Letter to the Editor with regard to the paper
ADSORPTION IN THE ACTIVATED SLUDGE PROCESS
by GA Ekama and G v R Marais published in *Water SA* 4 (1) 39—48, January 1978:

The purpose of this letter is to examine the mathematics of the method Ekama and Marais use to determine oxygen uptake rates and show the conditions under which their assumptions can lead to appreciable error. Since many investigators analyze data in the same manner as Ekama and Marais, this note may be of general interest.

The differential equation which describes the dissolved oxygen concentration in a well mixed flow reactor is

$$V\frac{d}{dt}(DO) = Q(DO_{IN} - DO) - VR \qquad (1)$$

$$DO = DO_O \text{ at } t = 0$$

If the volumetric flow rate, Q, the influent oxygen concentration, DO_{IN}, and the specific rate of oxygen depletion, R, are assumed to be constant during the course of an experiment, the solution to the above equation is

$$DO = \{DO_0 - DO_{IN} + \frac{VR}{Q}\}\, e^{-\frac{QT}{V}} + DO_{IN} - \frac{VR}{Q} \qquad (2)$$

Since the residence time, V/Q, is long compared to the time of an uptake experiment, the exponential term can be approximated with adequate accuracy as

$$e^{-\frac{\;}{V}} \doteq 1 - \frac{Qt}{V} \qquad (3)$$

the error being less than $\frac{1}{2}(\frac{Qt}{V})^2$.

Combining Eq. 2 and 3 yields

$$DO = DO_0 - (DO_0 - DO_{IN} + \frac{VR}{Q})(\frac{Qt}{V}) \qquad (4)$$

The slope of the linear plot of DO vs t is thus

$$\text{slope} = -\{\frac{Q}{V}(DO_O - DO_{IN}) + R\} \qquad (5)$$

The error involved in stating that the slope equals minus R depends on the relative magnitudes of the terms. In a system undergoing endogenous respiration (slope ∿ 10—15 mg O_2/l/h), the term $\frac{Q}{V}(DO_O - DO_{IN})$ will be roughly 20% of R.

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Ekama and Marais reply as follows:

We thank Mr. Fox for his comments on the method of determining oxygen consumption rates in a completely mixed activated sludge reactor. We are in complete agreement that the biological oxygen consumption rate is not necessarily correctly given by equating it to the dissolved oxygen versus time plot. The error that may be introduced by doing so results from a difference in the relative masses of dissolved oxygen entering the reactor via the influent and recycle flows and that leaving the reactor via the effluent flow.

We were aware of this source of error but found that in our experiments it was usually less than 5% of the slope of the dissolved oxygen versus time plot if a number of precautions were taken:

Usually there was dissolved oxygen in the influent sewage, up to 7 mg O/*l* — the influent was aerated for about 15 min prior to feeding to strip sulphides which may have been produced due to storage of the sewage. Also, the experimental units were fitted with special settling tanks which reduced sludge retention to a minimum. Consequently, the recycle stream contained dissolved oxygen concentrations which were approximately equal to those in the reactor. With these provisions the error due to differences between input and output masses of dissolved oxygen were minimized. Furthermore, high influent flows increase the error term but also increase the oxygen consumption rate. In contrast, low oxygen consumption rates associated with endogenous respiration usually occur when the influent flow is zero. Consequently, the relative error remained more or less constant.

Although in our series of experiments the errors introduced in the oxygen consumption rate determination by measuring the rate in the reactor were kept at a low level, large errors may be readily incurred in small reactors receiving high flows, as predicted by Mr. Fox. For example, the utmost care must be exercised to obtain reliable oxygen consumption rate measurements in the contact reactor of a contact stabilization process. We found that neglect of the precautions described above lead to errors of up to 40% of the correct value.

A further point to note is that the response rate of the dissolved oxygen probe should be about 10 times the measured oxygen consumption rate if the error due to lag in probe response is to be minimized. The analysis below indicates that this requirement is met, using the dissolved oxygen probes produced by Yellow Springs Instruments Co.

If one makes the reasonable assumption that the difference between the indicated dissolved oxygen concentration and

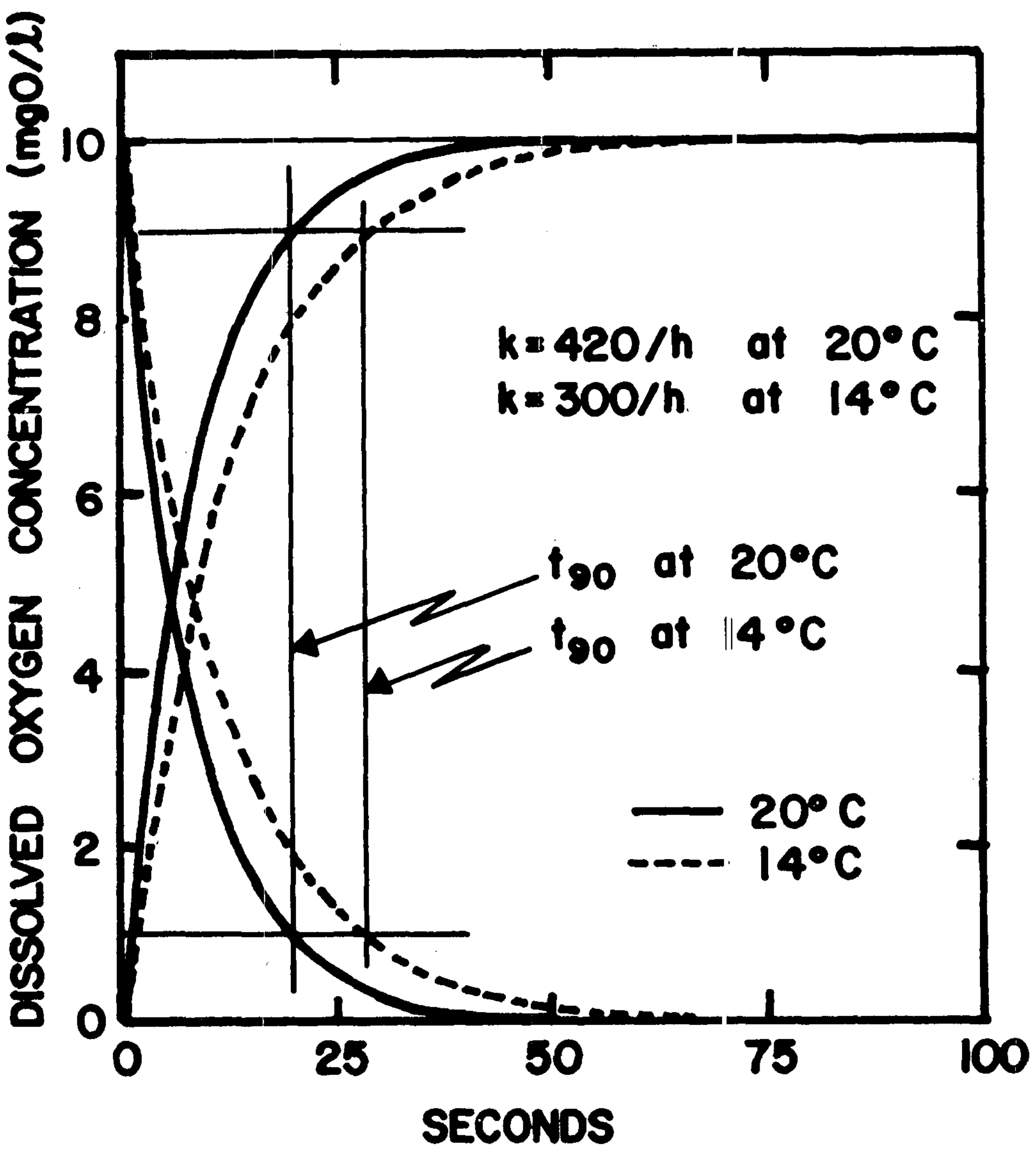

Figure 1
Determination of probe characteristic k

the real concentration dies away exponentially, then the response of the probe may be modelled by

$$\frac{dO_i}{dt} = -k(O_i - O_r) \quad (1)$$

where

O = dissolved oxygen concentration (mg O/l)
subscripts i and r refer to indicated and real concentrations respectively
k = probe characteristic (/h)
t = time (hours)

If it is assumed that the real dissolved oxygen concentration in the reactor in which the oxygen consumption rate is measured changes linearly with time, i.e.

$$O_{r,t} = O_{r,o} - Rt \quad (2)$$

where

second subscript o refers to the value at time = O, and R = oxygen consumption rate (mg O/l/h)

then

$$\frac{dO_i}{dt} = -k(O_i - O_{r,o} + Rt) \quad (3)$$

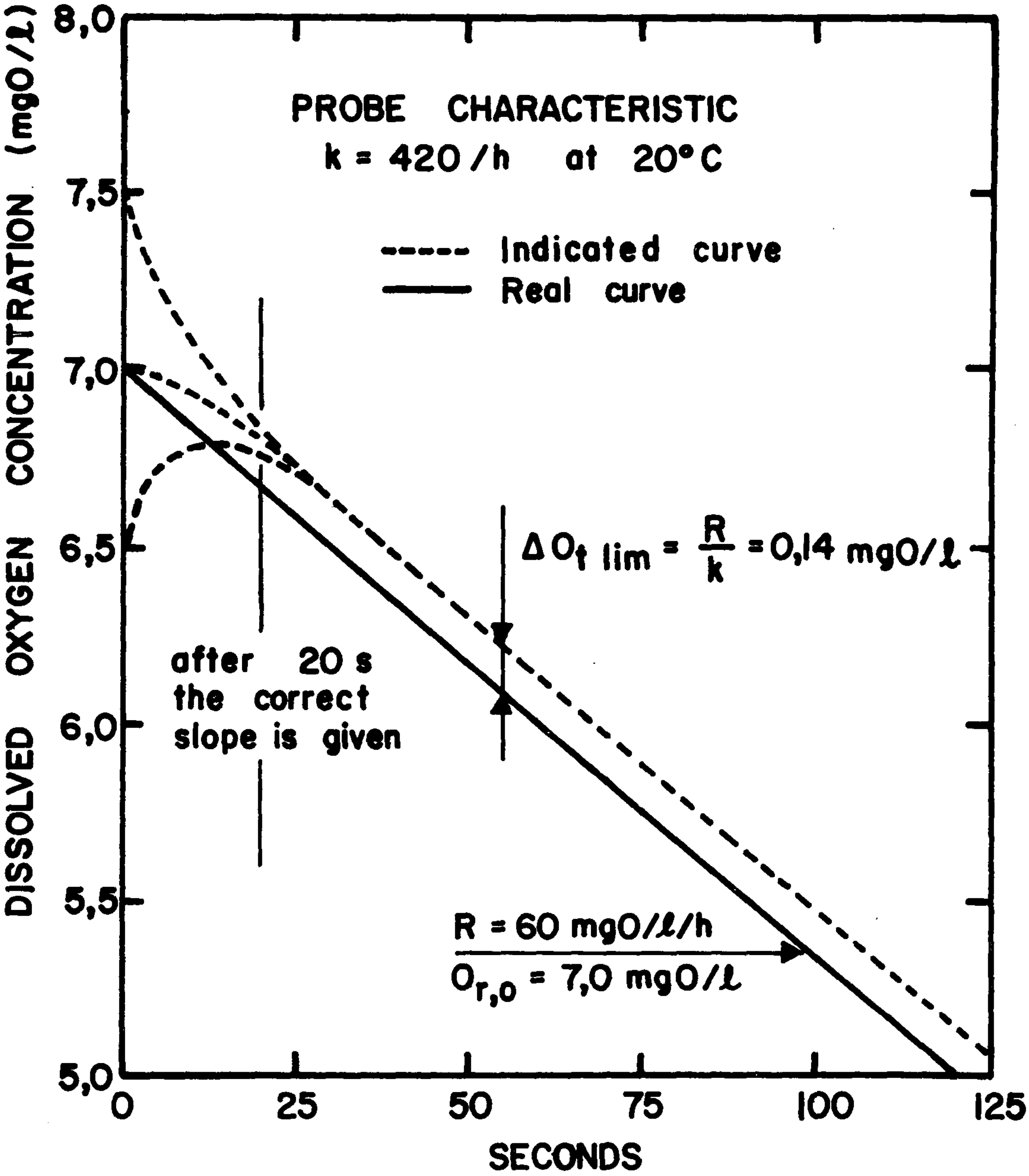

Figure 2
Comparison of indicated and real dissolved oxygen concentration during the oxygen consumption rate test

which has the solution

$$(O_i + Rt - O_{r,o} - R/k) = Ae^{-kt}$$

Now, at t = O; $O_i = O_{i,o}$

$$\therefore A = O_{i,o} - O_{r,o} - \frac{R}{k}$$

Now let $O_{i,o} - O_{r,o} = \Delta O_o$, i.e. an initial gap term

then

$$A = \Delta O_0 - \frac{R}{k}$$

Let $O_{i,t} - O_{r,t} = \Delta O_t$ i.e. gap at time t

i.e. $O_{i,t} - O_{r,o} + Rt = \Delta O_t$

then

$$\Delta O_t - \frac{R}{k} = (\Delta O_o - \frac{R}{k})e^{kt}$$

$$\text{i.e.} \Delta O_t = \frac{R}{k}(1 - e^{kt}) + \Delta O_o e^{kt} \qquad (4)$$

i.e. a relaxation term plus an initial gap term.

The probe characteristic may be determined as follows:

Assume the oxygen consumption rate is zero, i.e.

$$R = O$$

then $\Delta O_t/\Delta O_o = e^{-kt}$ (5)

Let t_{90} be the time at which the gap has reduced by 90% of the initial gap,

i.e. $\Delta 0_{t_{90}} / \Delta 0_0 = 0{,}10$

then $k = 2{,}3/t_{90}$

Hence, k may be determined by recording the change in dissolved oxygen concentration when the probe is transferred very quickly from a high concentration, say 9 mg O/l, to a low concentration, say zero mg O/l, (or from a low concentration to a high concentration).

Based on the above procedure we found that for YSI dissolved oxygen probes

$k = 420/h$ at 20°C

$k = 300/h$ at 14°C

With these values of k, the theoretical curves of dissolved oxygen versus time, Eq. (5) (Fig. 1), and the experimentally measured curves, compare very well and indicate the validity of the basic assumption in Eq. (1).

Accepting Eq. (4), Fig. 2 shows the changes in the indicated dissolved oxygen concentration ($O_{i,t}$) versus time for different initial indicated values ($O_{i,o}$), compared to the real concentration ($O_{r,t}$) versus time with an initial real concentration ($O_{r,o}$) of 7 mg O/l at an oxygen consumption rate (R) of 60 mg $O/l/h$ at 20°C. It can be seen that (i) after approximately 20 s the indicated dissolved oxygen concentration versus time slope is parallel to the real slope but displaced a constant R/k mg O/l above it, (ii) over a reasonable range, i.e. 7 to 3 mg O/l, the correct slope will be given even at the higher oxygen consumption rates.

The above analysis shows that the YSI dissolved oxygen probes are satisfactory for measuring oxygen consumption rates as these rates in the activated sludge process seldom exceed 15% of the probe response rate.

In conclusion, we again wish to thank Mr. Fox for drawing attention to the problems of measuring oxygen consumption rates in completely mixed reactors which we omitted in our paper.

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