Least-squares analysis of settling data under discrete settling conditions

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Abstract

This article presents a least-squares analysis of settling data obtained from four different quiescent settling column tests. In the study the settling data are modelled using a polynomial function with depth and time as model variables. The model parameters are estimated by solving a set of five linear simultaneous equations. The effects of sampling times and depths on the estimated value of settling efficiency are presented.

Introduction

Sedimentation is a process by which the removal of solid particles from a suspension is achieved through settling under gravity. The design of a settling tank is commonly done with the aid of quiescent settling column tests (Camp, 1946; O'Connor and Eckenfelder, 1958). Typically, a quiescent settling column test is conducted by introducing the relevant water or wastewater into a column which has several sampling ports located along the column. The suspended solids contained in the water or wastewater are allowed to settle under quiescent conditions. Samples of water or wastewater are then withdrawn from the various sampling ports along the depth of the settling column at a number of selected time intervals. The concentrations of suspended solids in these samples are then determined in the laboratory. As the cost and effort required for conducting a settling column test increases with the total number of samples taken, it would be desirable to minimize the number of samples. In view of the above, it is worth while to examine the effects of the frequency of sampling and the number of sampling ports along the column depth on the estimated value of settling efficiency. In this article, the effects of these two variables on the estimated value of settling efficiency are examined by the use of an efficient least-squares technique. This technique is derived from the model developed by Berthouex and Stevens (1982).

Development of least squares model

Typically, the data obtained from a quiescent settling column test are analysed by the use of a graphical method. Recently, Berthouex and Stevens (1982) have proposed an innovative approach whereby the settling data are analysed by using the MINITAB statistical package (Ryan *et al.*, 1976). The equation describing the concentration profile in a settling column is given by Berthouex and Stevens (1982) as:

$$C(z,t) = a + bz + ct + dt^2 + ezt \qquad (1)$$

in which t = time; z = settling depth measured from the water surface; C(z,t) = concentration of suspended solids at depth z and time t; and a, b, c, d, and e = empirical constants.

Based on Eq. 1, one can write the least-squares equation as follows:

$$\min SSE = \sum_{i=1}^{N} [C_i - C(z_i, t_i)]^2 \qquad (2)$$

in which SSE = sum of square errors around the calibrated concentration profile; n = total number of samples taken from the settling column during the settling test; C_i = concentration of suspended solids for sample i; and z_i and t_i = column depth and sampling time associated with the measurement of C_i.

Substituing Eq. 1 into Eq. 2 yields:

$$\min SSE = \sum_{i=1}^{n} (C_i - a - bz_i - ct_i - dt_i^2 - ez_i t_i)^2 \qquad (3)$$

The values of the empirical constants associated with Eq. 3 can be estimated by established statistical packages. Alternatively, one can also estimate these values by setting the partial derivatives of SSE with respect to a, b, c, d, and e to zero, from which a set of five linear simultaneous equations (with the five constants as unkown) is obtained. The values of the five constants then can be found by solving the set of simultaneous equations. This procedure can be implemented on an advanced programmable calculator or a microcomputer.

Once the values of a, b, c, d, and e are estimated, the fraction of suspended solids removed, E, at time T in a tank of active settling zone depth Z can be computed from Eq. 4 (Berthouex and Stevens, 1982).

$$E(T) = 1 - \frac{1}{ZC_0} \int_0^Z C(z,t)dz \Big|_{t=T} \qquad (4)$$

$$= 1 - \frac{1}{ZC_0} \left(aZ + \frac{bZ^2}{2} + cTZ + dT^2Z + \frac{eZ^2T}{2}\right) \qquad (5)$$

in which C_0 = concentration of suspended solids at t = o.

In the analysis presented in this paper, the settling efficiency corresponding to a given time T, E(T) is obtained by the method of solving the set of simultaneous equations mentioned above.

Effect of sampling time

In general, the result estimated from a fitted model will be affected by the number of observations used to calibrate the

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model. This observation has been reflected also in the study of Berthouex and Stevens (1982). They reported that the calibrated model did not approximate the data adequately when the complete time range of the data collected was used for analysis. In order to investigate the effect of the data time range, four sets of settling data were taken from the literature (Adams *et al.*, 1981; Berthouex and Stevens, 1982; Sundstrom and Klei, 1979; and Tchobanoglous and Matsumoto, 1979) for detailed analysis. In this analysis, the time ranges were varied by changing the lower and upper bounds of the time range as indicated in the first two columns of Tables 1 to 4.

Table 1 shows the results of analysis using the data given by Berthouex and Stevens (1982); all removal efficiencies are calculated for depth 1,22 m and time 60 min. Detailed information pertaining to the test set-up is given in the table. As shown in Table 1, the removal efficiencies associated with the time ranges of 0 to 120 and 0 to 60 differ significantly from those associated with time ranges of 10 to 120; 20 to 120; 40 to 120; 20 to 60 and 40 to 60 respectively. This suggests that the data collected from the early phase of the settling test may bias the estimated value of settling efficiency. This appears to indicate that one should not take samples during the early phase of the settling test (the initial concentration still has to be measured). Furthermore, in Table 1 it can be seen that the removal efficiencies associated with the time ranges of 20 to 120 (with 4 sampling times), 40 to 120 (with 3 sampling times), 20 and 60 (with 3 sampling times), and 40 to 60 min (with 2 sampling times) are fairly consistent with one another. This indicates that as few as 2 to 3 sampling times appear sufficient for accurate estimates of the settling efficiency.

TABLE 1
EFFECT OF SAMPLING TIMES USING DATA FROM BERTHOUEX AND STEVENS (1982)[a,b,c]

Time Range, in minutes			
From	To	R^2	E (%)
0	120	0,82	99,5
10	120	0,92	90,6
20	120	0,98	87,8
40	120	0,99	86,8
60	120	d	d
0	60	0,93	83,4
10	60	0,98	85,4
20	60	0,99	86,4
40	60	0,99	86,3
0	40	0,98	21,7
10	40	0,99	54,4
20	40	0,98	108,3

[a] The settling test was conducted by using a column with 3 sampling ports (Z = 0,61, 1,22 and 1,83 m) covering a time range of 0 to 120 min. Altogether there were 18 observations available. The wastewater used had an initial suspended solids concentration of 560 mg/ℓ.
[b] All removal efficiencies were computed at depth Z = 1,22 m and time t = 60 min.
[c] The removal efficiency obtained by Berthouex and Stevens (1982) via the graphical method was 89,4%.
[d] Failed to give an estimate. (The matrix of the simultaneous equations is singular.)

Table 1 also indicates that the results associated with the time ranges of 0 to 40; 10 to 40, and 20 to 40 min are not satisfactory. Moreover, the analysis for the time range of 60

TABLE 2
EFFECT OF SAMPLING TIMES USING DATA FROM TCHOBANOGLOUS AND MATSUMOTO (1979)[a,b,c]

Time Range, in minutes			
From	To	R^2	E (%)
20	80	0,98	81,5
30	80	0,97	81,6
40	80	0,97	81,3
50	80	0,98	81,1
60	80	0,99	81,6
70	80	0,99	80,3
20	70	0,97	81,4
30	70	0,97	81,5
40	70	0,97	81,5
50	70	0,98	81,3
60	70	0,99	81,6
20	60	0,97	81,6
30	60	0,97	81,4
40	60	0,97	81,7
50	60	0,98	81,6
20	50	0,96	82,8
30	50	0,97	80,5
40	50	0,97	85,1
20	40	0,95	83,2
30	40	0,95	94,3

[a] The settling test was conducted by using a column with 5 sampling ports (Z = 0,5, 1,0, 1,5, 2,0 and 2,5 m) covering a time range of 20 to 80 min. Altogether there were 23 observations available. The wastewater used had an initial suspended solids concentration of 500 mg/ℓ.
[b] All removal efficiencies were computed at depth Z = 3 m and time t = 60 min.
[c] The removal efficiency obtained by Tchobanoglous and Matsumoto (1979) via the graphical method was 81,5%.

TABLE 3
EFFECT OF SAMPLING TIMES USING DATA FROM ADAMS *ET AL.* (1981)[a,b,c]

Time Range, in minutes			
From	To	R^2	E (%)
5	120	0,95	49,8
5	90	0,96	50,9
5	60	0,96	52,6
5	40	0,94	53,5
5	20	0,94	50,7
5	10	0,95	378,9
10	120	0,96	51,7
10	90	0,97	52,3
10	60	0,97	53,1
10	40	0,97	52,7
10	20	0,96	60,5
20	120	0,94	53,0
20	90	0,94	52,6
20	60	0,95	52,7
20	40	0,94	51,3
40	120	0,92	58,6
40	90	0,88	56,8
40	60	0,94	43,0
60	120	0,88	62,6
60	90	0,83	62,3

[a] The settling test was conducted by using a column with 3 sampling ports (Z = 0,61, 1,22, and 1,83 m) covering a time range of 5 to 120 min. Altogether there were 21 observations available. The wastewater used had an initial suspended solids concentration of 300 mg/ℓ.
[b] All removal efficiencies were computed at depth Z = 1,83 m and time t = 23 min.
[c] The removal efficiency obtained by Adams *et al.* (1981) via the graphical method was 54,5%.

to 120 min even failed to give an estimate of the settling efficiency (the matrix of the set of five simultaneous equations is singular). Unlike the other analyses in Table 1 discussed above, this latter group of analyses requires extrapolation of results; i.e. the time at which the settling efficiency is required lies outside the time range. The poor estimate of the settling efficiency when extrapolation is required confirms that extrapolation of results should be avoided. The last sampling time should either be equal to or longer than the proposed detention time of the sedimentation tank to be designed.

Table 2 shows the results of analyses using the data given by Tchobanoglous and Matsumoto (1979); all removal efficiencies are calculated for depth 3 m and time 60 min. The results shown in Table 2 indicate that the settling efficiencies associated with the time ranges of 50 to 60; 60 to 70, and 70 to 80 min (i.e. each with 2 sampling times and no extrapolation). yield approximately the same as that associated with 20 to 80 min time range (with 7 sampling times and no extrapolation). This confirms the conclusion drawn from the analyses in Table 1 that as few as 2 to 3 sampling times are sufficient for accurate estimates of settling efficiencies provided no extrapolation is required. Interestingly the settling efficiencies associated with time ranges of 20 to 50 and 30 to 50 min (i.e. with 4 and 3 sampling times and including extrapolation) are consistent with the other analyses in Table 2 discussed above. However, the results associated with the time range of 30 to 40 min (i.e. 2 sampling times with extrapolation) are not consistent. This indicates that extrapolation of results sometimes may lead to satisfactory results. This conclusion also can be made from the results presented in Table 3. As there is no way of knowing a priori when extrapolation of results will be satisfactory, one should not carry out extrapolation as a normal procedure. Therefore, when the detention time of the proposed sedimentation tank is uncertain, it is essential that the last sampling time is planned such that extrapolation of results would not be necessary.

Table 4 shows the results of the analysis using the data given by Sundstrom and Klei (1979). The results shown in this table follow a similar pattern to those in Tables 1, 2, and 3. However, the settling efficiency associated with the time range of 40 to 60 min (with 2 sampling times and no extrapolation) was about 9% higher than the others. This suggests that two sampling times are not adequate for this set of data. Consequently, although two sampling times worked well for the data used in Tables 1, 2, and 3, this might not be adequate for all data sets - 3 sampling times should be accepted as a minimum for estimating settling efficiency.

TABLE 4
EFFECT OF SAMPLING TIMES USING DATA FROM SUNDSTROM AND KLEI (1979)[a,b,c]

Time Range, in minutes			
From	To	R^2	E (%)
10	80	0,99	60,6
20	80	0,99	60,1
30	80	0,98	60,2
40	80	0,99	60,5
60	80	0,98	60,6
10	60	0,99	61,0
20	60	0,99	60,7
30	60	0,98	60,8
40	60	0,98	69,2
10	40	0,99	59,5
20	40	0,98	61,9
30	40	0,96	68,6
10	30	0,98	52,6

[a] The settling test was conducted by using a column with 3 sampling ports (Z = 0,6, 1,2, and 1,8 m) covering a time range of 10 to 80 min. Altogether there were 18 observations available. The wastewater used had an initial suspended solids concentration of 500 mg/ℓ.
[b] All removal efficiencies were computed at depth Z = 1,8 m and time t = 50 min.
[c] The removal efficiency obtained by Sundstrom and Klei (1979) via the graphical method was 62,5%.

TABLE 5
EFFECT OF SAMPLING DEPTHS USING DATA FROM TCHOBANOGLOUS AND MATSUMOTO (1979)[a]

Time Range, in minutes		Settling Efficiency, in per cent[b]						
From	To	(1)	(2)	(3)	(4)	(5)	(6)	(7)
20	80	81,5	81,3	81,7	81,3	79,7	79,5	79,7
30	80	81,6	81,4	81,7	81,3	79,7	79,5	79,7
40	80	81,3	80,9	81,6	81,2	80,2	79,4	80,2
50	80	81,1	80,5	81,6	81,1	80,0	79,6	80,0
60	80	81,6	80,0	82,3	81,7	80,0	80,0	80,0
20	70	81,4	81,2	82,0	81,3	79,7	79,6	79,6
30	70	81,5	81,4	82,0	81,3	79,7	79,6	79,7
40	70	81,5	81,1	82,0	81,5	80,2	79,7	80,2
50	70	81,3	80,1	81,9	81,4	80,0	79,5	80,0
20	60	81,6	80,6	82,3	81,7	80,4	80,2	80,4
30	60	81,4	80,2	82,3	81,7	80,4	80,2	80,3
40	60	81,7	80,7	82,3	81,7	80,4	80,1	80,2

[a] The data set used in this table is the same as that used in Table 2.
[b] Settling depths for the results shown in Column:

1 : Z = 0,5, 1,0, 1,5, 2,0, and 2,5 m
2 : Z = 0,5, 1,5 and 2,5 m
3 : Z = 1,0, 2,0, and 2,5 m
4 : Z = 1,0, 1,5, and 2,5 m
5 : Z = 0,5, 1,0, and 1,5 m
6 : Z = 1,5, and 2,5 m
7 : Z = 1,0, and 1,5 m

Effect of sampling depth

In addition to sampling times, sampling depths may also have some effect on the estimated value of settling efficiency. In order to investigate the effect of sampling depth, the same sets of data used for the above analyses were again employed. However, in this analysis, the number of sampling depths and positions were varied rather than keeping them fixed as in the cases of the previous analyses.

Table 5 shows the results of analysis using the data of Tchobanoglous and Matsumoto (1979); all removal efficiencies are calculated for depth 3 m and time 60 min. As shown in the Table, the settling efficiencies calculated from a varying number and combination of sampling depths (from 5 i.e. at depths 0,5; 1,0; 1,5; 2,0 and 2,5 m (column 1) to 2 i.e. at depths 1,0 and 1,5 m (column 7) are fairly consistent with one another. Interestingly, the results obtained from two sampling depths at 1,0 and 1,5 m (column 7) are also quite consistent with those of the others even though the settling efficiency was evaluated at a depth of 3 m, which requires considerable extrapolation with respect to sampling depth.

This indicates that as few as two sampling depths appear adequate for estimating the settling efficiency and the exact locations of sampling points do not seem to be important. Analysis of the other sets of data confirmed this result; one that is gratifying because it simplifies the task of engineers in designing settling column tests.

Conclusion

The effects of sampling times and sampling depths on the estimated value of settling efficiency were investigated by a least-squares analysis using four sets of settling data taken from literature. In this study, it was found that as few as three sampling times are adequate for estimating settling efficiency; the three sampling times should cover the detention time of the proposed sedimentation tank and the first sampling time should not take place in the early phase of the settling test; as few as two sampling depths are adequate for estimating settling efficiency; and the depths of the sampling points (along the column) do not have a significant influence on the estimate of the settling efficiency.

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References

ADAMS, C.E., FORD, D.L., and ECKENFELDER, W.W. (1981) *Development of design and operational criteria for wastewater treatment.* Enviro Press, Nashville, USA.

BERTHOUEX, P M., and STEVENS, D.K. (1982) Computer analysis of settling test data. *J. Environ. Engr. Div. ASCE* **108** 1065-1069.

CAMP, T.R. (1946) Sedimentation and the design of settling tanks. *Trans. ASCE III* 895-936.

O'CONNOR, D.J , and ECKENFELDER, W.W. (1958) Evaluation and laboratory settling data for process design. In: *Biological treatment of sewage and industrial wastes,* Vol. II, McCabe, J. and Eckenfelder, W.W. (Eds) Reinhold Publishing Co., New York, USA.

RYAN, T.A., JOINER, B.L., and RYAN, B.F. (1976) *Minitab student handbook.* Duxbury Press, North Scituate, Mass., USA.

SUNDSTROM, D.W., and KLEI, H.E. (1979) *Wastewater treatment.* Prentice-Hall, Englewood Cliffs, USA.

TCHOBANOGLOUS, G., and MATSUMOTO, M.R. (1979) *Solution manual to accompany wastewater engineering: treatment, disposal and re-use.* McGraw Hill Book Company, New York, USA.