A computer code for the calculation of the relative transmissivity distribution in an aquifer for steady state ground-water levels

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Abstract

A computer code to solve the inverse problem for transmissivities in an aquifer has been developed. The underlying concept of the model is that the spatial distribution of ground-water levels which is in a pseudo-steady state, is purely a function of the transmissivity distribution over the aquifer. The results of an application of this model to the Atlantis aquifer are discussed. The calculated transmissivities, with a constant S-value, were then used to calibrate the model. The results are promising.

Introduction

The demand for ground water for municipal and agricultural use has grown steadily during the past decade, leading to an ever-increasing demand for more information on ground-water modelling techniques. Ground-water management involves decision-making with respect to location, rate and time of pumping of ground water, recharging an aquifer, and forecasting the short and long-term consequences of such operations (Khan, 1986).

The hydraulic parameters, T and S, are usually determined by conducting pumping tests. The parameters so determined represent only that portion of the aquifer which lies within the range of influence of the well. Basically, a model is an approximation for transforming input (pumpage, recharge) into piezometric heads.

In the calibration of aquifer models, the desire for an automated adjustment process is sometimes in conflict with the need for subjective intervention during the calibrating process. This paper does not attempt to offer a new model, but instead looks at the question of adjustment of transmissivities to bring a model into some measure of calibration. The key problem in modelling is the calibration of the model, i.e. the adjustment of the parameters of the model to obtain a satisfactory match between estimated and historical aquifer response, based on available knowledge of stresses imposed on the aquifer in the past. However, this "inverse" problem is not unique (Simpson *et al.*, 1971).

It is possible to adjust parameters in an infinity of arbitrary ways to obtain the desired fit (Emsellem and De Marsily, 1971). The parameters that give a good match with the observed data, however, may not be the real parameters for the aquifer (Khan, 1980). Because of this, Labadie (1975) named the calibrated parameters surrogate parameters. It is obvious that to get a suitable model to predict the future behaviour of the aquifer the surrogate parameters must reflect, as nearly as feasible, the underlying physical structure of the aquifer.

The inverse problem of parameter identification in ground-water flow has been studied by a number of persons like Chang and Yeh (1975), Frind and Pinder (1973), Emsellem and De Marsily (1971), Lovell *et al.* (1972), Neuman (1973) and Khan (1986).

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The Basic model

The two-dimensional flow of ground water is governed by the well-known equation:

$$\frac{\partial}{\partial x}\left[T\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T\frac{\partial h}{\partial y}\right] = S\frac{\partial h}{\partial t} \qquad (1)$$

where:

h = pressure head (m)
S = storage coefficient
T = transmissivity (m^2/d)

Under steady state conditions, this equation is written as:

$$\frac{\partial}{\partial x}\left[T\frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial y}\left[T\frac{\partial h}{\partial y}\right] = 0 \qquad (2)$$

It is important to note that the solution of (2) is a function of T and the boundary conditions, but not of the S-value, whereas Eq. (1) is a function of T and S.

Application of Darcy's Law to each stream tube (Fig. 1) in the aquifer yields:

$$Q_{inflow} = Q_{outflow} = TiL \qquad (3)$$

where:

T = transmissivity at the inflow side of the stream tube (m^2/d)
i = ground-water gradient at the inflow elements
L = width of the tube (m)

If the width of the stream tube is a constant and the T-value at the inflow side (or outflow side) is known, the T-distribution over the whole stream tube can be calculated from Eq. (3).

Each stream tube can be discretised into a number of triangles. The ground-water gradient at each node of a triangle can be calculated by using the following equation:

$$i_x = \frac{h_1(y_2 - y_3) + h_2(y_3 - y_1) + h_3(y_1 - y_2)}{D}$$

$$i_y = \frac{h_1(x_3 - x_2) + h_2(x_1 - x_3) + h_3(x_2 - x_1)}{D} \qquad (4)$$

where:

$$D = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$$

h_i = pressure head at node i

(x_i, y_i) = coordinate of node i

Once the gradient for each node is known, the calculation of the corresponding T-value at the node is simple.

It is, however, not necessary to construct stream tubes for an aquifer, as long as it is remembered that the calculated T-values for nodal mesh points are only applicable for areas where the widths of the stream tubes are constant. Program TCAL (Fig. 2) can be used for the calculations of these T-values.

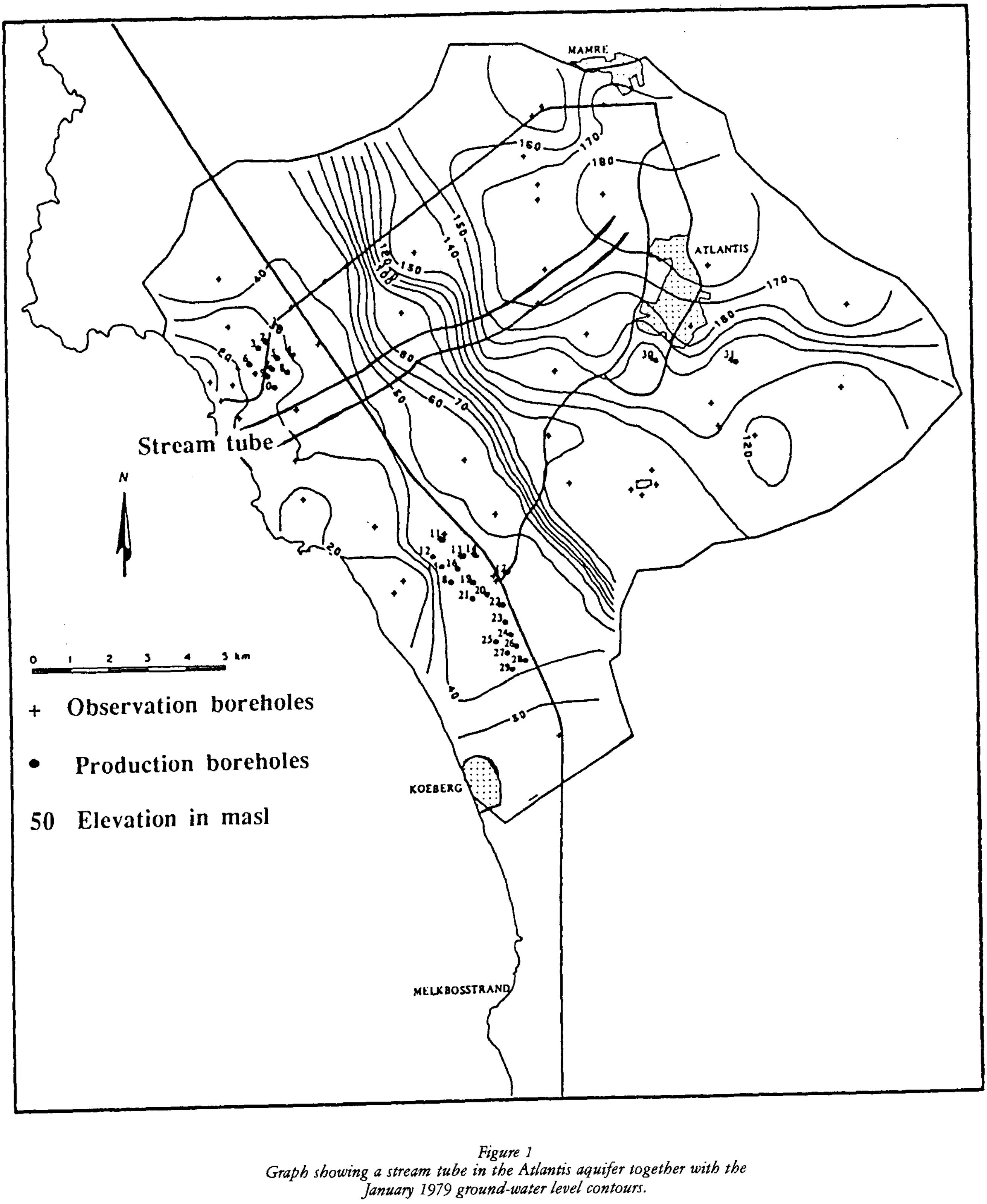

Figure 1
Graph showing a stream tube in the Atlantis aquifer together with the January 1979 ground-water level contours.

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C     PROGRAM TO CALCULATE THE TRANSMISSIVITY DISTRIBUTION IN AN
C     AQUIFER FOR STEADY STATE GROUND-WATER LEVELS
      ________________________________________________________________
C     INPUT PARAMETERS
C     1. N,NE — WHERE N = NUMBER OF NODES AND NE = NUMBER OF ELEMENTS
C     2. NODE NUMBER, X,Y,GROUND-WATER LEVEL FOR EACH NODE
C     3. ELEMENT NUMBER AND INCIDENCES OF EACH ELEMENT
C
      CHARACTER*20 FNAME
        DIMENSION X1(1000), Y1(1000), WL(500), DIST(1000), G1(1000)
        DIMENSION X(500), Y(500), IN(1000,3), XI(3), YI(3), ZI(3)
        DIMENSION G2(30), INODE(500)
        OPEN (8,FILE = '8.TXT', STATUS = 'UNKNOWN')
      WRITE (*,*) 'ENTER DATA FILE NAME'
      READ (*, 113) FNAME
113   FORMAT (A20)
      OPEN (5, FILE = FNAME)
        NUM = 4
        WRITE (*,*) 'ENTER A T-VALUE AND GRADIENT I FOR THIS T-VALUE'
        READ (*,*) T,GI
        Q = T*GI
C
C     READ NUMBER OF NODES AND NUMBER OF ELEMENTS
C
        READ(5,*)N,NE
C
C     READ NODE NUMBER, X,Y, AND WATER LEVEL
C
        DO 1 I = 1,N
        READ (5,*) INODE(I), X(I), Y(I), WL(I)
  1   CONTINUE
C
C     READ ELEMENT NUMBER AND THE INCIDENCES OF THE ELEMENT
C
      DO 2 I = 1,NE
      READ (5,*)NNN, (IN(I, J), J = 1,3)
  2   CONTINUE
C
C     BEGIN LOOP FOR GRADIENT CALCULATIONS
C
      DO 100 L = 1,NE
      VX = 0.
      VY = 0.
      DO 51 I = 1,3
      J = IN(L,I)
        ZI(I) = WL(J)
      XI(I) = X(J)
 51   YI(I) = Y(J)
      DD = XI(2) *YI(3)-XI(3)*YI(2)-XI(1)*(YI(3)-YI(2)) + YI(1)*(XI(3)-XI(2))
      GX = ZI(1)*(YI(2)-YI(3)) + ZI(2)*(YI(3)-YI(1)) + ZI(3)*(YI(1)-YI(2))
      GY = ZI(1)*(XI(3)-XI(2)) + ZI(2)*(XI(1)-XI(3)) + ZI(3)* (XI(2)-XI(1))
      GX = - GX/DD
      GY = - GY/DD
      GXY = SQRT(GX*GX + GY*GY)
      XX = (X(IN(L,1)) + X(IN(L,2)) + X(IN(L,3)))/3.
      YY = (Y(IN(L,1)) + Y(IN(L,2)) + Y(IN(L,3)))/3.
        G1(L) = GXY
        X1(L) = XX
        Y1(L) = YY
100   CONTINUE
C
C     END OF GRADIENT CALCULATIONS
C
C     START TO CALCULATE A T-VALUE FOR EACH NODE
C
        DO 500 I = 1,N
        SOM = O.
      XX = X(I)
      YY = Y(I)
      DO 501 K = 1,NE
      DIST(K) = SQRT((X1(K)—XX)**2 + (Y1(K)-YY)**2)
```

Figure 2
Listing of program TCAL.

Figure 2 (continued)

```
501   CONTINUE
      DO 502 J = 1,NUM
      IC = 1
      DO 503 JK = 2,NE
      IF (DIST(JK).LT.DIST(IC))IC = JK
503   CONTINUE
      G2(J) = G1(IC)
      DIST(IC) = 1.0E20
502   CONTINUE
      DO 505 JJ = 1,NUM
      SOM = SOM + G2(JJ)
505   CONTINUE
      SOM = SOM/NUM
      IF(SOM.EQ.0.)SOM = .001
      TRANS = Q/SOM
      WRITE(8,510)I,X(I),Y(I),TRANS
510   FORMAT(I5,2F10.0,F10.2)
500   CONTINUE
      STOP
        END
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The Atlantis aquifer as case study

A mathematical finite element model was developed at the Institute for Ground-water Studies to describe ground-water flow in the Atlantis aquifer (Müller and Botha, 1986). The main objective of the model was to:

- simulate the physical conditions prevailing in the aquifer, and
- optimise the future development of the production fields.

The main physiographical feature which influences the flow of ground water in the area is a coastal flat running parallel to the coast with a width of approximately 5 km. The inland part of the aquifer, in the vicinity of Atlantis and Wesfleur, is separated from the coastal flat by a sudden rise in bed-rock topography, also manifested in the present surface topography. The geological features of the area are of importance only in so far as they form the physical flow medium and the boundaries of the aquifer. The lateral boundaries are defined by the coastline in the west, and by the pinching out of the water-bearing sand formations in the north, east and south.

The hydraulic conductivity varies between 0 and 20 m/d (mean = 10 m/d) with an average saturated aquifer thickness of 20 m, while the S-values are in the order of 17 per cent.

The ground-water level contour maps for different times show the same general behaviour, which implies that the ground water is in a pseudo-steady state condition. Fig. 1 shows the ground-water levels for January 1979.

Müller and Botha (1986) calibrated the Atlantis finite element method between the period January 1979 and January 1980. This procedure was a tedious job of more than a month's trial and error procedure, until a good fit between observed and simulated values was found. During the calibration period, several deficiencies were identified. Among these were:

- too large elements in regions of steep ground-water gradients; and
- deficiencies in data regarding initial heads and hydraulic parameters.

For the present investigation, the same finite element mesh as used by Müller and Botha (Fig. 3) was used, with the same initial ground-water levels. Müller and Botha (1986) found it impossible to calibrate the model in the vicinity of steep water-table gradients in the aquifer (Fig. 1). They stated that it can be corrected for by using a mesh with finer elements in these regions, so as to control the flux of ground water more effectively. In these regions, observed and actual water levels differ up to 6 m after one year.

By using Müller and Botha's mesh (Fig. 3) and initial water levels, program TCAL was used to calculate the T-distribution of the Atlantis aquifer. Each element in the mesh used by Müller and Botha was transformed into two triangular elements for use by program TCAL. By using these T-values, the best match possible between the observed and simulated water levels was obtained (Fig. 4). This figure confirms the statement of Müller and Botha (1986) that, to obtain an excellent fit between simulated and actual water levels, a finer mesh must be used in the vicinity of the steep water-table gradients.

It is obvious from Fig. 4 that a better match has been obtained with program TCAL than with the trial and error process used by Müller and Botha.

Since the coding of program TCAL, it has been applied for the calculation of the transmissivity distribution in the Dewetsdorp and De Aar aquifers in the Karoo Basin in South Africa, with very good results.

Summary and conclusions

An indirect approach to the inverse problem in ground water has been coded for an IBM-compatible computer. The concept of surrogate T-values, rather than actual T-values, was used.

The concept of surrogate parameters is an important feature in the calibration of a model. This concept was first used by Labadie (1975) and then by Khan (1980). It can be said with certainty that all ground-water models use surrogate rather than actual parameters, because flow through porous media is a highly complex phenomenon. The ground-water flow equation is based on a number of assumptions, so that calibrated hydraulic parameters act as approximations (surrogates) for the actual ones.

The indirect approach discussed in this paper, was tested on the Atlantis aquifer as a case study. A good comparison between observed and calculated water levels was found.

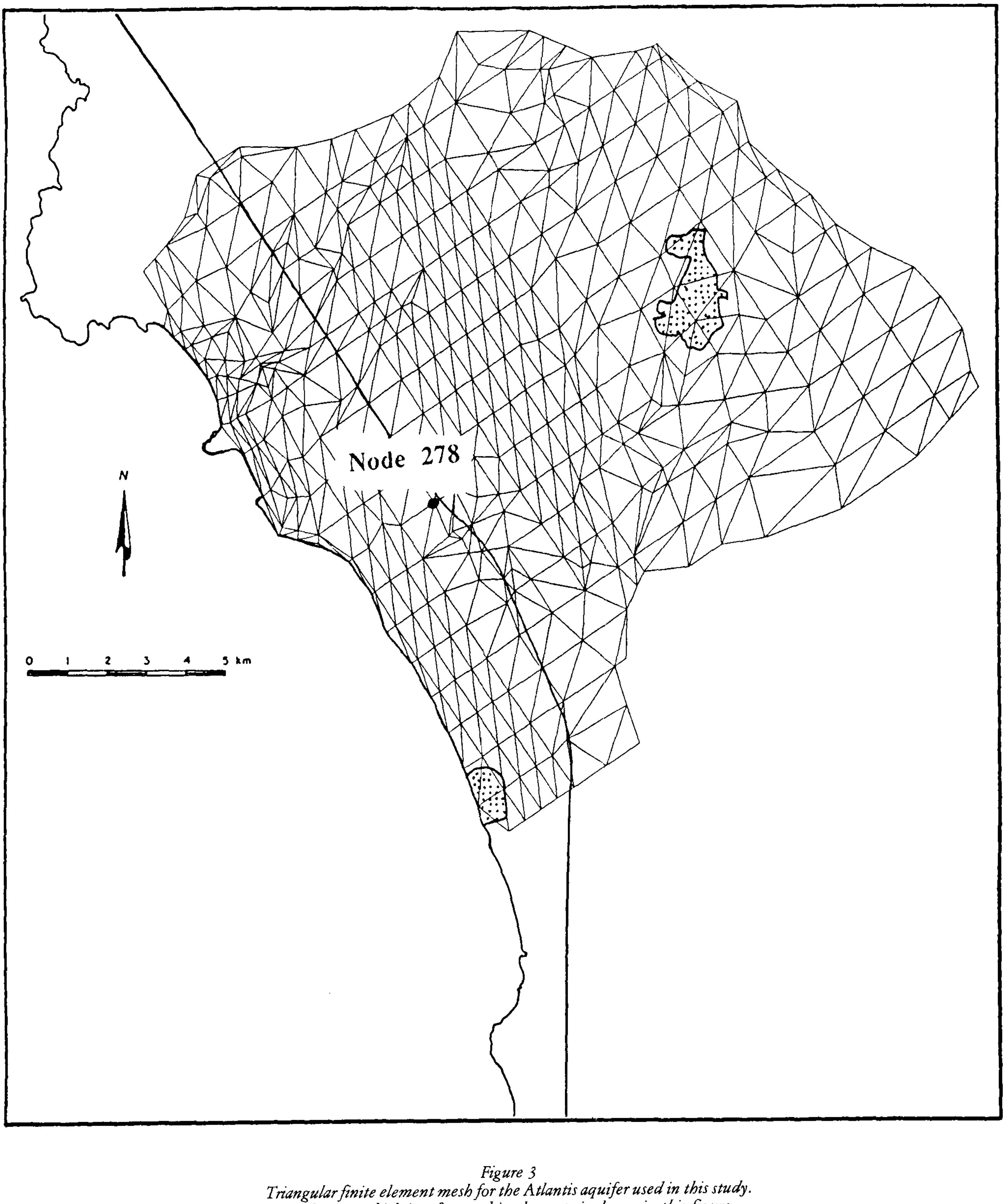

Figure 3
Triangular finite element mesh for the Atlantis aquifer used in this study. Node 278, which is referenced in the text, is shown in this figure.

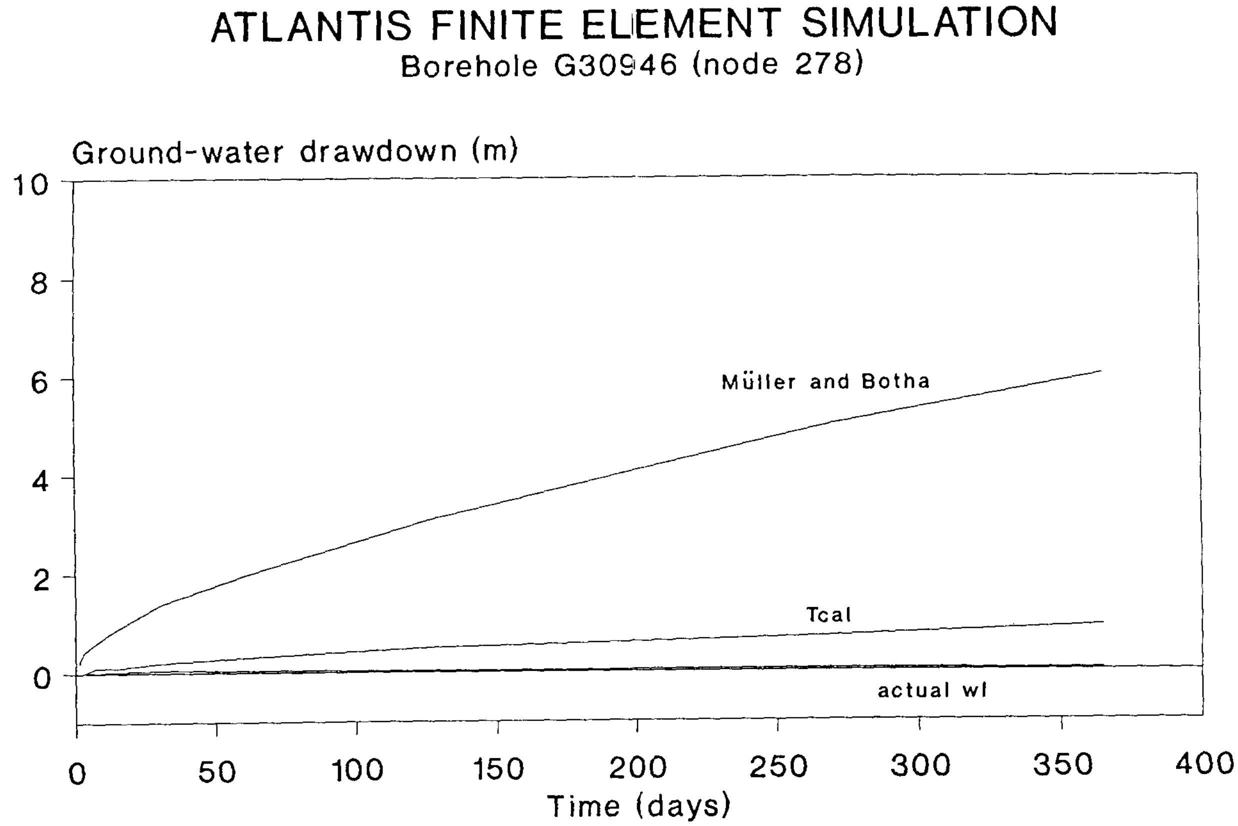

Figure 4
Actual drawdown in observation borehole G30946 in the vicinity of a steep water-table gradient as well as the simulated drawdown obtained with the proposed method and the Müller and Botha model.

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