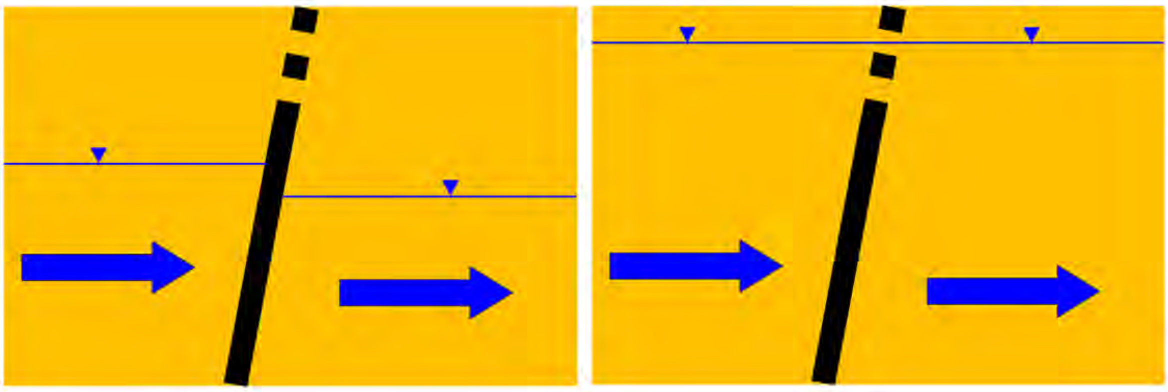

Figure 4-6 Side on view of regional water flow in the presence of a dyke structure. Left-hand side indicates the situation if the water level is below the weathered zone of the dolerite dyke. Right-hand side indicates the situation if the water level is above the weathered zone.

The scenarios presented in Figure 4-5 and Figure 4-6 can have a significant impact on the interpretation of regional hydraulic conductivity values. As presented in the Conceptual Model chapter the influence of high hydraulic conductivity zones in the vicinity of a regional low hydraulic conductivity area can change the behaviour of the estimated or measured bulk flow parameters. Although the dominant formation in the area has a lower hydraulic conductivity value, it is the high hydraulic conductivity zones which are targeted for water supply well fields. To illustrate this observation data obtained from the WRC project

(Murray et al., 2011) and is collated in Table 4-7. Note transmissivity values have been converted to hydraulic conductivity values (D = 100 m). The “Lower K” values were calculated based on the harmonic mean of thirty three percent of the lowest hydraulic conductivity values of the sample population. The “Upper K” values were calculated from thirty three percent of the highest hydraulic conductivity values and the arithmetic mean of the sample population. Finally, the “Middle K” values were calculated based on the median of the remaining thirty three percent of the sample population values. This was done for the matrix, sills, dykes, sill margins and alluvium for the respective formation from which pump test data was collected.

It is interesting to evaluate the collected data in Table 4-7 and compare it to the results obtained in Table 4-6. The data presented in Table 4-7 has a “Lower K” mean hydraulic conductivity value for the matrix in the order of 0.010 m/d with values that range from 0.003-0.039 m/d. The “Middle K” mean value for the matrix is 0.018 m/d with values that range from 0.006-0.073 m/d. It is remarkable that the “Lower K” and “Middle K” mean values does not differ dramatically, an effective increase of one hundred percent on estimated values. This could be indicative that these two sample sets do not have significant differences and most likely represent the same population, i.e. similar geological formations.

Thus an estimate of the regional hydraulic conductivity of the area can be estimated to be thirty percent of the harmonic mean value (0.012 m/d) tabulated in Table 4-7.

Once the higher conductive zones are considered, the progression of mean values follows a similar trend as that observed in Table 4-6. However, it should be noted that the difference in the hydraulic conductivity between the country rock or matrix and the fracture network would be far greater than three orders in magnitude, i.e. 1×10^{-3} vs. 1×10^{3} m/d respectively. This could be the only explanation why an order of increase in the hydraulic conductivity is observed as one moves from the “Lower K” through to the “High K” values in the respective sequences. The observed change can in part be an artefact from the mean value calculation method (Table 4-5) in which each method emphasises a certain section of the data.

Table 4-7 Hydraulic conductivity values compared to geological formation.

Formation	Lower K (m/d) Range						Middle K (m/d) Range						Upper K (m/d) Range					
	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin	Matrix	Dyke	Sill	Sill margin	Alluvium & intergranular	Alluvial basin
Adelaide-1	0.020	0.020	0.018	0.013	0.002	0.113	0.037	0.422	0.363	0.349	0.053	0.325	0.274	2.822	2.099	4.198	0.439	2.548
Adelaide-2	0.012	0.017	0.010	0.010	0.002	0.113	0.022	0.268	0.218	0.207	0.053	0.325	0.166	1.756	1.116	1.672	0.439	2.548
Adelaide-3	0.005	0.013	0.008	0.008	0.002	0.113	0.009	0.149	0.113	0.151	0.053	0.325	0.085	0.836	0.510	0.842	0.439	2.548
Adelaide-4	0.039	0.040	0.026	0.013	0.002	0.113	0.073	0.814	0.700	0.349	0.053	0.325	0.537	4.832	3.886	4.198	0.439	2.548
Adelaide-5	0.008	0.020	0.012	0.013	0.002	0.113	0.015	0.188	0.147	0.349	0.053	0.325	0.111	1.277	0.869	4.198	0.439	2.548
Burgersdorp	0.011	0.016	0.010	0.040	0.002	0.113	0.021	0.256	0.207	0.396	0.053	0.325	0.156	1.790	0.925	2.222	0.439	2.548
Clarens	0.009	0.022	0.014	0.025	0.002	0.113	0.017	0.204	0.161	0.073	0.053	0.325	0.122	1.102	0.703	0.545	0.439	2.548
Drakensburg	0.007	0.013	0.007	0.007	0.002	0.113	0.013	0.166	0.128	0.128	0.053	0.325	0.099	0.910	0.719	0.719	0.439	2.548
Dwyka-1	0.008	0.011	0.006	0.004	0.002	0.113	0.015	0.179	0.139	0.147	0.053	0.325	0.111	1.439	1.000	1.119	0.439	2.548
Dwyka-2	0.008	0.020	0.012	0.007	0.002	0.113	0.015	0.188	0.147	0.051	0.053	0.325	0.111	1.332	0.700	0.791	0.439	2.548
Dwyka-3	0.005	0.010	0.006	0.004	0.002	0.113	0.009	0.116	0.086	0.147	0.053	0.325	0.064	0.883	0.544	1.119	0.439	2.548
Ecca	0.010	0.025	0.015	0.026	0.002	0.113	0.019	0.225	0.179	0.179	0.053	0.325	0.135	1.213	0.700	0.950	0.439	2.548
Elliot	0.007	0.015	0.009	0.025	0.002	0.113	0.013	0.164	0.126	0.073	0.053	0.325	0.095	1.234	0.709	0.545	0.439	2.548
Katberg	0.013	0.021	0.013	0.011	0.002	0.113	0.024	0.274	0.223	0.147	0.053	0.325	0.175	1.695	0.990	1.582	0.439	2.548
Molteno	0.008	0.018	0.011	0.008	0.002	0.113	0.015	0.188	0.147	0.134	0.053	0.325	0.111	1.276	0.860	0.857	0.439	2.548
Msikaba	0.003	0.022	0.014	0.014	0.002	0.113	0.006	0.032	0.021	0.021	0.053	0.325	0.044	0.704	0.274	0.274	0.439	2.548
Pietermaritzburg	0.013	0.027	0.017	0.017	0.002	0.113	0.024	0.271	0.221	0.251	0.053	0.325	0.175	1.440	0.792	1.386	0.439	2.548
Prince Albert-1	0.007	0.012	0.007	0.018	0.002	0.113	0.013	0.155	0.119	0.145	0.053	0.325	0.091	1.373	0.843	1.586	0.439	2.548
Prince Albert-2	0.005	0.013	0.007	0.018	0.002	0.113	0.009	0.119	0.088	0.145	0.053	0.325	0.072	0.573	0.481	1.586	0.439	2.548
Tierberg 1	0.010	0.067	0.047	0.008	0.002	0.113	0.019	0.222	0.177	0.155	0.053	0.325	0.135	2.039	1.531	1.772	0.439	2.548
Tierberg-2	0.010	0.016	0.009	0.008	0.002	0.113	0.019	0.225	0.179	0.155	0.053	0.325	0.135	1.758	0.993	1.772	0.439	2.548
Tierberg-3	0.010	0.014	0.009	0.006	0.002	0.113	0.019	0.225	0.179	0.145	0.053	0.325	0.140	1.170	0.858	1.114	0.439	2.548
Volksrust	0.007	0.017	0.010	0.012	0.002	0.113	0.013	0.161	0.124	0.098	0.053	0.325	0.095	0.747	0.440	0.531	0.439	2.548
Vryheid-1	0.004	0.008	0.005	0.006	0.002	0.113	0.007	0.094	0.069	0.084	0.053	0.325	0.054	0.624	0.396	0.623	0.439	2.548
Vryheid-2	0.005	0.013	0.007	0.006	0.002	0.113	0.009	0.128	0.096	0.084	0.053	0.325	0.072	0.775	0.411	0.623	0.439	2.548
Vryheid-3 / -4	0.009	0.019	0.012	0.022	0.002	0.113	0.017	0.201	0.158	0.143	0.053	0.325	0.119	0.948	0.605	1.047	0.439	2.548
Waterford-1 / -2	0.014	0.020	0.013	0.019	0.002	0.113	0.026	0.302	0.248	0.226	0.053	0.325	0.190	2.417	1.342	1.988	0.439	2.548
Whitehill	0.009	0.018	0.011	0.012	0.002	0.113	0.017	0.184	0.143	0.483	0.053	0.325	0.130	1.220	0.869	1.221	0.439	2.548

In this section the problems in using documented data from field studies were highlighted as well as the method by which average values were calculated for the respective formations. The following section will incorporate field studies as well as the national groundwater database search of selected areas.

4.5 Combining Measured and Published Hydraulic Conductivity Values

In South Africa commercial exploitation of groundwater resources are regulated by law to such an extent that production boreholes are required to be registered at the Department of Water Affairs. A recent study conducted at the Institute for Groundwater Studies centred on the application of this data to estimate regional transmissivity values (Figure 4-7). The values were obtained from Groundwater Resource Directed Management (GRDM) Database in which borehole yields were converted to transmissivity values following a method suggested by Hughes and Parsons (Hughes et al., 2007). Another set of data values were obtained from a current research project on climate change within the Institute for Groundwater Studies under the funding of the Department of Water Affairs (Figure 4-8). The two sets of data will be used in conjunction with field results to discuss the evaluation method of estimating regional transmissivity values.

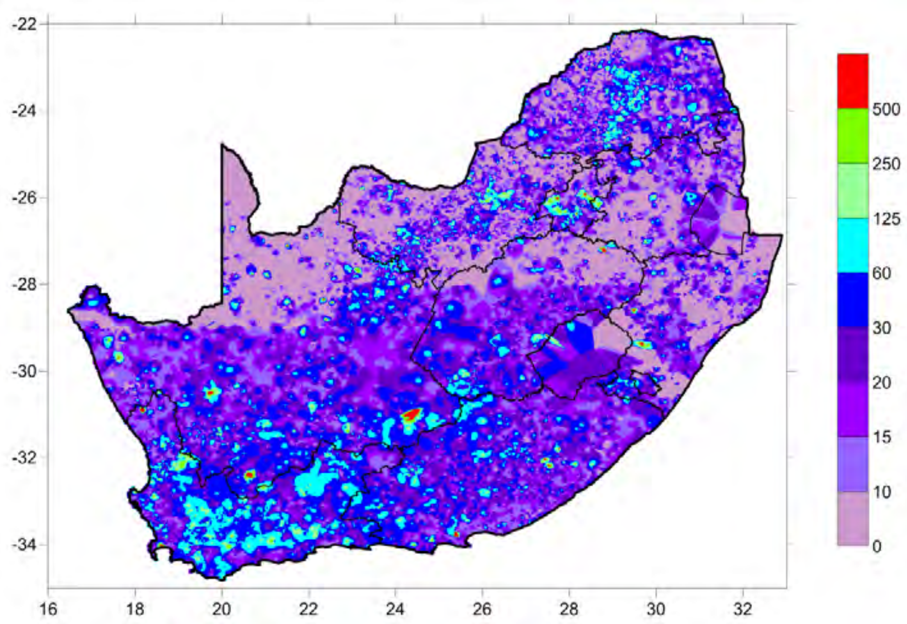

Figure 4-7 Regional transmissivity map as constructed from GRDM data (Hughes et al., 2007).

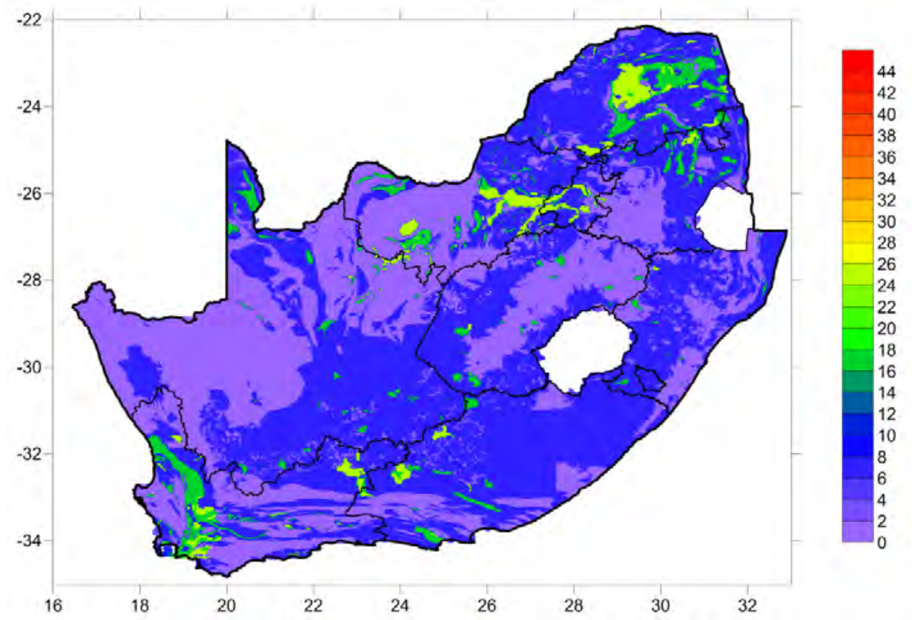

Figure 4-8 Regional transmissivity map as constructed from climate change data.

The difference in the estimated transmissivity values can be clearly observed from the respective scale bars. The maximum transmissivity values for the GRDM model approaches 500 m^2/d while in contrast the climate change model has a maximum of 46 m^2/d. The most significant difference between the two systems is that firstly geology was incorporated in the climate change model as well as a harmonic mean value based on pump tests in the respective areas. Roughly an order difference exists between the two model systems; this is analogous to the difference between the arithmetic and geometric mean values as compared to the harmonic mean assessment from the previous sections.

4.5.1 Krugersdrift Test Site Evaluation

Since it is difficult to evaluate the efficacy of these methods at such a large scale, a recently developed test site near the Krugersdrift Dam was used as a starting point for the evaluation of estimated transmissivity values. Both regional approximations for the transmissivity values are known for the area as well as a set of pump test results. The site location is presented in Figure 4-9 while the pump test data is given in Table 4-8. The estimated regional transmissivity value for this area for the GRDM and climate change models are 15-25 and 5 m^2/d, respectively. In the instance of the GRDM model a differentiation in expected transmissivity values were observed, with site A1 having an approximate transmissivity of 25 m^2/d and site A2 a value of 15 m^2/d.

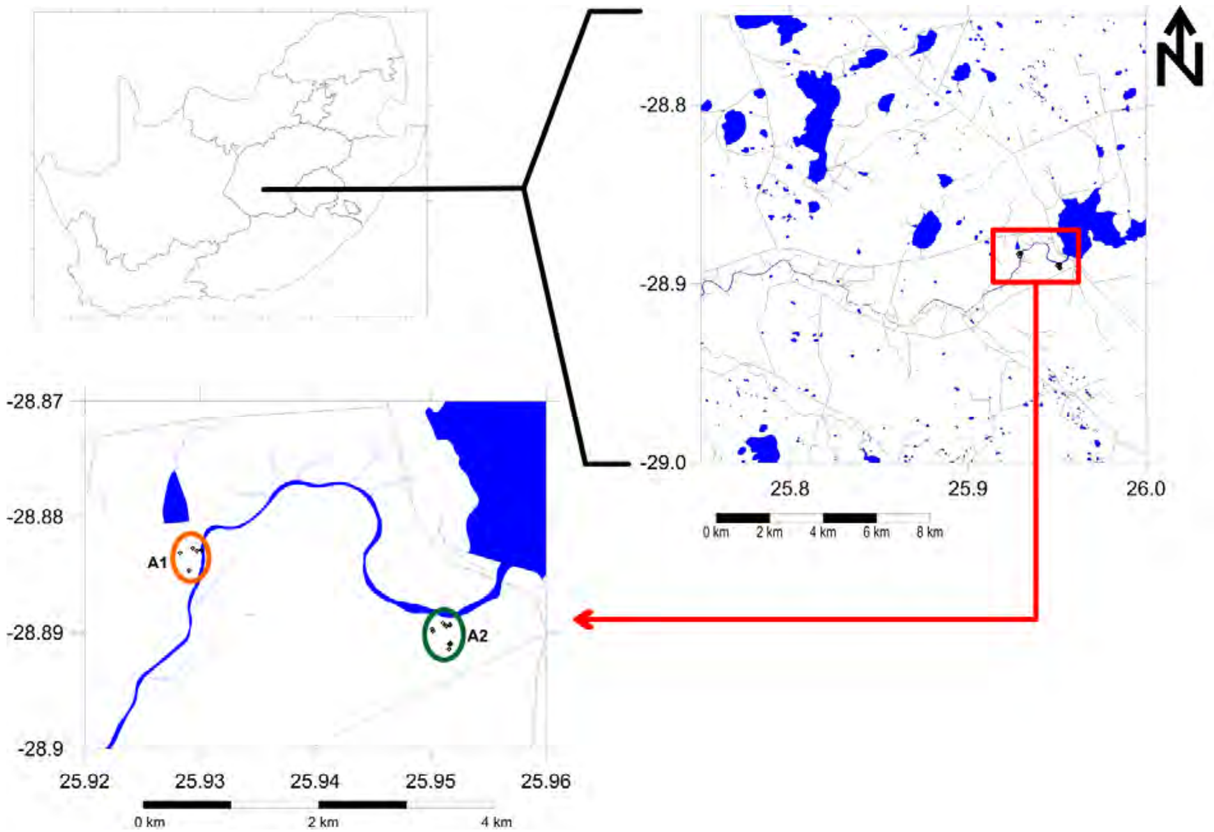

Figure 4-9 Krugersdrift test site located within the Free State province, South Africa. The two areas where pump test data were obtained is indicated in the bottom left hand section as A1 and A2 respectively.

Table 4-8 Selected transmissivity values from pump tests conducted at Site A1 and A2.

Borehole ID	T (m2/d)	Borehole ID	T (m2/d)
A1_1	142	A2_4	23
A1_2	0.3	A2_5	45
A1_3	164	A2_6	87
A1_4	130	A2_7	44
A1_5	138	A2_8	49
A1_6	0.7	A2_9	2
A2_1	65	A2_10	269
A2_2	110	A2_11	58
A2_3	42		

Considering the pump test data from Table 4-8 a clear spread of transmissivity values for the respective sites can be observed. Site A1 has transmissivity values which range from 164.00-0.30 m^2/d, if a regional estimate of the transmissivity value for this area is based on the above observations a harmonic mean value of 1.25 m^2/d would be obtained. Scale the value with thirty percent would nominally increase the value to 1.63 m^2/d. This would result in an estimate well below that determined by the GRDM and climate change model. Although the climate change model gave an approximation which is in the same order. Site 2 has considerably more data for the area and the transmissivity values range from 269.0-2.00 m^2/d. If it is considered on its own a harmonic mean value of 15.95 m^2/d is obtained and the scaled value would be 20.74 m^2/d. The value calculated is three times greater than the climate change model but almost exactly the same as that of the GRDM model. Finally, considering all the data the for the two test sites a harmonic mean transmissivity value of 3.10 m^2/d is calculated that approximates the climate change model values.

It is clear from the above that the number of samples considered plays a critical part in the evaluation of the regional transmissivity of the area. Secondly, the inclusion of pump test data with low transmissivity values is critical in determining a balanced regional bulk flow value. As noted previously a quarter of the values obtained should represent the matrix or host formation to give an effective reflection of the regional transmissivity.

4.5.2 Campus Test Site – The Riemann Evaluation

The campus test site has featured in a number of studies in which the Institute for Groundwater Studies has taken part in over the last four decades. The site has been developed with the sole purpose of investigating hydrogeological properties in Karoo type aquifers (Botha et al., 1998, Dennis et al., 2010), as such a number of boreholes have been drilled in a relatively small area (Figure 4-10).

One investigation by Riemann (Van Tonder et al., 2001, Riemann, 2002) probed the influence of fracture zones on the observed transmissivity values obtained. A variety of physical methods were employed, i.e. slug, multi-rate, constant head and constant discharge tests. The results were analysed by various techniques and transmissivity values were reported for both the fracture and the surrounding matrix material.

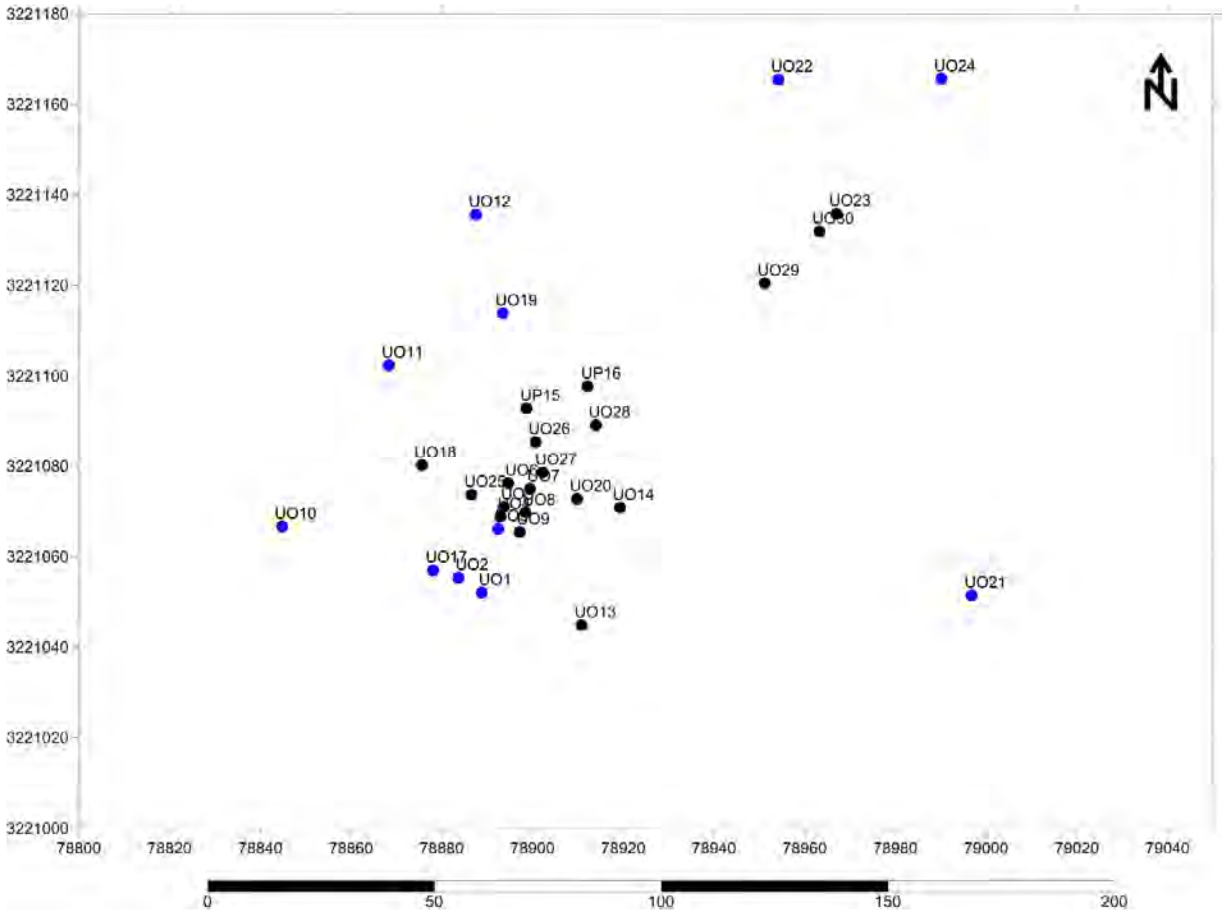

Figure 4-10 Campus test site with borehole locations shown in the map. Blue dots indicate boreholes located in the matrix and black dots represent boreholes that intersect the bedding plane fracture zone (Riemann, 2002).

It was stated in Riemann (Riemann, 2002) that two types of models are typically applied in fractured-rock aquifers which are the single fracture model (Huitt, 1955, Snow, 1965, Wilkes, 1999, Witherspoon et al., 1980) and the double porosity model (Moench, 1984, Moench, 1988, Dougherty and Babu, 1984). It could be deduced from the discussion that each of these models suffered from specific types of shortcomings, most notably an accurate description of the fracture geometry and the interconnectivity network (Riemann, 2002). Due to these deficiencies, porous media methods are still commonly applied in fractured and dual porosity medium aquifers (Kruseman and De Ridder, 1991). To overcome these weaknesses in the analytical models, a numerical model was used to evaluate the aquifer system and determine an approximate transmissivity value (Chiang and Riemann, 2001).

The campus test site is underlain by a series of mudstones and sandstones from the Adelaide Subgroup (Beaufort Group). Mapping of geological outcrops around the campus site reveals the existence of extensional and shearing fractures. The most dominant type of fractures in these sediments are sub-horizontal bedding plane fractures, orthogonal and diagonal fractures with dominant north-west, north-east and east-west trends (Riemann, 2002).

A dominant black shale layer at approximately thirteen meters below surface level forms a low permeable layer that separates the top mudstone and bottom sandstone layers from each other. The three aquifer systems are present at the campus test site. The top phreatic aquifer occurs within the upper mudstone layers of the campus test site. This aquifer is separated from the middle and main aquifer, which occurs in a sandstone layer between eight and ten meters thick, by a layer of carbonaceous shale with a thickness of half a meter to four meters thick (Dennis et al., 2010). The bottom aquifer occurs in the mudstone layers which are more than one hundred meters thick and which is in turn underlain by a sandstone unit (Botha et al., 1998, Riemann, 2002). In each of the boreholes tested in this case study (UO23, 26, 27, 28, 29, 30) the fracture zone occurred from twenty to twenty four meters below surface. The aperture of the fracture was estimated to be in the order of two millimetres (Riemann, 2002), it should be highlighted that calcification does occur in these fracture zones. Thus the aperture would vary significantly in the area due to the calcification of the fractures zone as well as the irregular distribution of the horizontal bedding plane fracture.

Pump test data analysis of the tested boreholes (UO5, 23, 26, 27, 28, 29, 30) were analysed by means of the Cooper-Jacob method (Kruseman and De Ridder, 1991) as well as a numerical model (NWMP, 2006, Barker, 1988). During the investigation of Riemann boreholes UO5 and UO26 were the main discharge points respectively. Initially none of the boreholes were equipped with piezometers, packers or equipment which could cause preferential flow. This scenario represented the first set of data points. Subsequently, piezometers were installed which focused either on the fracture zone or the matrix formation. In this instance this represents the second scenario in which preferential measurements are made.

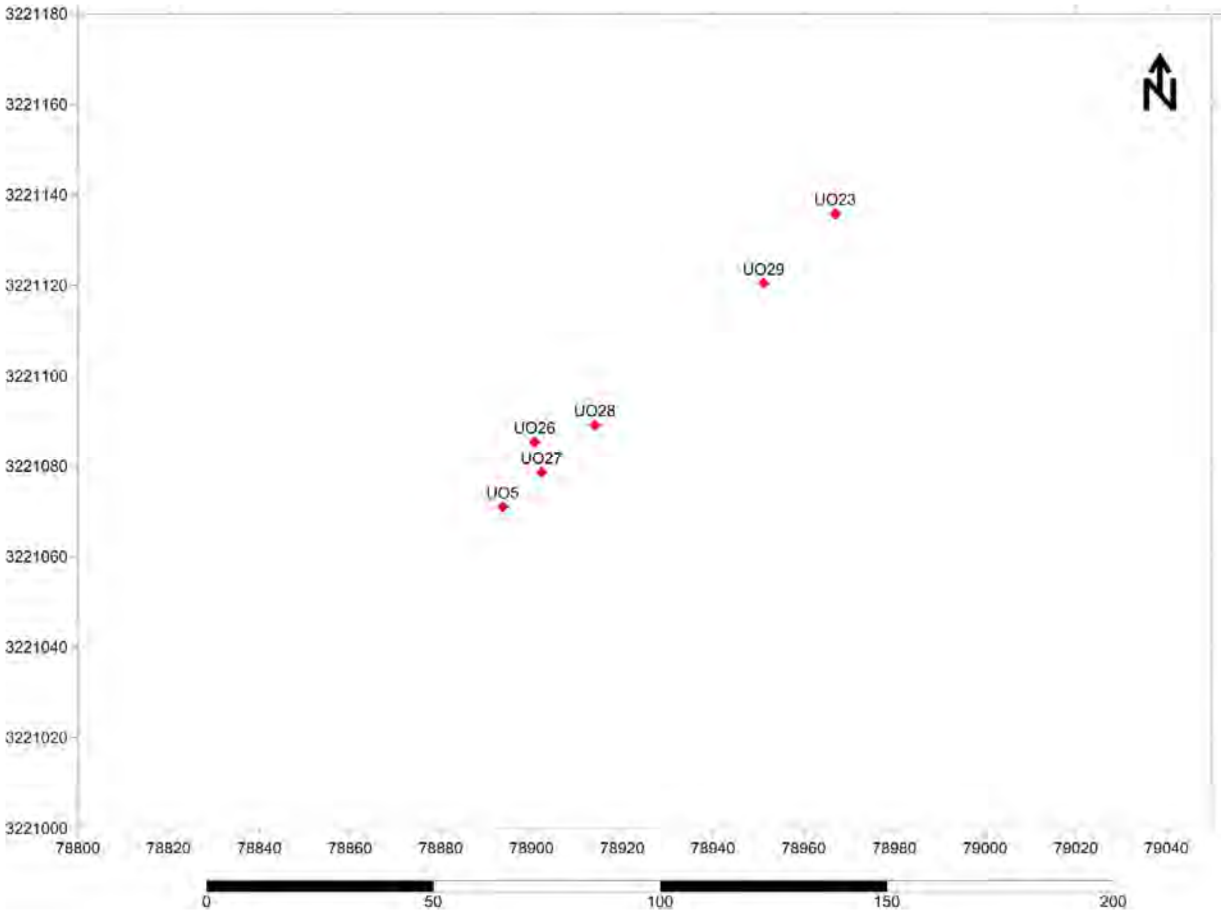

Figure 4-11 Pump test observation points at the campus test site (Riemann, 2002).

During the initial test phase (no piezometers) the following transmissivity values were obtained for the total aquifer system and the fracture zone (Table 4-9). It is interesting to note that the transmissivity values for both the Cooper-Jacob and Theis methods are nearly similar for the matrix while the estimated fracture zone transmissivity is nearly an order larger. The pump test was performed over a thirteen hour time period with a constant discharge rate of 0.71 l/s. The boreholes were monitored continuously throughout the test and data fitted to obtain values reported in Table 4-9.

Considering the preceding results obtained, it is expected that the transmissivity of the country rock or matrix formation should be less than 10 m^2/d. However since pump tests represent the mean value of the total aquifer system, the effect of a mean value between these two extreme values in the aquifer is observed. The result is that although a high transmissivity zone exists, the low transmissivity zones eventually control the bulk flow parameters in the aquifer.

Table 4-9 Representative transmissivity values from Riemann (Riemann, 2002) for the September 2000 data set.

Borehole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO26	10.6	10.0	Discharge point
UO27	10.6	10.0	T-values (fracture zone + matrix)
UO28	10.5	10.0	
UO29	10.7	11.0	
Fracture	755.0		Cooper-Jacob II for fracture zone

Similar results were obtained if UO5 was the abstraction point, only a small change in the estimated total transmissivity could be observed. However a significant change is observed in the projected transmissivity of the fracture zone (Table 4-10). This could most likely be ascribed to the sensitivity of the Cooper-Jacob II method to observation borehole data in calculating this value.

Table 4-10 Representative transmissivity values from Riemann (Riemann, 2002) for the September 2000 data set.

Borehole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO5	13.4	13.0	Discharge point
UO27c	13.8	14.0	T-values (fracture zone + matrix)
UO23c	13.7	13.0	
Fracture	534.0		Cooper-Jacob II for fracture zone

A numerical model was constructed in the previous study (Riemann, 2002), the model was calibrated by means of inverse modelling techniques. The estimated transmissivity values for the matrix and fracture zone was 9.5 m^2/d and 720 m^2/d, respectively. The obtained values are similar to those calculated from the respective pump tests. Interestingly, if the harmonic mean of the two values is taken a mean value of 18.8 m^2/d is obtained which compares favourably with the transmissivity values obtained in Table 4-10.

The second constant discharge test with installed piezometers on the Campus Test Site was conducted at borehole UO5 with a discharge rate of 2 l/s for a period of 24 hours. The piezometers in boreholes UO23, UO27, UO28 and UO29 were used to observe the water level in the fracture zone and the rock matrix, respectively. The water levels in boreholes UO6, UO7, UO20, UO26 and UO30 were also measured during the period of the test. The drawdown behaviour in the fracture-piezometers and the boreholes (Table 4-11) are similar to the previously reported values in Table 4-10. This would indicate that although piezometers were installed only to observe the fracture zone with a high transmissivity value, the influence of the matrix can be detected even in these analysis (Riemann, 2002). Thus, the observed transmissivity value from the pump test is an mean value of both the fracture zone and the formation matrix transmissivity value.

Table 4-11 Results of the evaluated aquifer parameter for the constant rate test UO5, July 2000, obtained from the fracture-piezometers.

Bore-hole	Cooper-Jacob	Theis	Comment
	T (m^2/d)	T (m^2/d)	
UO5	10.6	9.0	Discharge point
UO27c	10.7	11.0	T-values (fracture zone + matrix)
UO28c	10.6	11.0	
UO29c	10.6	11.0	
UO23c	10.6	10.0	
All BH	743.0		Cooper-Jacob II

In all instances in the above discussion the inclusion of bedding plane fracture did not substantially changed the observed transmissivity value. The difference between pump testing an uncased borehole, which results in an average transmissivity value of both the fracture zone and the matrix, and the use of piezometers to exclude the matrix, does not reduce the influence of the matrix formation on the obtained transmissivity value. This would indicate that the fracture network is not as extensive and that the water derived from the aquifer is supplied by the formation which has a higher storativity value than the fracture. Riemann (Riemann, 2002) reported various analysis techniques and experimental methods to evaluate pump tests results, however the transmissivity of the host rock or matrix significantly reduced the calculated transmissivity value. It is most likely that the host rock or matrix has a lower transmissivity value (5-8 m^2/d) than that reported by research on the campus study site (Riemann, 2002).

4.6 Conclusion

In this chapter the effect of random selection on regional hydraulic conductivity has been presented, the effect of sample size and mean value determination method has been discussed. The harmonic mean consistently underestimated the regional hydraulic conductivity; however this method was not prone to sample size and outlier values. In a second approach random biased sampling was done on the regional map, showing the effect of directed drilling programs on the hydraulic conductivity values obtained. The effect of biasing was clearly demonstrated in the mean values obtained and the harmonic mean gave the most sensible results of all the mean values. It should be highlighted that the more biasing that was introduced into the system, the more unlikely it was that an actual mean value for the regional hydraulic conductivity could be determined.

Since a purely theoretical approach was taken in the previous two sections and evaluation of documented average field data was considered. The field data contained three different mean values, i.e. low, middle and high hydraulic conductivity estimates. It was observed that the low hydraulic conductivity values had a remarkable similarity with the theoretical data sets of the previous sections. Nonetheless the higher hydraulic conductivity values of the high estimate indicated that the higher hydraulic conductivity zones are more than three orders greater than the less conductive zones.

Finally, comparisons between transmissivity mean value models and actual field data were done at a test site. The number of boreholes that were required to calculate a representative mean transmissivity value in an area depended on the number of low yielding boreholes that were included. The influence of distance on the transmissivity was also given with the two local sites in the area providing two different transmissivity mean values that differed by more than three times.

This section clearly illustrated that to determine a mean value for an area that is representative of a region; low transmissivity boreholes should be included in the evaluation. If this is not done then the mean value obtained is skewed and represents only the upper stratum of transmissivity or hydraulic conductivity values. This in effect has little use in estimating the regional mean transmissivity or hydraulic conductivity of an area.

Chapter 5 Discussion

5.1 Introduction

In this section the preceding chapters' observations will be combined and discussed in such a manner that a sensible conclusion about transmissivity or hydraulic conductivity can be determined. In general two flow systems are of interest to hydrogeologists. In the first instance (Figure 5-1 A) there is no stress applied to the aquifer (natural flow) and the second depicts a convergent flow field (Figure 5-1 B) when groundwater is abstracted from the aquifer. Natural flow is of interest when an estimate of groundwater fluxes towards, says a river, is necessary. In contrast in the instance of the convergent case, the interest is to estimate the impact of abstraction on the water level or to determine a sustainable yield for a borehole or well-field. Figure 5-1 shows the location of four boreholes and also the location of a waste site. The location of different size fractures is indicated and will be used in the following sections.

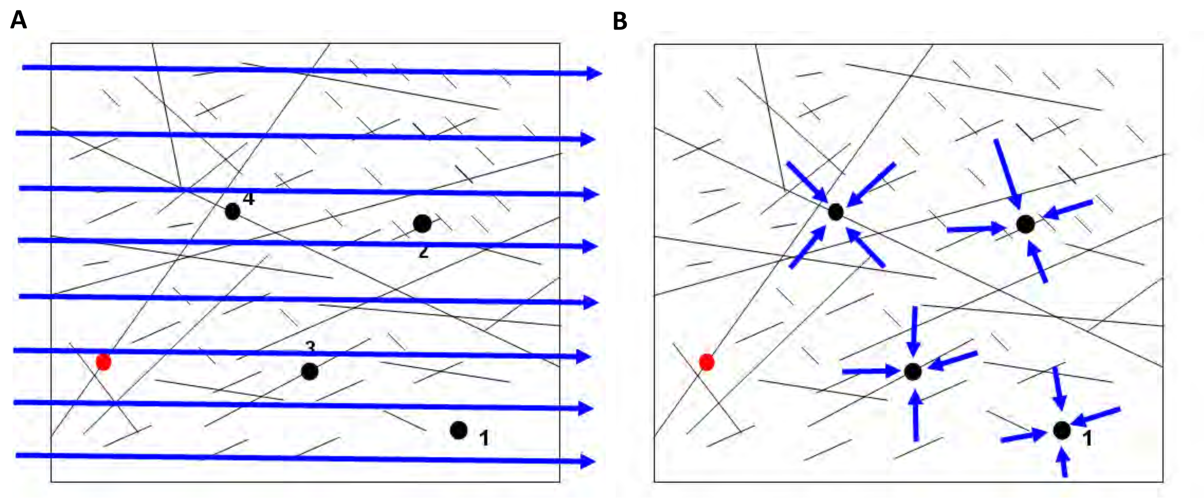

Figure 5-1 Plan views of two typical flow regimes in an aquifer system, (A) natural uniform flow and (B) convergent flow. The black dots represent boreholes and the red dot a pollution source.

In either scenario illustrated in Figure 5-1, the most important parameter to estimate is the hydraulic conductivity (K) or transmissivity (T). For natural flow (Figure 5-1 A), Darcy's Law is used to estimate the flux over the area. In the instance of convergent flow (Figure 5-1 B) the sustainable abstraction rate can be estimated with analytical or numerical methods. In either instance an estimate of transmissivity or hydraulic conductivity is required and the field methods that can be conducted to estimate these values are typically obtained from field measurements (slug tests, packer tests, borehole flow meter tests, borehole dilution tests and pumping tests).

The most commonly used method to estimate the transmissivity value for each borehole is to perform a pump test with the subsequent analyses of the drawdown data with an analytical model. Assuming that the correct model was used and that the estimated transmissivity values are "correct" (Figure 5-1 A). The boreholes located in Figure 5-1 A is placed in such a method that borehole 1 is located in the country rock or matrix, borehole 2 has a limited fracture network, borehole 3 is a more extensive fracture network and finally that borehole 4 is in a well-connected fracture network. Then the following hypothetical values can be assigned to each of the individual boreholes; T_1 = 1; T_2 = 5; T_3 = 20 and T_4 = 100 m^2/d, respectively.

It then follows that:

- Average transmissivity values can be calculated for the area, i.e. arithmetic mean (33.5 m^2/d); the geometric or log mean (10 m^2/d) and the harmonic mean (3.5 m^2/d).

- If a lognormal distribution is assumed, i.e. the distribution follows a bell-shape (Gaussian) curve for the logarithmic transformed values, then the geometric mean is 10 m^2/d with standard deviation of 7.1. Typically stochastic modellers use the geometric mean and variance in their models to address uncertainty.

- Estimating the ratio of [volume of influenced fracture material] **/** [total influenced volume of material]; it follows from visual inspection, that this ratio and thus estimated transmissivity value, will always decrease with affected volume (or time during the pumping test). This ratio is a factor of the ratio between the fracture transmissivity (T_f) and the matrix transmissivity (T_m), e.g. Figure 5-2 C and D shows examples of how the volume of material can change with pumping test time for different transmissivity value of T_f and T_m. Figure 5-2 C illustrates that the volume of fracture material divided by the total volume of material decreases with time and thus that the representative transmissivity equals a decrease in value with time in a heterogeneous aquifer. Figure 5-3 shows the decrease with representative transmissivity with time.

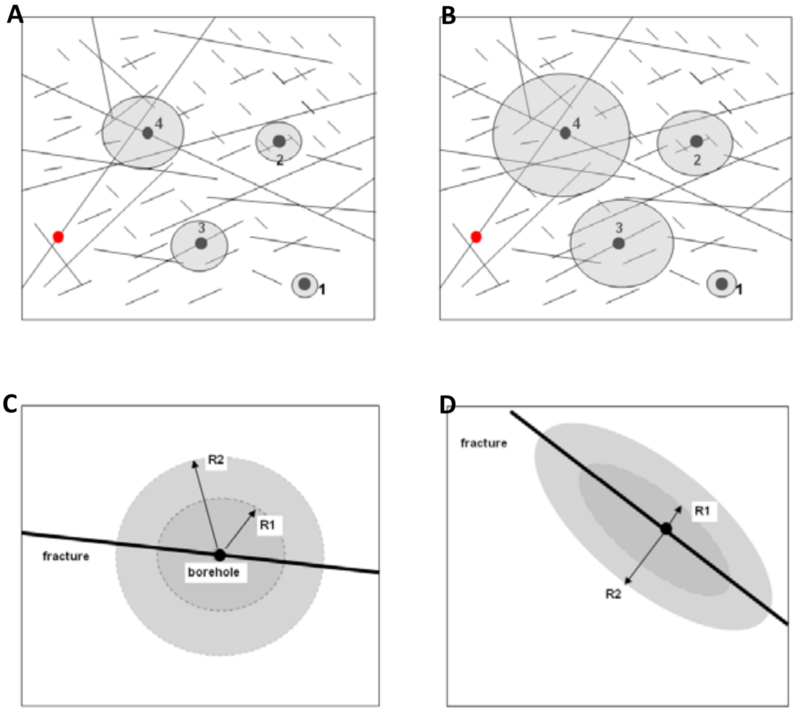

Figure 5-2 Relative two dimensional influence of pumping tests at the 4 boreholes with time, (a) short time and (b) longer time. Just from a practical inspection of the location of the boreholes it can be deducted that $T_1 < T_2 < T_3 < T_4$ and (c) increase of volume material as seen by pumping test for a homogeneous aquifer and (d) for a heterogeneous aquifer.

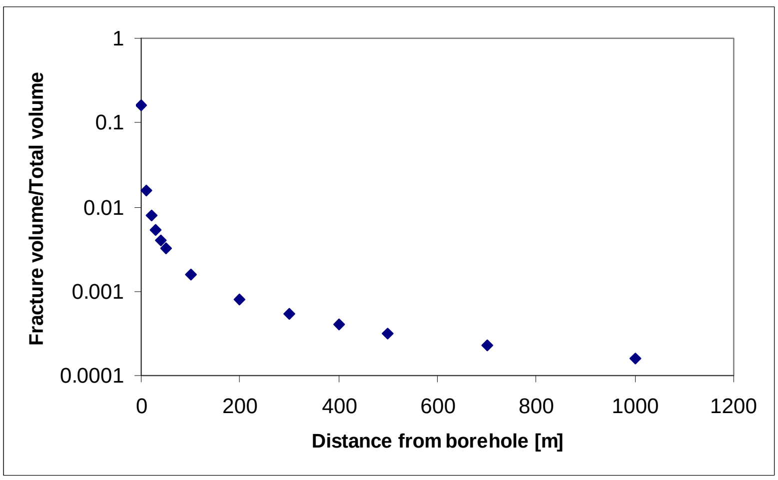

Figure 5-3 Ratio of volume of fracture material to the total volume material with increase from abstraction borehole.

Next consider an oversimplified situation (Figure 5-5) in which a high transmissivity block (T = 1000 m^2/d) was included in an aquifer with a general background transmissivity (T = 20 m^2/d). It is further assumed that the storage coefficient is constant in the area (S = 0.01) and the area is surrounded by a no-flow boundary. Finally the water is abstracted from a borehole situated in the low transmissivity area (T = 20 m^2/d) at a rate of 100 m^3/d (Figure 5-4). The influence of the large transmissivity zone is extremely small on the representative transmissivity value, which implies that the change in drawdown is also extremely small.

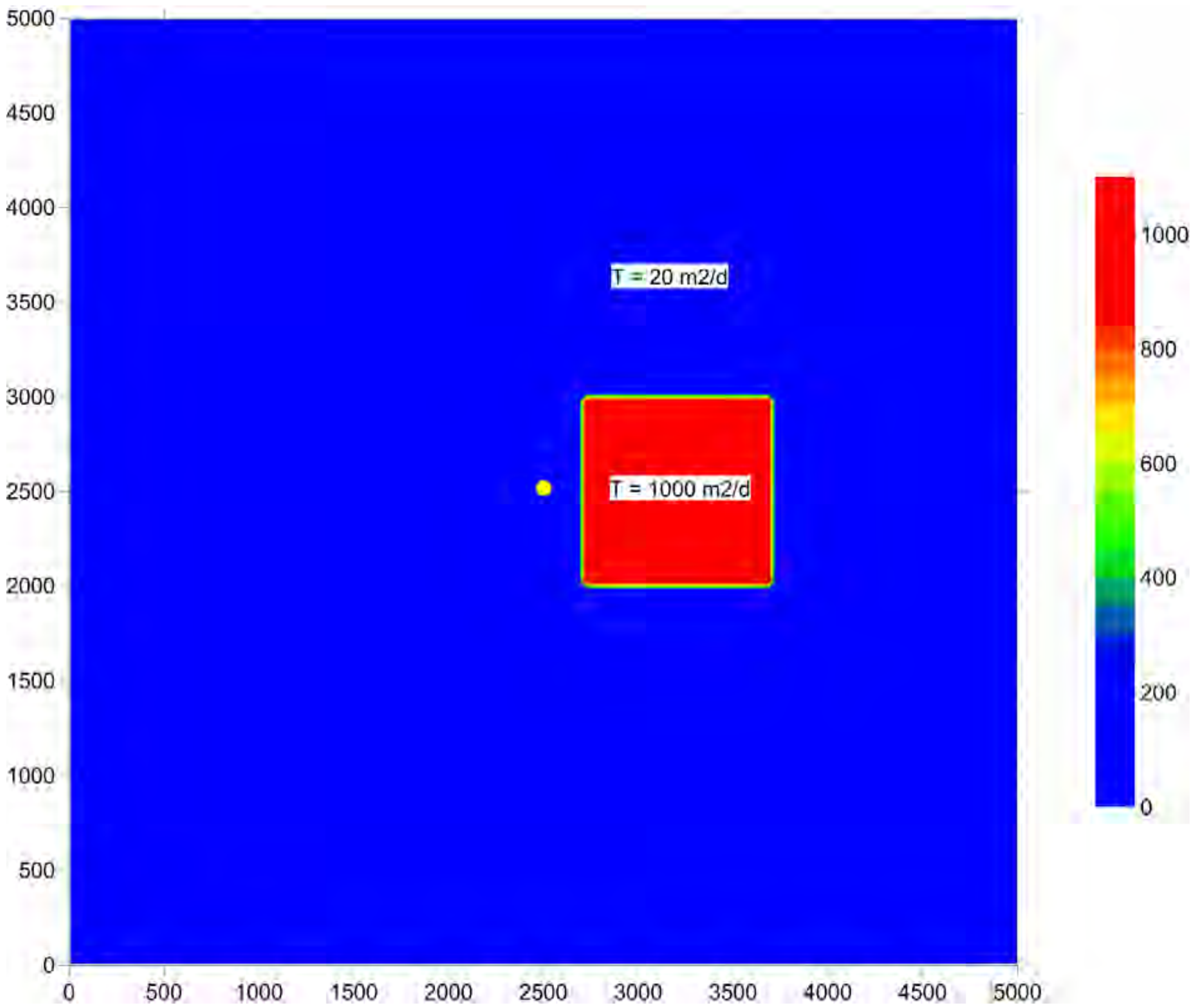

Figure 5-4 High transmissivity zone of 1000 m^2/d situated in an aquifer with transmissivity of 20 m^2/d. The black dot shows the position of a borehole.

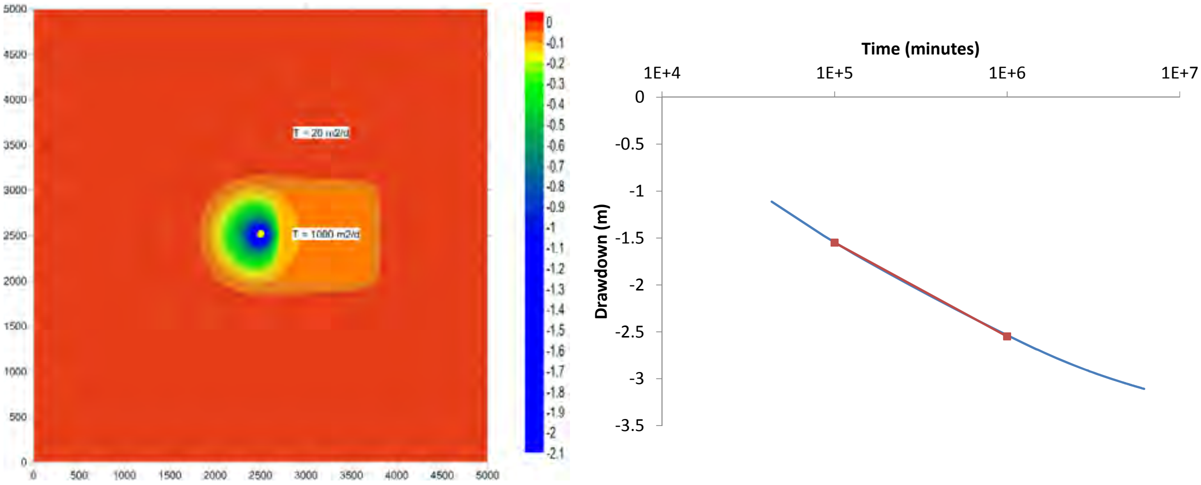

Figure 5-5 The figure on the left shows the expected drawdown in the area after 4320 days. On the right hand side a drawdown curve from which it can be seen that the high transmissivity block has nearly zero influence on the drawdown with an estimated transmissivity (22 m^2/d) value for the whole area. The influence of the no-flow boundary (decreasing water level) can clearly be observed in the latter part of the curve.

Even if four high transmissivity zones (Figure 5-6) are included, the influence on the representative transmissivity is still very small (Figure 5-7).

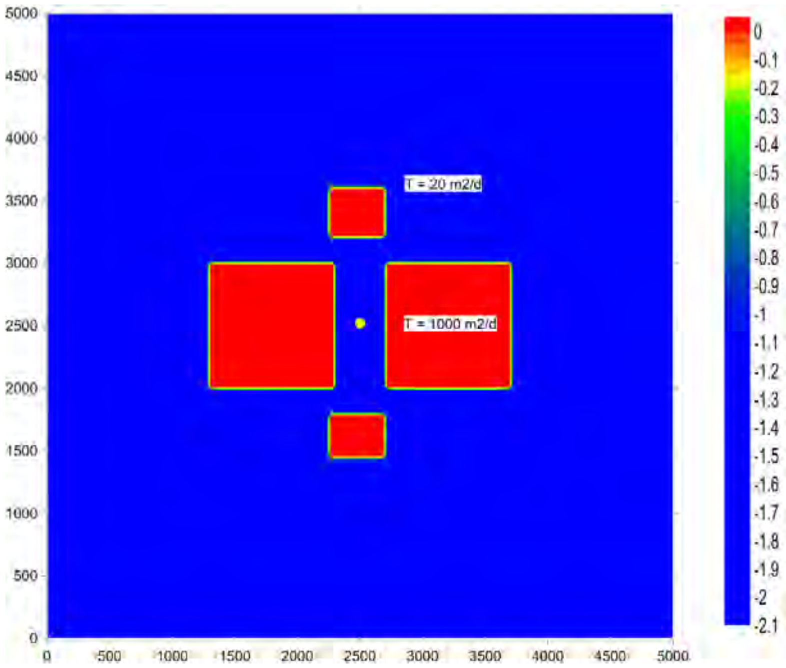

Figure 5-6 Inclusion of four high transmissivity zones in a model area.

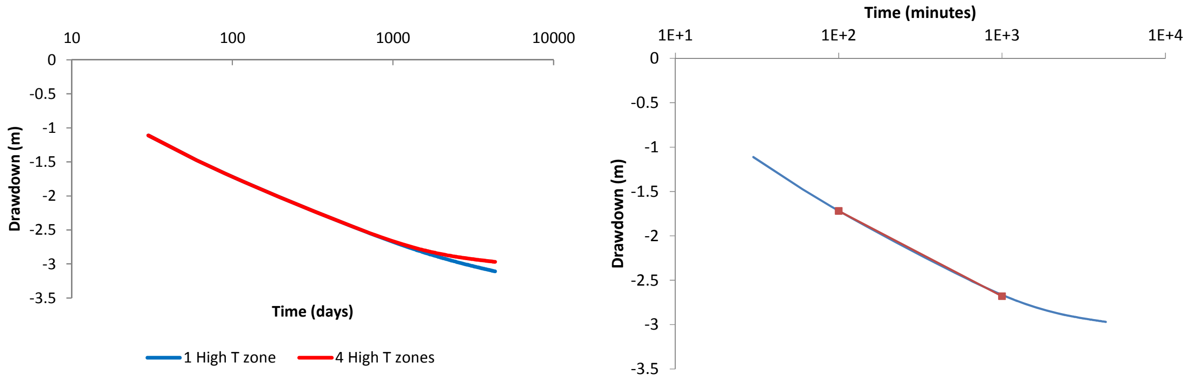

Figure 5-7 Influence of four high transmissivity zones on the drawdown and estimated representative transmissivity value.

If the borehole is moved to inside one of the high transmissivity zones of 1000 m^2/d, the representative transmissivity value is 50 m^2/d, which indicates a dramatic decrease in transmissivity with time (Figure 5-8).

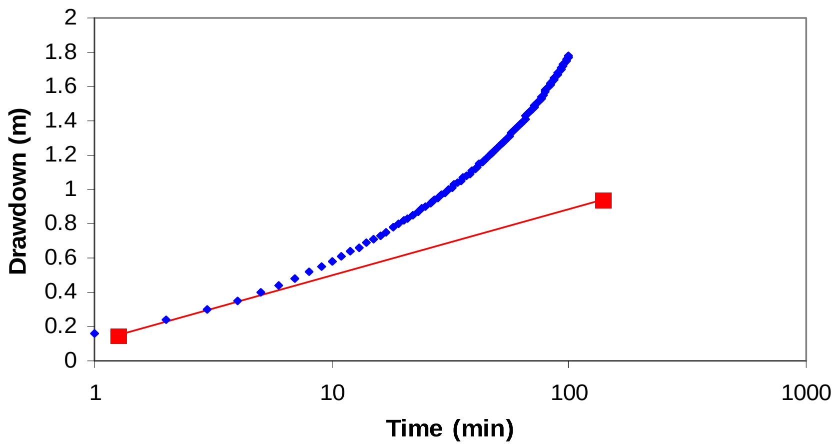

Figure 5-8 Representative transmissivity value = 50 m^2/d if abstraction takes place in one of the four high transmissivity zones of 1000 m^2/d. The influence of the no-flow boundaries can clearly be seen at late times.

In summary the following observation can be made:

- The borehole situated in the lower transmissivity zone of 20 m^2/d results in the following averages being observed for the geometric mean (20, 1000) = 141 m^2/d and harmonic mean (20, 1000) = 39 m^2/d, respectively, while the true or representative transmissivity for the area is 25 m^2/d.

- The borehole situated in the high transmissivity zone of 1000 m^2/d results in the following averages being observed for the geometric mean (20, 1000, 1000, 1000, 1000) = 457 m^2/d and the harmonic mean = 92 m^2/d, while the true or representative transmissivity for the area is 50 m^2/d.

- In the cases where abstraction took place in the preceding example, it is evident that even the harmonic mean (39 m^2/d) is larger than the estimated representative transmissivity value (25 m^2/d) of the total aquifer as determined by curve fitting methods.

A question that can be asked is: will the representative transmissivity value differ if the aquifer changed from a confined aquifer to a water table aquifer? To investigate this affect, the horizontal and vertical heterogeneities need to be considered and accounted for in the solution.

5.2 Horizontal Heterogeneity

To illustrate the impact of horizontal heterogeneities on a system, different fracture networks were constructed (Figure 5-9 and Figure 5-10). Fracture characteristics shown on these maps include: fracture length, fracture aperture and fracture connectivity.

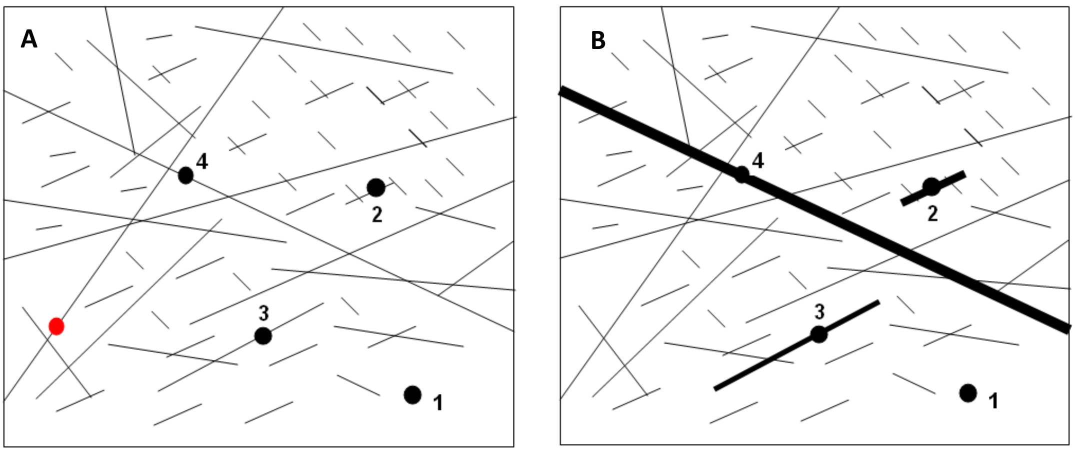

Figure 5-9 (A) Four boreholes drilled at different locations in a 1 km^2 block radius, with the red dot indicating the pollution source and (B) the thickness of the lines is representative of the transmissivity (T) values observed within the area. Borehole 1 has the lowest transmissivity value, while borehole 4 the highest transmissivity value.

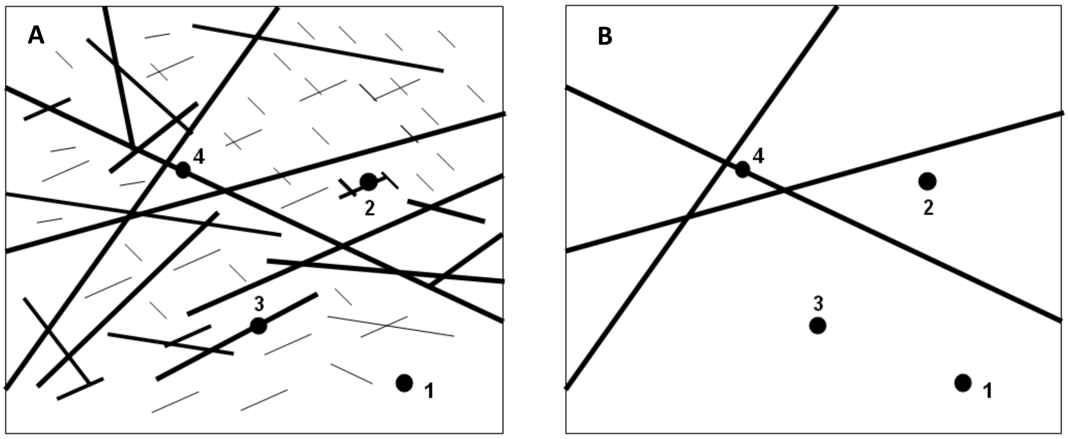

Figure 5-10 (a) Dark lines indicate the variable connectivity-length of the four boreholes in respect to the fractures and (b) fractures connecting the left side of the block to the right side.

A visual inspection of Figure 5-9 and Figure 5-10 yields:

- As the thickness of the lines represents the apertures of the different fractures, each of the boreholes are drilled into a fracture with a different yield and thus a different transmissivity value:
    - Borehole 1 has the lowest transmissivity value because only the matrix was intersected;
    - The transmissivity values of the other boreholes are in the order: $T_2 < T_3 < T_4$;

- Any irrigation farmer will choose to have a borehole such as borehole 4 because of the more favourable fracture characteristics of this borehole compared to the other three;

- The "internal" boundaries of the fracture, i.e. the effective length (extent) of the fracture and the connectivity to other fractures are greater in borehole 4, which makes borehole 4 superior to the other boreholes in terms of sustainability (ignoring for the moment the depth of the main water strike);

- The transmissivity values in the vicinity of each borehole varies with orders of magnitude (aperture thickness);

- If boreholes 2, 3 and 4 are pumped, the representative transmissivity value of each borehole will decrease with time;

- Movement of pollution (red dot) along the different fractures: the fracture that intersects borehole 4 is the most vulnerable to pollution;

- Finally, what will happen if borehole 1 is pumped? Will the representative (equivalent) transmissivity increase with time?

For a geohydrologist, two main questions arise:

(1) Under natural conditions and assuming that flow is from left to right when considering Figure 5-1 and Figure 5-2, what will be the groundwater flux across the outflow boundary? To estimate this flux, the geohydrologist will have to use some kind of average for the block in Figure 5-1 or divide the block into smaller parallel strips normal to the flow direction and estimate the representative transmissivity of each of these strips and then apply Darcy's Law.

(2) Under converging conditions (transient state abstraction from boreholes), what is the representative or average transmissivity value to use to estimate the drawdown in the borehole after a long time?

Before answering these two questions, one has to investigate the effect of the vertical dimension.

5.3 Vertical Heterogeneity

Figure 5-11 shows a borehole that is intersected by two main water-yielding fractures, i.e. Fracture 1 and 2. Both fractures have the same aperture and thus the same transmissivity value (close to the borehole).

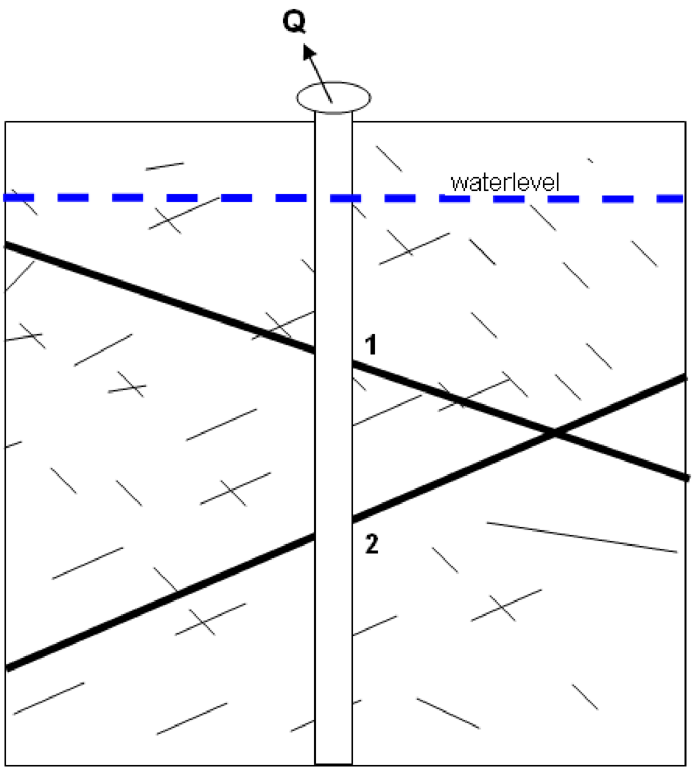

Figure 5-11 Vertical section of borehole that intersects two main water-yielding fractures.

From a visual inspection of Figure 5-11, the following can be deduced:

- If abstraction takes place, fractures 1 and 2 will deliver equal amounts of water to the borehole, thus $T_1 = T_2$ and $T_{total} = T_1 + T_2$ (for early times at least);
- Fracture 1 will be dewatered first and then $T_{total} = T_2$;
- If a farmer has to choose between one of the fractures he would likely choose fracture 2 because it delivers water to the borehole for a longer time than fracture 1. Secondly more drawdown is available;
- It is thus clear that transmissivity (representative) will always decreases with time.

5.3.1 Different Scales

Heterogeneity, as expressed as transmissivity (T) or hydraulic conductivity (K), is the single most significant feature in fractured-rock aquifers but unfortunately it can vary with orders of magnitude (even more than 12 orders) over very short distances (Sanchez-Vila et al., 2006). The estimation of a representative hydraulic conductivity which controls the average behaviour of groundwater flow within an aquifer at a given scale has been searched for over the past few decades. It is the goal of up-scaling to be able to move from different types of volume. Typically the scale volumes of interest can be defined as the small (< 0.5 m), intermediate (aquifer test scale) and the large (aquifer) scale systems. By using a representative hydraulic conductivity a parameter needs to be estimated that controls the average behaviour of groundwater flow within an aquifer at a given scale.

Three related concepts are defined**:**

- Effective hydraulic conductivity, which relates the ensemble averages of flux and head gradient;

- Equivalent conductivity, which relates the spatial averages of flux and head gradient within a given volume of an aquifer; and

- Interpreted conductivity, which is derived from interpretation of field data.

In geohydrological studies that involve complex environmental questions, scale inconsistency is very common. The scale at which transport and flow phenomena in the porous media are best described is usually very different from, i.e. larger than the scale at which measurements are made and interpreted, but also very different from the scale (smaller) required for management decisions. When focusing on the modelling of groundwater flow and transport the following spatial scales are often distinguished: (Bierkens and Van der Gaast, 1998):

- The *pore scale* (10^{-6}-10^{-2} m); the scale at which flow and transport through porous media is described in terms of forces and mass fluxes within the fluid phase and the solid phase and between these phases. Groundwater flow for instance is described by the Navier-Stokes equations;

- The *core scale* (10^{-1}-100 m); the scale at which flow and transport are described in terms of continuity equations and simplified flux equations such as Darcy's law and Fick's law. The minimum scale at which these simplified flux equations are valid is called the representative elementary volume (REV). This is exactly the scale at which measurements of hydraulic properties are performed on samples from drilling cores;

- The *model block scale* (10^{1}-10^{2} m); the scale of blocks or elements of numerical flow and transport models;

- The *local scale* (10^{2}-10^{3} m); the scale at which groundwater flow and -transport is considered as three-dimensional. Examples of local scale groundwater problems are pollution and remediation studies around waste sites and the assessment of travel time distributions in protection areas around drinking water wells;

- The *regional scale* (horizontal dimension 10^{3}-10^{5} m); the scale at which the subsoil is divided into permeable layers (aquifers) and less permeable layers (aquitards). The groundwater flow through aquifers is considered to be mainly horizontal and the flow through the aquitards mainly vertical. The lowering of water tables due to groundwater abstraction is often modelled at this scale. The assumption that the flow in aquifers is predominantly horizontal is valid when the horizontal extent of the model domain is much larger than the vertical extent. This is usually the case for large model areas (horizontal 10^{3}-10^{5} m) in alluvial and marine deposits (vertical extent of the model domain 10^{2} m).

There is however one further problem with scaling an average value across all of these scale factors does not exist. If only the macroscopic scale is considered, a scale that is large in comparison with the grain or pore size of the porous medium, the flow through the medium can be defined as a continuous phenomenon through space. In general it can also be assumed for dual porosity aquifer systems that the property being observed is also dependent on location.

The question of representative values has in part been described in the late 1950's (Hubbert, 1956) in which a differentiation between macroscopic and microscopic scales were made. At the macroscopic level the scale is large compared to the grain or pore size of the porous medium and is in effect a continuous flow phenomenon in three dimensions. Since the properties vary with location it was proposed to quantify the value at the macroscopic level. In the paper it was illustrated by using porosity as a macro observable parameter (Hubbert, 1956).

The following is a direct quote from the paper but due to its remarkable applicability to the problem at hand it was deemed necessary to reproduce it here (Hubbert, 1956).

"Suppose that we are interested in the porosity at a particular point. About this point we take a finite volume element ΔV, which is large as compared with the grain or pore size of the rock. Within this volume element the average porosity is defined to be

$$\overline{f} = \frac{\Delta V_t}{\Delta V}$$

where ΔV_r is the pore volume within ΔV. We then allow ΔV to contract about the point P and note the value of $\overline{f}$ as ΔV diminishes. If we plot $\overline{f}$ as a function of ΔV (Fig. 5), it will approach smoothly a limiting value as ΔV diminishes until ΔV approaches the grain or pore size of the solid. At this stage $\overline{f}$ will begin to vary erratically and will ultimately attain the value of either 1 or 0, depending upon whether P falls within the void or the solid space."

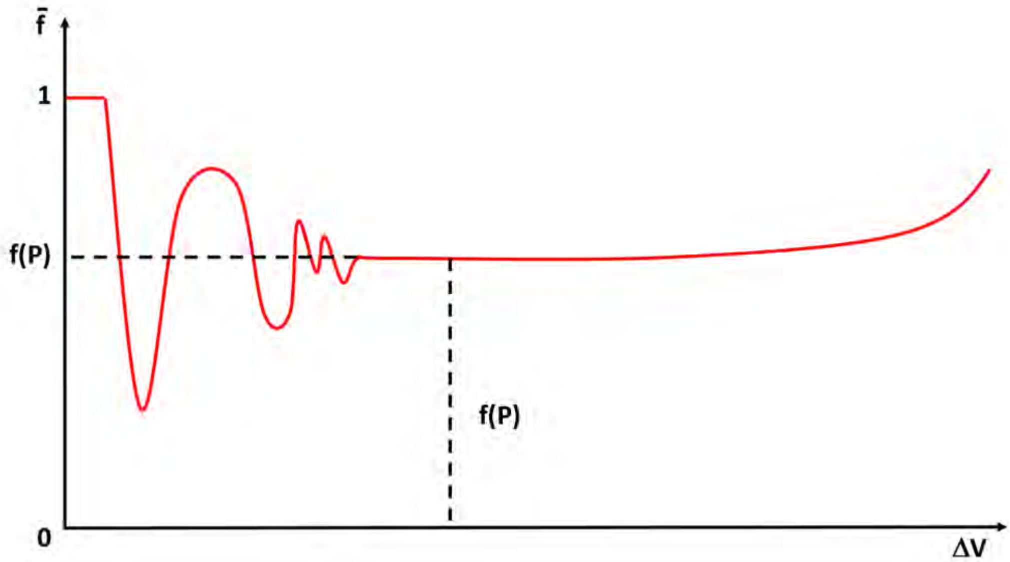

Figure 5-12 Method of defining point values of macroscopic quantities illustrated with porosity f (Hubbert, 1956).

"However, if we extrapolate the smooth part of the curve of $\overline{f}$ vs. ∆V to its limit as ∆V tends to zero, we shall obtain an unambiguous value of f at the point P. We thus define the value of the porosity f at the point P to be

$$f(P) = extrap \lim_{\Delta V \to 0} \frac{\Delta V_t}{\Delta V}$$

where "extrap lim" signifies the extrapolated limit as obtained in the manner just described."

In the paper it was stated that similar operations for a point value of any other macroscopic property can be obtained. The proof was extended to include lengths, volumes and areas. In a similar method transmissivity of hydraulic conductivity can be defined.

$$Q(P) = \lim_{\Delta T \to 0} \overline{Q}(\Delta T)$$

$$Q(P) = \lim_{\Delta K \to 0} \overline{Q}(\Delta K)$$

5.3.2 Fracture Connectivity

"The estimation of the bulk hydraulic conductivity of the fractured rock at the Mirror Lake site over the increasingly larger dimensions indicate that the connectivity of the fractures is important in characterizing the bulk hydraulic properties of the rock. If the highest conductivity fractures are not connected over any significant distance, then the hydraulic conductivity of the fractures that connect the more transmissive fractures are the ones that will control the bulk hydraulic properties of the rock at large dimensions. At other fractured rock sites with different geologic controls on fracture properties, the connectivity of fractures may yield different trends in the estimates of the hydraulic connectivity over increasingly larger physical dimensions." – Figure 5-13 (Shapiro et al., 2007)

During the investigation Berkowitz (Berkowitz, 1995) aimed to describe the significance of fracture network connectivity. This was based on the observation, that seemingly dense fracture networks could have little or no interconnectivity. This would result in highly fractured rocks having small transmissivities and hydraulic conductance capabilities. In his work he further described what he called the backbone of a fracture network. This can be described as being the highways on a high definition road map. If you take away all the urban and industrial dead ends, you are left with the highways that connect point A to point B. The highways, backbone, are the fractures that represent the interconnected fractures that allow the movement of water within an aquifer.

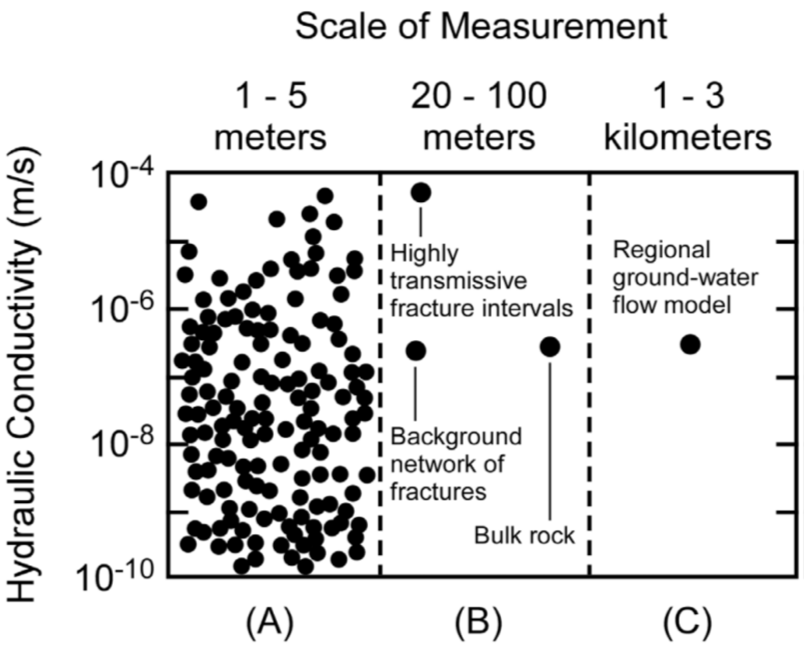

Figure 5-13 Hydraulic conductivity in the fractured bedrock of the Mirror Lake watershed and its vicinity as estimated over increasingly larger physical dimensions from (A) discrete-interval, single hole hydraulic tests, (B) cross-hole hydraulic tests, and (C) regional ground-water flow modelling (Hsieh, 1998).

5.4 Estimation of Representative Transmissivities

In practice, two methods are primarily used to estimate representative transmissivity (or hydraulic conductivity values), namely the 1) stochastic models (expected average value and variance) and the 2) deterministic models (analytical and numerical). Oil engineers are in favour of deterministic methods, while groundwater people prefer the stochastic approach. In the stochastic approach, the representative transmissivity value is known as the effective transmissivity and in the deterministic way is known as the equivalent transmissivity.

The geostatistical way (stochastic modelling) is used and advocated by a group of (mainly) groundwater experts and includes names like Neuman, Dagan, de Marsily, Sanchez-Villa, Carrera, Matheron, Journel, Gelhar, Kitanidis, Kinzelbach, Rubin, Freeze, Indelman and Gomez-Hernandez. The deterministic group includes mainly experts from the oil industry like Raghavan, Bourdet and Gringarten. Well-known experts in the groundwater field who supported the deterministic group are Darcy, Theis, Meinzer, Cooper, Jacob, Todd, Bear and Hunt.

As discussed by Dagan (Dagan, 1997), the choice is always affected by how uncertainty is viewed. Deterministic approaches are based on viewing parameters as unique, but uncertain, local quantities at some given scale. Stochastic approaches are based on viewing reality as one among the ensemble of possible spatial distributions (realizations) of the parameter. In the random method the approach has been to model the natural logarithm $Y = \ln T$ of aquifer transmissivity (T) as a statistically homogeneous, multivariate Gaussian random field with a given variance and spatial correlation function.

From a theoretical standpoint, the groundwater community—which has invested a considerable amount of research in stochastic techniques—has provided very important findings on the impact of aquifer heterogeneity on well hydraulics. The main issue was: “What is the significance of estimated parameters of a heterogeneous aquifer when based on the ‘best’ fit between a homogeneous model and measured data?” Another related question was: “Is it possible to infer some statistical parameters, e.g. T, K and S, describing the heterogeneity from an aquifer test?”. The answers have been on one hand disappointing. Inferring the degree of heterogeneity (variance, covariance function) from well testing is extremely difficult unless a large amount of interference tests be conducted (Noetinger and Gautier, 1998, Sánchez-Vila et al., 1999). On the other hand, these studies have shown that the parameters that are identified by well test interpretation can have a clear theoretical meaning: in many cases they are the equivalent transmissivities of the medium that would be obtained by up-scaling under uniform flow (Meier et al., 1998, Indelman, 2003).

Two related hydraulic conductivity parameters are defined as (i) effective hydraulic conductivity, K(eff), (used by the geostatistical group), which relates the ensemble averages of flux and head gradient and (ii) equivalent hydraulic conductivity, K(eq), (used by the deterministic group), which relates the spatial averages of flux and head gradient within a given volume of an aquifer.

Effective parameter estimation is based on averaging over the ensemble of realizations. Thus the definition of effective hydraulic conductivity is obtained from the generalization of Darcy’s law that results from relating the expected values of specific discharge and head gradient

$$\langle q \rangle = -K(eff)\ \nabla \langle h \rangle$$

where the angle brackets indicate ensemble averaging in the probability space of hydraulic conductivity, that is, averaging all the possible head and specific discharge fields that could be obtained with the ensemble of hydraulic conductivity fields. If the value that relates both expectations, K(eff), exists, it is a second-order tensor and is called the effective hydraulic conductivity. The tensorial nature of K(eff) may be a consequence of statistical anisotropy, boundary conditions, or domain geometry.

Equivalent parameter estimation relies on averaging in physical space. The resulting representative parameter is termed equivalent. Alternative terminologies that are often employed are block-averaged or volume-averaged parameters. Several authors refer to these as up-scaled parameters, since these are usually representative of some average behaviour observed over blocks larger than the support scale. Along these lines an equivalent hydraulic conductivity, K(eq), can be defined by means of an averaged version of the effective K definition given previously:

$$\mathbf{q} = -K(eq)\ \nabla \mathbf{h}$$

Here the letter in bold, refers to averaging in the spatial (volumetric) sense.

Considerable uncertainty has been voiced regarding the potential scale-dependence of hydraulic conductivity.

- Data from many sources seem to imply an increasing hydraulic conductivity, *K*, with increasing scale of measurement (Brace, 1984, Bradbury and Muldoon, 1990, Schulze-Makuch, 1996, Sánchez-Vila et al., 1999).
- Other apparently solid, theoretical derivations (Hunt, 2006) as well as numerical simulations (Hunt, 2006, Paleologos et al., 1996) appear to imply the contrary. Conclusions from some of the experiments have been contested for various reasons, such as whether the borehole intervals were sufficient, (Butler and Healey, 1998) whether turbulence was present in the bore-holes (Lee and Lee, 1999) or whether juxtaposition of different measurement methods led to interpretive errors (Zlotnik et al., 2000). Thus it is possible that some of the experimental results are spurious.

As demonstrated earlier a geohydrologist is usually interested in two types of flows, namely (a) natural mean parallel flow and (b) transient convergent flow (abstraction).

5.4.1 Mean Parallel Flow: Representative Transmissivity Value

5.4.1.1 Stochastic Model to estimate K(eff)

For the stochastic model, it is assumed that the logK value is distributed Gaussian. In stats the average of log K is known as the geometric mean.

The equivalent hydraulic conductivity of a flow system is that which would give the same flow rate with the given boundary conditions if the soil were uniform. It can be shown (Youngs, 1983) that the equivalent hydraulic conductivity *K(eq)* of a rectangular block of soil with variable hydraulic conductivity *K(x,* y, z) when flow takes place through it as a result of equipotentials being applied at two opposite faces, lies between the arithmetic mean K_a and the harmonic mean K_h of the conductivities of the soil elements making up the block. Thus

$$\mathbf{K_h < K(eff) < K_a}$$

The geometric mean value K_g is often taken as the value to be used in flow computations, since $\mathbf{K_h < K_g < K_a}$.

Alternatively the following equation can be used (Youngs, 1983)

$$\mathbf{K(eff) = [(K_a)^2\ K_h]^{1/3}}$$

For lognormal hydraulic conductivity fields:

$$\mathbf{K(eff) = K_g\ exp[(variance\ of\ K)/2]}$$

Assuming that the univariate distribution of K is lognormal, another alternative equation is:

$$\mathbf{K(eff) = (K_a)^{n-1/n} (K_h)^{1/n}}$$

Where n=flow dimension

5.4.1.2 Deterministic model to estimate K(eq)

Equivalent conductivity is heavily influenced by boundary conditions. The most restrictive bounds to equivalent conductivities are the same than for the stochastic method:

$$\mathbf{K_h < K(eq)_{ii} < K_a}$$

Where ii = deviation direction from parallel flow.

This implies that the equivalent value for any direction is bounded between the harmonic and the arithmetic mean of the local K values within the block. The difference between this equation and the effective K one, is that now averages are taken in the physical rather than in the probability space.

A combination of the limits of (100) can be used to obtain a back-of-the-envelope calculation of K(eq) in a given block. The idea is based on an extension of Matheron's equation (20a) to the real space, thus writing

$$\mathbf{K(eq) = (K_a)^{n-1/n} (K_h)^{1/n}}$$

While this equation is formally equal to the geostatistical one, the conceptual difference is, again, that averages are now taken within the block. Gue´rillot et al. [1990], used a simple combination of the K_{min} and K_{max} values for K(eq):

$$\mathbf{K(eq) = (K_{min})^{0.5} (K_{max})^{0.5}}$$

Since the work of Journel et al. [1986] many authors have used a combined analytical-numerical method to estimate equivalent parameters based on a power-averaging formula. Gomez Hernandez and Gorelick [1989] present the discrete version of a spatial average p norm as an estimator of K(eq) for very large blocks (large enough so that the equivalent value can also be viewed as a pseudo-effective value), K_b according to

$$K(eq) \approx K_b = \left[1/N \sum_{i-1}^{N} K_i^p\right]^{1/p}$$

Where N = number of elements within the large block.

5.4.2 Convergent Flow: Representative T-Value

5.4.2.1 Geostatistical model: 2D

A common approach has been to model the natural logarithm Y = ln T of aquifer transmissivity (T) as a statistically homogeneous, multivariate Gaussian random field with a given variance and spatial correlation function. There are many cases when a two-dimensional approach can be considered a proper modelling choice.

Dagan [1989] considered a prescribed flow rate boundary condition at the well and by simple asymptotic considerations concluded that if a well in an unbounded aquifer pumps at a constant deterministic rate, then the T_{eff} near the well is equal to T_H, while T_{eff} far from the well is equal to its mean uniform flow counterpart, i.e. the geometric mean, T_G. Contrariwise, if hydraulic head is prescribed at the well, the pseudo-effective transmissivity at the well should become equal to TA, thus recovering the near-well limit of Matheron [1967]. These limits have been verified by subsequent studies and were complemented by closed form solutions for the space-dependent pseudo-effective conductivity for unbounded [Indelman and Abramovich, 1994b] and bounded [Sanchez-Vila, 1997; Riva et al., 2001] domains.

An important assumption for the geostatistical model is that the T(fracture) and T(matrix) must not differ with order of magnitude. Thus, the assumption is that the flow is predominantly radial in the weak sense. The weak sense is crucial, since this assumption clearly does not hold point wise in the vicinity of the borehole, wherein flow is controlled by the local T values. Away from the borehole, the hydraulic head gradient varies slowly in space, mainly driven by the distance to the borehole.

In the geostatistical model the effective transmissivity is thus the geometric mean and the transmissivity grows always with support volume, which is rarely correct as real pumping test data has shown.

5.4.2.2 Deterministic model: 2D

Cardwell and Parsons (1945) method considers steady state, radial flow to a well in an aquifer in the form of a circle. The method shows that the equivalent transmissivity (T_e) is bounded by the weighted and harmonic means of the transmissivity of different aquifer regions.

$$\frac{\int_\Omega \frac{dx}{r^2(x)}}{\int_\Omega \frac{dx}{T(x)r^2(x)}} \leq T_e \leq \frac{\int_\Omega \frac{T(x)dx}{r^2(x)}}{\int_\Omega \frac{d(x)}{r^2(x)}}$$

Where Ω = region of aquifer bounded by the outer boundary and the well bore

$r(x)$ = distance from point **x** to the central well, and

$d(x) = r(x)drd\theta$ is an infinitesimal area

Hence they considered a deterministic approach to estimate the transmissivity of a porous medium.

Figure 5-14 illustrates (Sanchez-Villa, 2006) the general behaviour of effective and equivalent transmissivity estimates with the stochastic and deterministic methods.

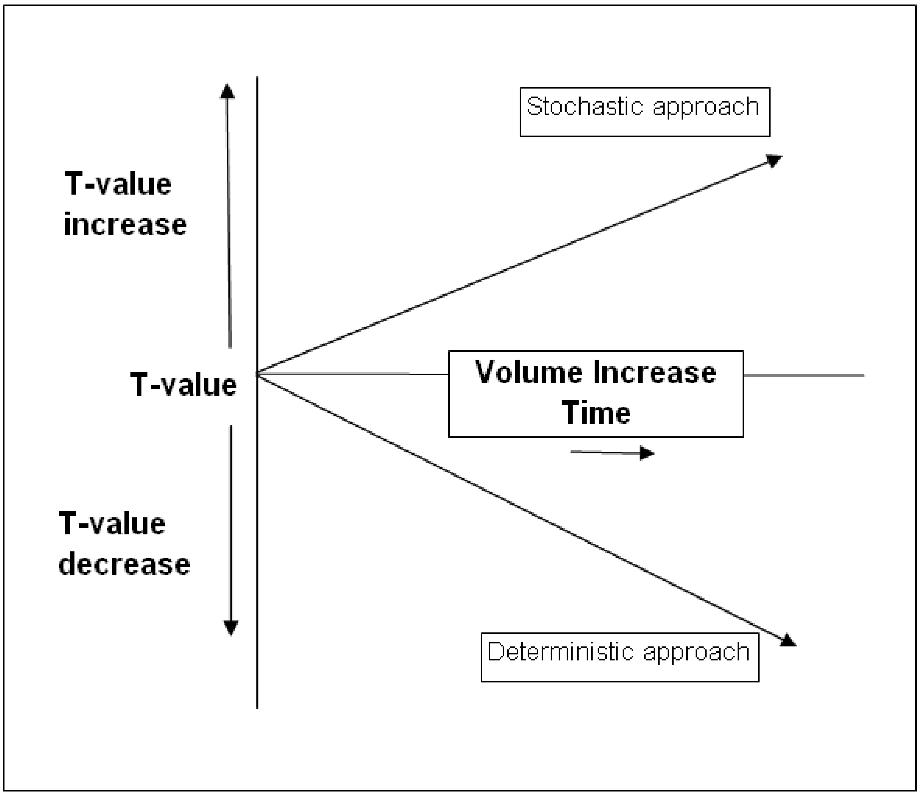

Figure 5-14 General behaviour of effective (stochastic) and equivalent (deterministic) transmissivity estimates with an increase in aquifer volume.

5.5 Conclusion

In this chapter the impact of estimating the correct transmissivity value was illustrated from the perspective of local and regional points of view. The influence of high transmissivity zones on a pumping borehole in a low transmissivity zone was clearly illustrated in that no real significant effect could be observed. The estimation of regional transmissivity values are in effect dominated by the location of the borehole in the different transmissivity zones and the observed effective transmissivity value obtained. The impact of horizontal heterogeneities and different fracture networks were discussed and the influence these features have on the actual transmissivity value obtained, i.e. the influence of internal boundaries on pump test data. Scale effects were also addressed from a regional perspective, with a focus on apparent scaling and the actual regional transmissivity value which should be obtained.

The estimation of representative transmissivity values were discussed as seen from a stochastic modelling perspective as well as from the deterministic point of view. A comparison between main stream groundwater and oil industry specialists were noted in which both groups share the fundamental training but differ on the methodology of determining the observed transmissivity values. Finally, the discussion was focused on determining a mean transmissivity value and what is actually meant by this average by each group.

In the following section a roadmap of practices will be given as it should be implemented from a South African perspective.

Chapter 6 Way Forward

South Africa has a variety of aquifer systems that range from porous media, fractured hard rock, dolomitic to dual porosity aquifers. By far the most dominant aquifer system in based in the Karoo type sediments which are comprised by heterogeneous formations as well as compositional methods. This in a sense makes South Africa unique in that the water supply to rural communities is drawn from “weak” aquifer systems that are low yielding. In comparison to primary aquifers that are being exploited internationally for water supply to cities. The estimation of regional transmissivity values thus carries an additional burden in that if the regional transmissivity values for an area are overestimated, that inadequate provision would be made for water supply to the local community. This in turn could cause negative economic and social impacts in these effected communities. Thus in effect knowing the regional transmissivity value could give an estimate of extractable volume of water in an area to establish a continuous water resource. It should be kept in mind that management of aquifer systems not only depends on bulk flow parameters but also on managing the water level in these systems.

In essence using geostatistical methods are not advised if regional transmissivity values are required from a South African perspective. The reason behind this statement is that the distribution of transmissivity values in an area does not follow the basic precepts that are required for these methods to work. In general the values are discontinuous in distribution and statistically skewed. Furthermore the presence of transmissivity areas or points that differ significantly in magnitude, i.e. transmissivity of 1 and 100, can be located within 1 meter from each other. The explanation of this phenomenon is the presence of dolerite dykes, which create baked-fractured zones with exceptionally large transmissivity values compared to the extremely low transmissivity ranges of the surrounding country rock (shales, mudstone and siltstone).

As noted in previous chapters in which different scenarios were investigated, the calculation of a mean transmissivity value for a region is not a simple matter. Firstly, if databases are used a careful inspection of the reported data is required. In general in South Africa it is the practice to only report high yielding boreholes, this in effect only represents anomalies from a statistical perspective and thus makes any attempt at estimating regional transmissivity values impossible. It was found in this study that if approximately twenty five to thirty five percent of reported boreholes in an area was low yielding, an estimated transmissivity value could be obtained that represent the regional transmissivity.

A second major point of concern is the lack of data about low yielding or "dry" boreholes, these boreholes should be at least tested and noted in a database before being sealed it. The information obtained from these boreholes are in effect more valuable to management strategies than most of the current data out in the literature which reports moderate to high yielding boreholes.

Finally, if a mean value is to be calculated it is strongly advised to use the harmonic mean which overemphasises the contribution of the low transmissivity values in a region. This would result in realistic estimates of available water in an area since the host rock dominates the bulk flow parameters in a region over an extended time period of abstraction. To illustrate in part this point, contemplate the following situation in which the governing flow equation for three-dimensional saturated flow in saturated porous media is mathematically described.

$$\frac{\partial}{\partial X}\left(K_X \frac{\partial h}{\partial X}\right) - Q = S_S \frac{\partial h}{\partial t}$$

Where K_X is the hydraulic conductivity along the major component axis which is assumed to be orthogonal and parallel to the major axes of the main hydraulic conductivity vectors. The piezometric head (h) and volumetric flux per unit volume is represented by the term Q. The specific storage coefficient is defined as the volume of water released or taken up from storage per unit change in head per unit volume of porous material.

Considering the steady state condition ($\frac{\partial h}{\partial t}$ = 0) and that the transmissivity can be expressed as the product of the hydraulic conductivity and the thickness of the saturated zone (T = KD). Then a simplification can be made such that under steady state conditions the following would hold true.

$$\frac{\partial}{\partial X}\left(T_X \frac{\partial h}{\partial X}\right) - Q = 0$$

Thus during the calibration of the numerical model only two main parameters needs to be adjusted, i.e. transmissivity and recharge. In general calibration is performed by specifying initial estimates of recharge and hydraulic conductivity, and solving the model for steady-state flow conditions. These estimated input parameters were then varied in successive simulations until the steady state head solution most closely matched the calibration target water levels. After the model was calibrated to the water levels, the pumping tests were simulated with the model. This is defined as the standard procedure for calibrating a groundwater model (Anderson and Woessner, 1992).

As indicated the specified limits for the transmissivity of the respective areas are set. If this is taken into account with the knowledge that only high yielding boreholes are typically reported, then the implication is that the transmissivity values can be as much as two orders greater than that observed from the regional transmissivity value. Since this is case the general groundwater contours in an area can be simulated but due to the higher estimated regional transmissivity values a lower groundwater level would be observed (Figure 6-1).

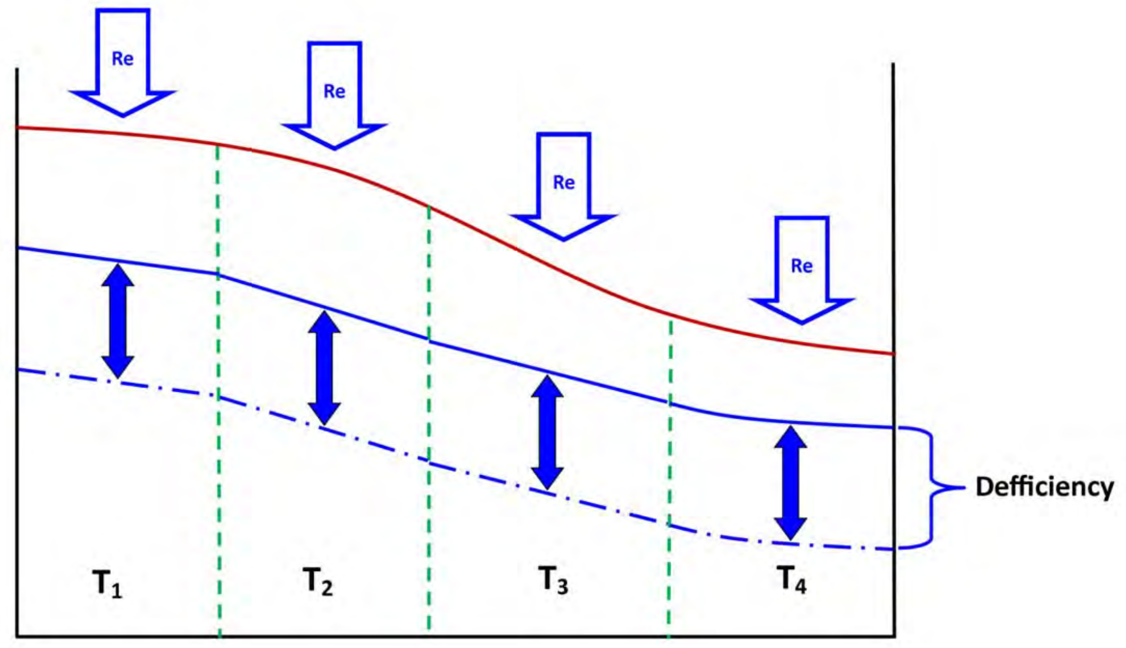

Figure 6-1 Illustrating the effect of transmissivity and recharge on steady state calibration methods.

To counter this reduced water level a higher recharge would be applied to offset this observation. This in turn would then result in a calibrated model in which the recharge would be significantly higher than what would naturally occur. The final knock-on of this modelled system would be that regional recharge values would be overestimated as well as the transmissivity values, resulting in a mismanaged aquifer system. A recent investigation by Van Wyk (Van Wyk, 2010) indicated that the estimated recharge for South Africa is far less than the values currently accepted. In the instance of South Africa this could have disastrous consequences since the country experiences periodic droughts which impacts on water demand. Consequently, if models are used to estimate recharge values, a more robust method must be employed to determine actual regional transmissivity values that include low yielding boreholes.

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