LONG-RUN HYDRAULIC AND ECONOMIC RISK SIMULATION AND OPTIMISATION OF WATER CURTAILMENTS

Report to the
WATER RESEARCH COMMISSION

by

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"Long-run hydraulic and economic risk simulation and optimisation of water curtailments"

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EXECUTIVE SUMMARY

LONG-RUN HYDRAULIC AND ECONOMIC RISK SIMULATION AND OPTIMISATION OF WATER CURTAILMENTS

BACKGROUND AND MOTIVATION

Irrigated agriculture has been identified as one of the major role players to enable rural communities to participate fully in the economic, social and political life of the country (NPC, 2011). The expectation is to increase the 1.5 million hectares currently under irrigation with another 500 000 ha (DAFF, 2015). Water availability is the major limiting factor that prohibits growth of this sector. The newly developed Irrigation Strategy for South Africa (DAFF, 2015) focusses on improving irrigation water management to improve the efficiency of water use thereby helping the National Development Plan 2030 to achieve its goals. In many instances irrigated agriculture, the single largest user of water in South Africa, is seen as a source of water by government because water use in the sector is characterised in some cases as highly inefficient (DWAF, 2004). Furthermore, some catchments are over allocated and compulsory licensing is currently used to reconcile the over-allocation within these catchments through water curtailments (DWA, 2012). Re-allocation of water is a sensitive issue. Providing decision support through the application of hydro-economic models is seen as essential to facilitate the process (Harou, Pulido-Velzquez, Rosenberg, Medelín-Azuara, Lund and Howit, 2009).

In response to the increasing need for integrated hydro-economic information to manage water more efficiently, the Water Research Commission (WRC) has funded a project on “The development and testing of an integrated hydro-economic model to evaluate the financial impact of curtailment decisions on a farm case study in the Crocodile catchment” (WRC report 1805/1/12). The integrated hydro-economic framework consists of a network model, also described as a node-and-channel model, (MIKE BASIN) that uses simulated rainfall-runoff from ACRU to represent water availability in the catchment, a FAO-56-based irrigation module to quantify irrigation water demand and an optimisation model that is used to determine irrigators’ responses to changes in water availability. The research showed that it is possible to incorporate water-related operating rules that were practised in the catchment, but which are not currently included in the Department of Water and Sanitation’s decision-making framework to more realistically model water availability. However, the mathematical programming approach used by the researchers lacked the necessary detail to model the essential soil-water-crop relationships necessary to quantify the hydrological impact of long-run dynamic adaptation strategies (changes in crop mix and irrigation technology choice) to water curtailments.

PROBLEM STATEMENT AND OBJECTIVES

Water managers are currently lacking good information about the hydro-economic impact of water re-allocation policies which hampers the implementation of these policies. The lack of good hydro-economic information stems from the unavailability of integrated hydro-economic models that are able to quantify the impact of institutional changes on water availability and the impact of irrigators' operational and investment responses to changes on the hydrology while ensuring the financial sustainability of irrigation farming in the long-run.

The general objective of this research is to develop and apply a long-run hydro-economic risk simulation and optimisation modelling framework to quantify the hydro-economic impact of water curtailments.

In order to achieve the general objective of the research the following specific objectives were set:

- To develop a farm-level water use simulation model and optimisation solution procedure to optimise the water use between multiple crops at farm level with the purpose of modelling the short-run response of irrigators to water curtailments.
- To develop a long-run financial risk simulation model and optimisation solution procedure to optimise the impact of structural changes (crop mix and irrigation technology investments) in response to water curtailments on irrigation farming profitability over the long-run.
- To develop methods and procedures to integrate the short-run and long-run response models with MIKE BASIN to optimise the dynamic response of irrigators to water curtailments on the financial feasibility of irrigation farming and return flows for the specific case study farms.

APPROACH AND METHODOLOGY

The success of the project strongly hinged on the ability of the research team to represent the soil-water-crop interrelationships in the economic models with more detail in order to more realistically model the linkages between hydrology and farm-level irrigation water use. Increasing the complexity with which the soil-water-crop system is represented in the economic optimisation model renders the use of standard mathematical programming solvers infeasible. A review of the literature indicated evolutionary algorithms as a feasible alternative to gradient-based solvers to optimise complex representations of biophysical systems. The conceptual framework for linking the hydrology, farm-level irrigation water use and the economic models with an evolutionary algorithm was developed through a workshop between research team members.

MIKE HYDRO BASIN was chosen as the model to reconcile irrigation water demand from the farm with the water availability in the catchment. During the second year of the project the MIKE OPERATIONS platform was launched by the Danish Hydrology Institute which allows for the seamless data transfer and running of exogenous models through the use of Python scripts. The MIKE OPERATIONS platform also included an evolutionary algorithm that could be used to optimise the parameters included in the setup of the model. An economic simulation model was developed to simulate the impact of changes in crop areas and irrigation water applications on total farm gross margins in the short-run while assuming the use of predetermined irrigation technology. The short-run economic water simulation model was integrated with MIKE HYDRO BASIN using Python scripts to form a tight coupling between the models which facilitate the use of the embedded evolutionary algorithm to optimise the system. Unfortunately, the number of variables that need to be controlled by the algorithm to optimise farm-level water use were too many and the solver failed. A decision was made to develop a tailor made algorithm that is able to optimise the water use of the farm using the output from the hydrology model to quantify water availability. Consequently, the tight coupling between the hydrology and the economic water use model was broken.

Optimising a simulation model with an evolutionary algorithm implies an iterative procedure whereby the appropriateness of alternative solutions are checked over many iterations. A tight coupling between the algorithm and the simulation model is desirable to speedup the solution process. The General Algebraic Modeling System (GAMS) was used to develop and solve the short-run and long-run economic models using a genetic algorithm in conjunction with gradient-based solvers. Both the short-run and long-run optimisation models include a FAO-56 type daily irrigation simulation model to model the timing and duration of water deficits on crop water use. The effects of non-uniform irrigation applications were modelled by replicating the water budgets for each irrigation field three times with each water budget receiving a different amount of irrigation water based on the uniformity of the specific system. In the long-run, irrigators were allowed to change their irrigation technology to improve water application efficiencies.

The models were applied to a representative irrigation farm below the Vanderkloof dam to determine the hydro-economic feasibility of imposing a water curtailment of 15%.

SUMMARY OF RESULTS AND CONCLUSIONS

The short-run modelling results on profitability showed that irrigation farming will still be profitable when water allocations are curtailed by 15%. Increasing irrigation application uniformity proves to be an important strategy to combat water curtailments. The impact of absolute risk aversion on the reduction in certainty equivalents was large. However, the reduction should be evaluated taking cognisance of the fact that the extreme level of absolute risk aversion corresponds to a lower confidence interval of 90%.

Evaluating the optimised responses to a water curtailment of 15% showed that management responses at the intensive margin dominates extensive margin responses. Intensive margin responses are associated with changes in irrigation scheduling while extensive margin responses are associated with changes in area irrigated to different crops. Cognisance should be taken of the fact that optimal intensive margin responses are conditional on knowing the soil-water status, the impact of water deficits on crop growth and yield, the impact of non-uniform water applications on crop yield, the distributional characterisation of future weather variables and the managerial ability to integrate all the information into an optimal decision. Results further showed that improving water use efficiency through optimal irrigation scheduling and more uniform water application rates may potentially have a negative impact on the hydrology through changes in return flows and increases in evapotranspiration. The impact of risk aversion on the intensive margin responses to a water curtailment seems negligible small. The last mentioned may be the direct result of the small changes in crop yields modelled for economically rational water application amounts in the area limiting phase of production where the production function is fairly flat. The conclusion is that the results should be interpreted with caution as the economic impact of a water curtailment is based on optimal response that may not be feasible within the managerial ability of irrigation farmers.

The long-run results showed that Irrigation farming might still be profitable over the long-run because the NPV for a water curtailment scenario is positive. The reduction in NPV due to a water curtailment is not proportional to the water curtailment which implies that irrigation farmers are able to reduce the impact of a water curtailment through management responses in the form of intensive margin management and irrigation technology adoption decisions. The long-run technology use decisions for the full water allocation showed that it is profitable to upgrade the nozzle packages of irrigation systems even though the systems might be close to the end of their economic lives. Interestingly, pivots are not replaced to sustain an irrigation area of 233.7 ha and the irrigation area is slightly less for a large part of the planning horizon. Risk aversion had no impact on the replacement strategy that was followed, however, less of the total irrigation area was upgraded to systems with higher irrigation application uniformities. The irrigation system replacement strategy that is followed under the curtailed water allocation scenario of the risk neutral decision makers is similar to that of the full water allocation. The only difference being that less pivots are replaced which causes the total irrigation area to be less when compared to the full water allocation scenario. Risk aversion causes a slight increase in irrigation area when considering the curtailed water allocation scenario.

The financial feasibility of irrigation technology replacements was evaluated using the cash flow ratio. Results showed the average cash flow ratio for the full water allocation scenario to be higher than the threshold used by commercial banks to loan money. A water curtailment reduced the magnitude of the ratio while increasing the probability of falling below the threshold of 1.15. Overall the impact of risk aversion was relatively small.

The importance of intensive margin management responses to combat water curtailments in the short- and long-run emphasises the need to understand the inefficiencies associated with agricultural water use better. Very few analyses are done in South Africa that explicitly take cognisance of the factors driving inefficiencies.

RECOMMENDATIONS FOR FUTURE RESEARCH

The research has made a significant contribution towards representing the biophysical aspects of crop production within economic models of optimal water resource use. The models, methods and procedures that were developed as part of this research open up the opportunity to conduct further research. Firstly, through the application and further development of the integrated hydro-economic framework to evaluate alternative policies to reallocate water at catchment scale and secondly, the application of the solution procedures developed in this research to evaluate water resource use using bio-economic modelling.

The following recommendations pertain to the application of the integrated hydro-economic modelling framework:

- The feasibility of upscaling the procedures developed for a specific case study farm to represent the water use of the irrigation sector in a catchment should be further investigated.
- A water accounting and auditing framework needs to be developed to give effect to water markets and capacity sharing.
- Extending the optimization algorithm to include operational variables will enable the optimization of the operating rules in the catchment.
- The application of the hydro-economic framework allows for the evaluation of farm-level variability such as heterogeneity in soils, cash reserves, farm structure, etc. on the hydro-economic feasibility of alternative water reallocation scenarios.

The following recommendations pertain to the improved modelling of bio-economic systems through the use of the optimisation methods developed in this research:

- The ability of the optimization solution procedure to optimize discontinuous systems allows for the optimization of irrigation strategies that could be applied in real life compared to the optimal irrigation schedules optimized in this research. Optimizing irrigation strategies will improve the creditability of the results.
- The solution procedure could be linked to any biophysical model that is used simulate the impact of exogenous changes to a system. Modelling the economic impact of climate change could for example be improved through a better representation of the biophysical component of the economic model.
- The efficiency with which the solution procedure determines the near optimal solutions should be improved through the utilization of parallel processing and further algorithmic improvements.

INNOVATION

The research required a highly innovative approach to water resource management through the integration of a detailed representation of farm-level irrigation water management and technology adoption. The innovation stems from the ability of the research framework to integrate detailed hydrological simulations with daily irrigation water use decisions at farm-level that are optimised in the long-run. Consequently, irrigators' responses to changes in operating rules are modelled more accurately. Furthermore, the optimisation solution procedure is innovative. The solution procedure integrates both heuristics and standard optimisation algorithms to optimise the hydro-economic modelling system over the long-run.

LIST OF ACRONYMS AND ABBREVIATIONS

ACRU	Agricultural Catchment Research Unit hydrological model
CE	Certainty Equivalent
CropSyst	Cropping Systems Simulation Model
CU	Christiansen Uniformity
Dd	Depth deficit
DHI	Danish Hydraulic Institute
DMINLP	Dynamic Mixed Integer Nonlinear Programming
Dn	Depth net
Dp	Depth percolated
Dr	Soil water deficit in the root zone
ET	Evapotranspiration
ETa	Actual evapotranspiration
ETm	Potential evapotranspiration
EWR	Ecological Water Requirement
FWS	Field Water Supply
GA	Genetic Algorithm
GA-LP	Genetic Algorithm-Linear Programming
GAMS	General Algebraic Modeling System
HE	Hydro-Economic
IEM	Irrigation Efficiency Model
IRR	applied irrigation
LP	Linear Programming
MOTAD	Minimisation of Total Absolute Deviations
NPV	Net Present Value
NWRS	National Water Resource Strategy
RAW	Readily Available Water
SPSS	Statistical Package for the Social Sciences
STOMSA	STOchastic Model for South Africa
TAW	Total Available Water
WRC	Water Research Commission
WRPM	Water Resource Planning Model
WRYM	Water Resource Yield Model
Y	Yield

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CHAPTER 1: INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

The National Development Plan 2030 (NPC, 2011) identifies the need for rural communities to participate fully in the economic, social and political life of the country. In this regard irrigated agriculture has been identified as one of the major role players to achieve an integrated rural economy. The expectation is to increase the 1.5 million hectares currently under irrigation with another 500 000 ha (DAFF, 2015). Water availability is the major limiting factor in the growth of this sector.

Irrigated agriculture accounts for 62% of all surface and groundwater use in the country and its water use is in some cases characterised by high inefficiencies (DWAF, 2004b). In many instances, irrigated agriculture is seen as a source of water by government due to inefficiencies. The newly developed Irrigation Strategy for South Africa (DAFF, 2015) focusses on improving irrigation water management to improve the efficiency of water use, thereby helping the National Development Plan 2030 to achieve their goals. Some catchments are over allocated and compulsory licensing is used to reconcile the over-allocation within these catchments through water curtailments (DWA, 2012). The NWRS-2 highlights the need for a more sophisticated water management approach through decentralisation and stakeholder participation to optimise operational management of infrastructure to address sometimes conflicting water requirements (DWA, 2012). Such a more sophisticated approach to water management will require some form of hydro-economic modelling exercise (Harou, Pulido-Velzquez, Rosenberg, Medelín-Azuara, Lund and Howit, 2009). Providing decision support through appropriate hydro-economic modelling will provide the necessary information to optimise water use efficiency taking cognisance of the long-run sustainability of irrigated agriculture.

In response to the increasing need for integrated hydro-economic information to manage water more efficiently, the Water Research Commission (WRC) has funded a project on “The development and testing of an integrated hydro-economic model to evaluate the financial impact of curtailment decisions on a farm case study in the Crocodile catchment” (WRC report 1805/1/12). The integrated hydro-economic framework consists of a network model, also described as a node-and-channel model, (MIKE BASIN) that uses simulated rainfall-runoff from ACRU to represent water availability in the catchment, a FAO-56-based irrigation module to quantify irrigation water demand and an optimisation model that is used to determine irrigators’ responses to changes in water availability. The use of MIKE BASIN as a platform to model the interaction between water demand and supply proved to be very valuable as the research team was able to incorporate water-related operating rules that were practised in the catchment, but which are not currently included in the Department of Water and Sanitation’s decision-making framework. Accommodating these operating rules increased the creditability of the results which enhanced participation and discussions about the impact of alternative water management scenarios on the profitability of the case study irrigation farms with a specific location within the catchment. Although the hydrological modelling was satisfactorily done, the economic modelling has some shortcomings. The economic modelling did not consider any long-run dynamic adaptation strategies (changes in crop mix and irrigation technology choice). Furthermore, the water budget calculations necessary for modelling short-run responses were severely simplified to facilitate optimisation within a mathematical programming environment. As a result the research team was unable to model the impact of alternative adaptation strategies on the return flows of the case study farm satisfactorily.

Consequently, the research team identified the following to be critical issues that require further research in order to better model the impact of dynamic responses by irrigation farmers to catchment level water management scenarios in the long-run and the resulting hydrological consequences of on-farm adaptation strategies to water curtailments:

- The optimal response of farmers in the short-run should be more explicitly based on daily water budget calculations for improved modeling of the linkages between water availability, economic returns and the return flows of the farm.
- The economic modeling procedures and hydrological model integration need to be improved to facilitate long-run modeling of dynamic structural (crop mix and irrigation technology choice) responses of irrigation farmers to water curtailments while taking the assurance of water supply within a state contingent framework into account.

Ultimately the abovementioned developments will improve the hydro-economic linkages that are necessary to realistically model the impact of irrigator's response to catchment water management scenarios dynamically in the long-run. The main reason why the previous WRC project (WRC report 1805/1/12) on hydro-economic modelling did not achieve the above was because a mathematical programming approach was adopted to optimise water use. As a result several simplifications with respect to the soil water interactions and the dimension of the mathematical programming model structure were necessary to facilitate integrated hydro-economic optimisation. The problem was not so much the mathematical representation of the optimisation problem but rather the inability of the solvers to solve the nonlinear problem to a global optimal solution.

Evolutionary algorithms have been used extensively for decades to optimise complex model configurations through simulation-based optimisation routines. Recently several researchers (Schutze, Kloss, Lennartz, Al Bakri and Schmitz, 2012; Lehmann and Finger, 2014) have demonstrated the feasibility of evolutionary algorithms to optimise agricultural water use through the integration of irrigation simulation models and economic models. Application of such algorithms will overcome the problems experienced by the researchers to realistically model irrigation farmers' long-run response to changes in water availability dynamically.

1.2 PROBLEM STATEMENT AND OBJECTIVES

Water managers are currently lacking good information about the hydro-economic impact of water re-allocation policies which hampers the implementation of these policies. The lack of good hydro-economic information stems from the unavailability of integrated hydro-economic models that are able to quantify the impact of institutional changes on water availability and the impact of irrigators' operational and investment responses to changes on the hydrology while ensuring the financial sustainability of irrigation farming in the long-run.

The general objective of this research is to develop and apply a long-run hydro-economic risk simulation and optimisation modelling framework to quantify the hydro-economic impact of water curtailments.

In order to achieve the general objective of the research the following specific objectives were set:

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- To develop methods and procedures to integrate the short-run and long-run response models with MIKE BASIN to optimise the dynamic response of irrigators to water curtailments on the financial feasibility of irrigation farming and return flows for the specific case study farms.

1.3 STRUCTURE OF THE REPORT

Chapter 1 provides the background and motivation for the research as well as the problem statement and objectives of the research.

Chapter 2 provides an overview of the literature on hydro-economic modelling. Specific attention is given to the incorporation of irrigation system inefficiencies into economic models of agricultural irrigation water use. Modelling inefficiencies are important as it has a significant impact on the amount of water pumped, crop yield and the amount of percolation losses. The last part of the chapter is devoted alternative solution procedures to optimise complex hydro-economic models of agricultural water use.

Chapter 3 commences by providing a conceptual framework of the hydro-economic model linkages necessary to quantify the hydro-economic impact of water curtailments. The framework provides the basis for discussing the different models integrated into the framework as well as the hybrid solution procedure that was developed to optimise agricultural water use.

The results of applying the integrated hydro-economic model to evaluate the impact of water curtailments over the short-run and long-run are discussed in Chapter 4. The conclusions and recommendations for further research are given in Chapter 5.

CHAPTER 2:
LITERATURE REVIEW ON HYDRO-ECONOMIC MODELING

The literature review covers hydro-economic modelling, water resource assessment approaches, issues concerning crop water use optimisation and alternative methods to solve complex simulation models.

2.1 HYDRO-ECONOMIC MODELING

2.1.1 GENERAL DESCRIPTION

Mckinney, Cai, Rosegrant, Ringler and Scott (1999) stated that an interdisciplinary approach is needed to integrate natural and social aspects to improve the management of water resources. Hydro-economic (HE) models follow an interdisciplinary approach by integrating economic and hydrology aspects. HE models aim to use economic principles to direct the water allocation (Esteve, Varela-Ortega, Blanco-Gutierrez, and Downing, 2015) to different sectors within a catchment taking into account the hydrology of the catchment. HE models offer a complete image of what the economic behaviour and economic effects of water management policy and policy changes will have on water users (Varela-Ortega, Blanco-Gutierrez, Swartz and Downing, 2011). Consequently, HE models can assist policymakers to identify the most efficient and sustainable water management strategy within a catchment (Blanco-Gutierrez, Varela-Ortega, and Purkey, 2011).

According to Harou *et al.* (2009) HE models can be used in many research areas. A few examples of the research areas are operations planning and infrastructure expansion, water markets and allocation, adaptation pathways (e.g. climate change) and economic policy impact analysis. Harou *et al.* (2009) stated that HE models are mostly used in research that is based on a catchment or river basin. However, many researchers (Doulgeris, Georgiou, Papadimos and Papamichail, 2015; Ahrends, Mast, Rodgers and Kunstmann, 2008; Bharati, Rodgers, Erdenberger, Plotnikova, Shumilov, Vlek and Martin, 2008; Valrela-Ortega, Blanco-gutiérrez, Swartz and Downing, 2011; Grové, Frezghi, Pott and Lecler, 2012; Esteve, Valrela-Ortega, Blanco-Gutiérrez, Downing, 2015) focused on farm-level impact as irrigation farming is the largest water user in many catchments.

HE models can be developed using a holistic – or compartmental modelling approach. Cai, Mckinney, and Lasdon (2003) stated that a holistic modelling approach embeds both the hydrology and economics into one single unit. In other words, the information flow between the hydrology and economic models is endogenous. One of the most difficult problems to overcome with the holistic approach is to find a solver that is able to solve the model (Brouwer and Hofkes, 2008). The hydrology and irrigation water use relations in HE models are usually simplified in order to overcome the problem of finding a solver that is able to solve the mathematical programming model. Harou *et al.* (2009) argue that the simplifications modellers make to overcome hardware and software limitations lead to optimization models, which may not approximate the reality of the systems modelled. A benefit of the holistic modelling approach is that the inter-relationships between hydrologic and economic systems are modelled endogenously which consents a more effective combined environmental-economic analysis (Cai, 2008).

The compartment modelling approach, on the other hand, is represented by independent hydrological and economic modules where the output of the one module is used as input to the other module (Heinz, Pulido-Velazquez, Lund and Andreu, 2007). Thus, there exists a loose connection between the different components and the module operates independently from each other. Choice of mathematical formats to transform the data in order that both models can use each other’s output as inputs is of the utmost importance in the application of the compartment modeling approach (McKinney *et al.*, 1999). Therefore the main problem with the compartment approach is to find the right transformation of data and information between sub-models (Brouwer and Hofkes, 2008). Grové *et al.* (2012) stated that an advantage of this approach is that it can be used to study complex problems since it combines detailed hydrological and economic modelling systems together through data transfer activities. The amount of data that needs to be transferred to economic models to model dynamic irrigation water use at farm-level is significant (Grové and Oosthuizen, 2010). Heinz *et al.* (2007) stated not only can a HE model be compartmental or holistic, it can also be either a simulation model or an optimization model or both. McKinney *et al.* (1999) argued that it might be difficult to exchange information between hydrological and economic models because economic models are most often based on optimization techniques whereas hydrology models are most frequently based on simulation techniques.

HE simulation models do not rely on external solvers to determine the impact of changes to the hydro-economic system that is being modelled. Consequently, HE simulation models are excellent in analysing infrastructure operations (Pulido-Velazquez, Andreu, Sahuquillo and Pulido-Velazquez, 2008) and water allocation in complex river systems (Pulido-Velazquez *et al.,* 2008) which necessitates a more detailed and realistic representation of the hydro-economic interactions. Simulation models are best suited for answering “what if” questions with regards to changes in the representation of a hydro-economic system.

HE optimization models on the other hand help identify the best scenario and can be used for testing and refinement with a detailed simulation model. Optimization models use equations that represent the management and physical constraints of the river system to reach a specific mathematical objective (Harou *et al.*, 2009). Grové *et al.* (2012) stated that optimization models can be used to identify or propose potential actions that should be explored in an operational sense to improve the management of the overall hydrological system. The optimization models can also be used to predict the response of water uses based on a defined objective function. With both optimization and simulation having limitation and advantages. Loucks *et al.* (1981) (cited in Harou *et al.*, 2009); Lund and Ferreira (1996) (cited in Heinz *et al.*, 2007) stated optimization and simulation models performs well together. The optimization model identifies a promising solution and the simulation model is then used to refine and test the feasibility of the solution.

2.1.2 KEY CHALLENGES OF HYDRO-ECONOMIC MODELLING

To integrate the hydrology and economic models some difficulties have been encountered by researchers (McKinney *et al.*, 1999; Cai, 2008; Brouwer and Hofkes, 2008; Pulido-Velazquez *et al.,* 2008; Harou *et al.*, 2009; Grové *et al.,* 2012). Some of these difficulties are discussed in more detail below.

2.1.2.1 Spatial scales

The spatial domain for HE models ranges from a single household or farm to groups of countries. The most common spatial domain in HE modelling is regional, whereas analysis on an international and household or farm level can also be useful (Harou *et al.*, 2009). Cai (2008) argues that spatial scales of different scales could be matched through the identification of the key hydrologic relationship necessary for meaningful economic analysis and effective decision making with regards to the problem that is being modelled.

According to Cai (2008) distributed and lumped models are used for economic and water resources modelling. The spatial variability of the economic production activities and hydrologic processes are not considered within a lumped model. Typically a sector, e.g. irrigation, is modelled as one unit. Consequently, such an approach is not suitable to model the financial impact of adaptation strategies when considering changes in institutional rules governing water allocation to the sector. The same problem occurs in the semi-distributed approach (Harou *et al.*, 2009) where the water demand or production is modelled as a unit (Cai, 2008). With a distributed model one needs to choose an appropriated spatial aggregation for the simulation of the economic activities and hydrologic processes respectively (Cai, 2008). Such an approach provides for a much more detailed analysis of the water allocation problem. According to Harou *et al.* (2009), the node-link structure used in distributed models is well suited to link different scales. Hydrological models normally use a node-link structure were the spatial scale starts for the river basin and goes down to the field on the farm, this is called a top-down structure. On the other side a bottom-up structure is used by economic modelling thus the spatial scale starts from the field on the farm and working its way up to the whole basin (Cai, 2008).

The holistic modelling approach is often associated with larger spatial scales since it has an overall objective to improve the economic benefits of the whole catchment. Grové *et al.* (2012) argued that hydrological models are associated with larger decision-making units which in turn uses larger scales. However, the importance of representing smaller decision-making units (e.g. irrigators), within the overall hydro-economic framework, is of the utmost importance as the decisions made on a farm level will have a significant influence on the overall hydrology (Grové *et al.,* 2012). The compartmental modelling approach provides a more suitable vehicle for decision support at smaller spatial scales while taking the hydrology of the whole catchment into account (Grové *et al.,* 2012).

2.1.2.2 Temporal scales

Hydrologic simulation and the economic analysis generally use different temporal scales. Normally hydrologic simulation uses a smaller time interval (daily to years) to make the simulation as realistic as possible. Whereas the economic analysis uses larger time intervals (Seasonal or annual). The time interval depends on the purpose of the research. Longer time intervals are required if the purpose of the research is to determine the economic damage caused by soil salinity accumulation (Cai, 2008).

Cai (2008) stated that considering of temporal scales get more complex when more than one crop is included in the model. This complexity is due to the fact that there are different production seasons which require hydrologic system operation and water allocation to satisfy the water demand in each crop growth stage of the different crops. A further complicating factor is evident when the start and end time period used for financial calculations do not coincide with the constraint on the water quota.

2.1.2.3 Institutional representation

When setting up and integrating HE models at a river basin level one has to deal with institutions of stakeholders at several levels from national to catchment management agencies and to water users association (Grové *et al.*, 2012; Cai, 2008). Some examples of these rules are water rights, water allocation mechanisms, dam operating rules, water trade, international flow obligations, instream flow requirements, etc. (Grové *et al.*, 2012). The challenge comes in when one needs to represent all stakeholders' institutional rules. These rules can be represented heuristically in simulation models or as constraints in optimisation models. Probably the most important constraint on agricultural water use is the irrigators' water right which is enforced through his water quota. Water quota constraints on agricultural water use is not easily enforced in hydrological simulation models unless the demand is predetermined (Grové *et al.*, 2012). Consequently, many HE models include a loop to determine the hydro-institutional feasibility of an alternative.

2.1.3 DISCUSSION AND CONCLUSIONS

The specific problem that needs to be solved will most probably dictate whether a holistic (unified) or compartment modelling approach is going to be used as both these methods have merit for application to water resource management problems. The main emphasis of this project is to provide a better representation of on-farm crop water use in the short-run to more accurately determine changes in crop areas and irrigation practices on irrigation farming profitability and irrigation water use. Given irrigation farming is profitable, long-run responses might include adoption of better irrigation technology which will influence in-season water management and use. Modelling the impact of irrigation water management on crop yield necessitates daily water budget calculations to determine the onset and duration of water stress on crop yields. Consequently, simulation modelling is required. On the contrary, optimisation is best suited to predict responses to water curtailments and to enforce water quotas. A combined simulation optimisation approach is inevitable which requires investigation into methods to solve such complex systems.

2.3 CATCHMENT SCALE WATER RESOURCES ASSESSMENT

2.3.1 ASSESSMENT APPROACHES

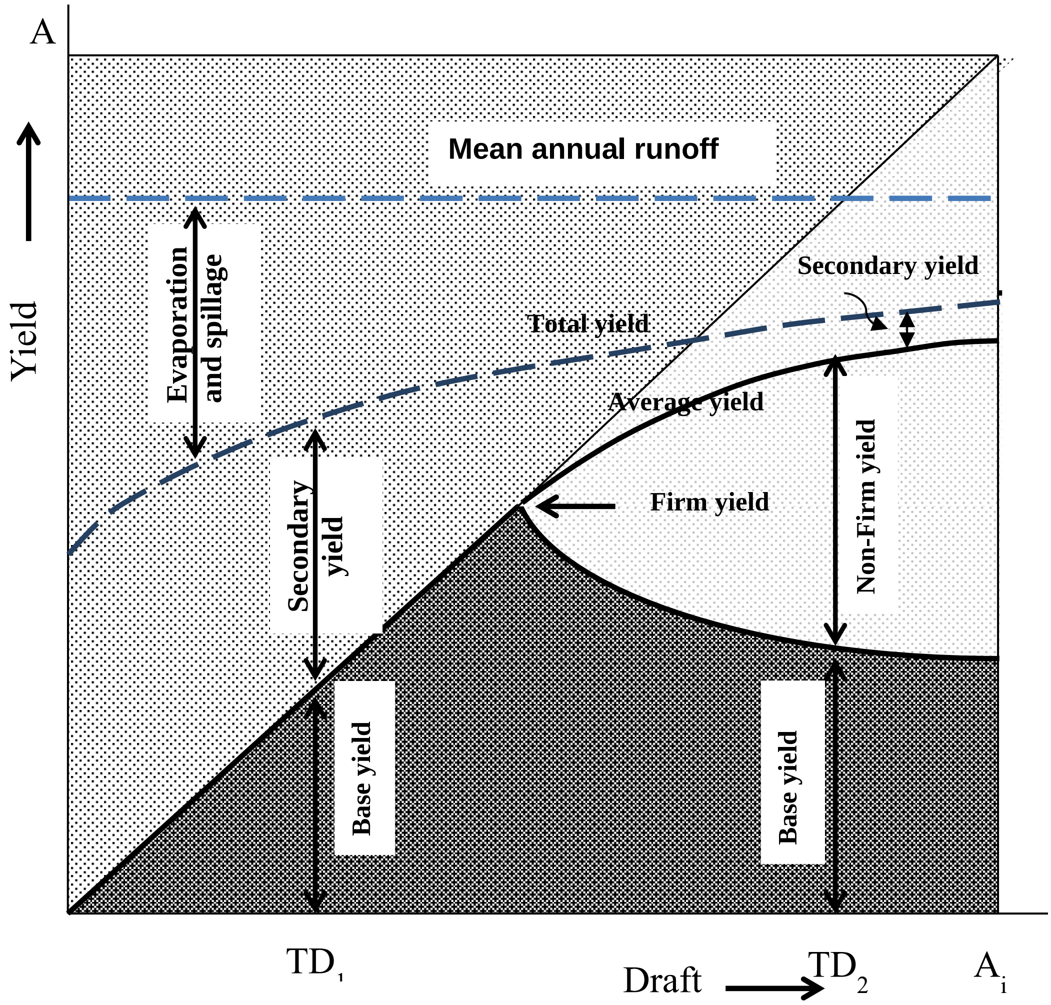
Currently, water resources in South Africa are proactively managed in order to supply water for human needs as well as the needs of other water-use sectors (e.g. mining, domestic, industrial, irrigation) and, where applicable, also international flow requirements. In certain catchments environmental water requirements (EWR) are being managed, largely via controlled releases from dams. However, in many run-of-river dominated catchments, water managers have yet to operationalise (give effect to) EWRs. The draft NWRS-2 acknowledges this fact and stresses the need for the EWRs to be upheld, as they together with Basic Human Needs are the only “right” to water recognised by the National Water Act of 1998 (Act 36 of 1998).

In a global context, South Africa has a very high variability of rainfall and runoff. The result is that, without dams (reservoirs), there are often times of more than enough (and sometimes too much) water and times of too little water. Water resource managers have had to develop sound water resource planning techniques to carefully work out how to cater for growing water demand in the face of the high variability of rainfall and runoff. Water managers in South Africa have, over the past 20-odd years, developed and refined the Water Resource Yield Model (WRYM) and Water Resource Planning Model (WRPM) which are used by water resource planners to help determine the water availability of catchments and how best to meet growing demands. These planning models replicate water resources, water users and the water apportionment rules that exist, which help govern assurance of water supply levels to various categories of water users. The planning models can be used to assess current levels of water availability (for current water use patterns, but can also be used to project into the future with consideration being given to changes in water-use patterns and flows (which may change due to climate change).

When undertaking water resource planning studies, two similar yet slightly differing approaches are often adopted. In the first option, the water planners only make use of historical river flow and weather information in their evaluation. This information is based on observed data, which may be processed (naturalised) to account for water abstractions and discharges that took place in the past. The second approach is to generate stochastic hydrological sequences (i.e. synthetically generated hydrological sequences), which are based on the naturalised historical sequence. The two approaches are explained in more details below. The second approach, which makes use of multiple flow sequences (i.e. a combination of the historical naturalised flow sequence as well as numerous synthetically generated stochastic flow sequences), enables water resource managers to undertake a more thorough probabilistic (risk-based) overview of water availability. This is explained in more detail below.

2.3.1.1 Assessment of water resources using naturalised observed historical data

The assessment of water resources using historical flows is performed to gain an understanding of the historical yield potential of a catchment with the current water resource infrastructure in place (e.g. dams &/or inter-basin transfer infrastructure, desalination & water reuse plants, etc.). Catchment historical firm yields are determined based on historically observed hydrological data sets. A shortcoming of this approach is that the accuracy of the assessment is highly dependent on the record length and the spatial coverage of the observed hydrological data. The hydrological data required for historical assessment of catchment resources is streamflow, rainfall, evaporation and historical water-use data. This data is mainly highly dispersed spatially and, where there is a record, the length of the documented data is often short. A typical output, which is a draft-yield curve, from the historical assessment of a catchment or water resource unit is shown in Figure 2.1.



Source: Basson, Allen, Pegram and Van Rooyen (1994)

Figure 2.1: Historical Draft Yield Graph

The total mean annual runoff is a combination of base yield, secondary yield, non-firm yield and the water lost due to evaporation and spillage from dams when they are over-full. A base yield is an amount that can be supplied with 100% reliability, while secondary and non-firm yield are yields that could be supplied at lesser percentages of reliability. The historical firm yield is the maximum yield that the catchment/ water resource unit can produce with 100% reliability based on historically observed data.

2.3.1.2 Assessment of water resources using historically observed and synthetically generated data

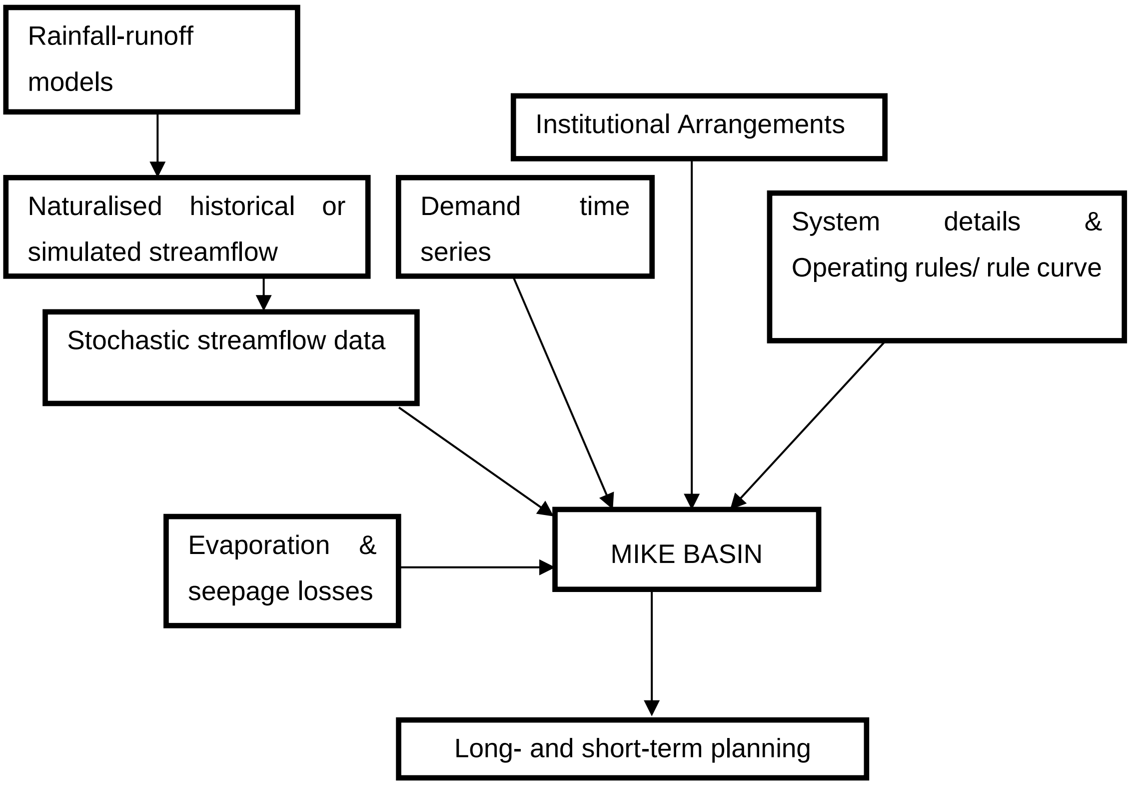
The assumption made in historical catchment water resources assessments is that the future hydrological annual or monthly series/sequence will be identical to the historical hydrological sequence. However, it is highly unlikely that historical hydrological series will be repeated in precisely the same sequence in the future. The sequencing of wet and dry cycles has an impact on the yield of catchments. A probabilistic risk-assessment approach has been adopted in South Africa, where the probabilistic approach uses stochastically generated (synthetically generated) streamflow values – with all the possible hydrological sequences (i.e. historical and stochastically-generated sequences) used as an input into the water allocation network models to assess all possible risk scenarios. The stochastic approach generates different flow sequences, but keeps statistical properties, such as the mean constant (i.e. the mean of the historical time series). Changing the flow sequences, changes the sequencing of wet and dry cycles. By using a number of separate flow sequences, water resource planners can plot out probabilities of exceedance (or non-exceedance).

The stochastic streamflow values are generated on the assumption that the historical annual/monthly streamflow series is stationary. The Stochastic Model of South Africa (STOMSA) is the stochastic generator software that was used in this project to generate stochastic streamflow values. STOMSA uses the Auto-Regressive Moving Average method, which simulates new total annual values that preserve the co-variance structure with the historical time series and is statistically similar to the original historical time series (Basson *et al.*, 1994). The software follows three steps when generating stochastic values – namely:

(i) Describes the characteristics of the marginal distribution of the annual flows,
(ii) Presents a time distribution that best represents the serial correlation exhibited by the normalise annual flows
(iii) Establishes cross-correlation between the normalised annual flows.

Besides from STOMSA, there are a number of other stochastic models that are used to generate synthetic streamflow values in South Africa. One of these models is a stochastic model developed by Hughes, Kapangaziwiri, Mallory, Wagener and Smithers (2011). The stochastic model determines the probability of future short-term flows based on the catchment's antecedent conditions. The model is being applied in the Crocodile Catchment (East) to determine the yield of the catchment.

The probable risk assessment approach to water resources allows water managers to manage resources at an informed risk level. The probabilistic risk-assessment framework to catchment water resource management is described in Figure 2.2.



Source: Basson *et al.* (1994)

Figure 2.2: Water resource modelling framework

Within the water resource modelling framework, a number of inputs are required for the network water-allocation model. These inputs are system details and operating rules; water institutional arrangements on how water shall be allocated; and evaporation and other water losses from the system and stochastic streamflow data. Each of the input elements to the water-allocation model is discussed below:

Institutional arrangements

A "water right" is defined as "the right to take and use water subject to terms and conditions of the grant" (Burchi and D'Andrea, 2003). An institutional arrangement [1]is where water rights of different water sectors are defined and, in South Africa's case, how water is allocated or distributed based on some priority water right (National Water Act, 1998). EWRs and basic human needs are allocated first priority, where other legitimate water claims are allocated on a sector priority basis. Domestic and industrial

[1] Institutional arrangement is legal framework, tools and mechanism with which catchments are operated.

water claims have higher priority than irrigation. The water allocation framework distributes water according to the institutional arrangement of the country.

System details and operating rules

In the water-allocation modelling framework, the system details are configured to represent the system ground truth. Catchments, reservoirs, rivers, channels and water users are configured and a relationship or link is created where a link exists on the ground. Reservoirs, rivers and channel operating rules are configured or detailed in the water-allocation modelling setup to realistically represent the operation of the catchment in the modelling framework.

Stochastic streamflow

Stochastic streamflow values are generated using a stochastic streamflow generator (e.g. STOMSA) from naturalised streamflow data/values. Naturalised streamflow refers to flows that would have occurred without an anthropogenic effect. The naturalised flows are estimated by subtracting the anthropogenic effect from the historical or simulated streamflow data. The anthropogenic effect could be summarised as past human activities in the catchment (e.g. past water use for economic and social benefits).

Traditionally, in South Africa, the anthropogenic effects are estimated using different models (e.g. past irrigation water use is determined using the WQT model). However, another approach to determine natural flows could be to use a suitable physical process-based rainfall-runoff. The naturalised flows are simulated by using historically observed rainfall and natural past landuse (e.g. Acocks Landover).

Demand time series

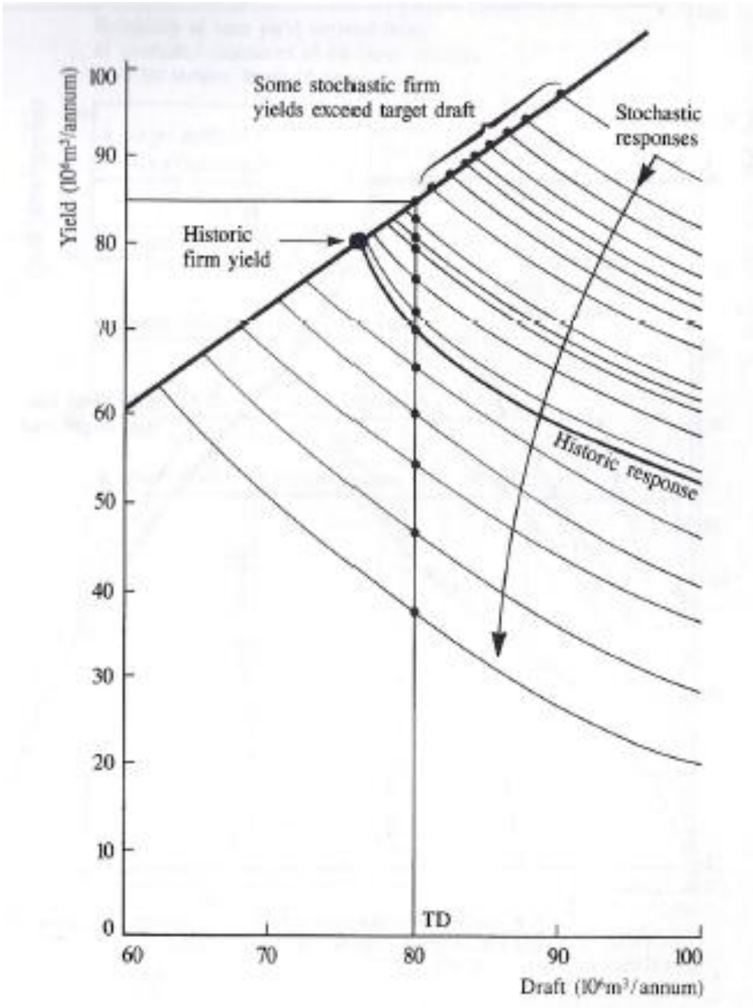
Demand time series are monthly/daily time series that are legitimate claims of the water users. In areas where the legitimate claim figure is available from a relevant authority (e.g. Irrigation board), the water allocation figure is used. However, as the licensing process is still in progress, the demand time series are estimated using different models (e.g. irrigation water demand is determined using WQT model). International Trans-boundary flows are also included as demand time series in the modelling framework.

Evaporation and Seepage losses

Significant water is lost from reservoirs, rivers and channels through evaporation and seepage. It is important to account for the water lost from the system in the modelling framework. The losses are represented as time series in the modelling framework and, in arid and sub-arid climate areas, evaporation losses have a big impact on catchment yield.

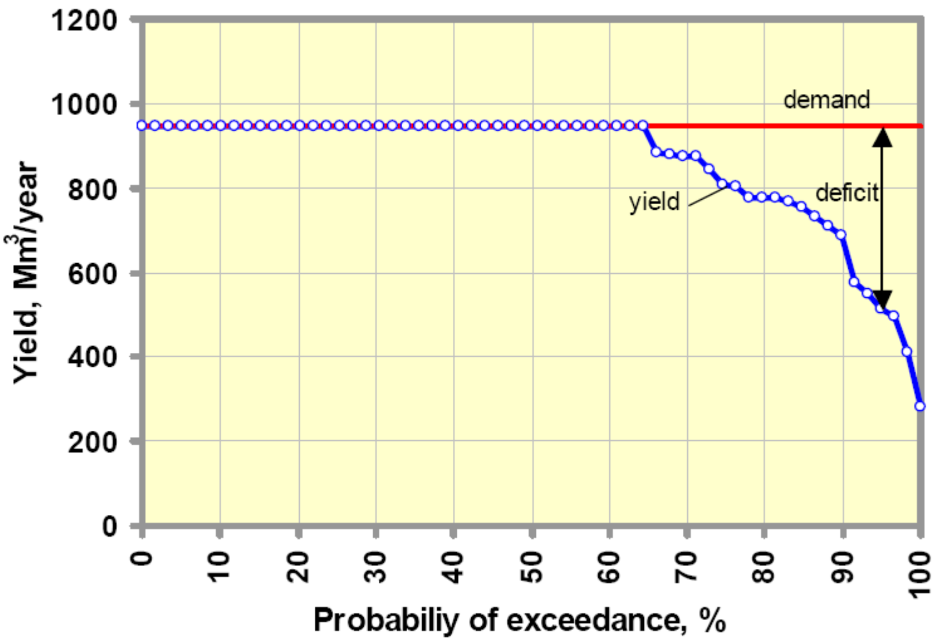
Framework outputs

The outputs from the probabilistic approaches are stochastic yield-target draft graphs and short- or long-term yield curves, as shown in Figure 2.3 and Figure 2.4.



Source: Basson *et al.* (1994)

Figure 2.3: Stochastic Draft-yield curve



Source: Findikakis, El Afti and Stephens (2013)

Figure 2.4: Probability of exceedance of a system/catchment

A yield is water resource amount that is available from the catchment or the system, a demand is a water amount that users are requesting from the system and deficit is a difference between the water demand and the available yield. The different stochastic response in Figure 2.3 are summarised to produce a graphical representation like the one portrayed in Figure 2.4.

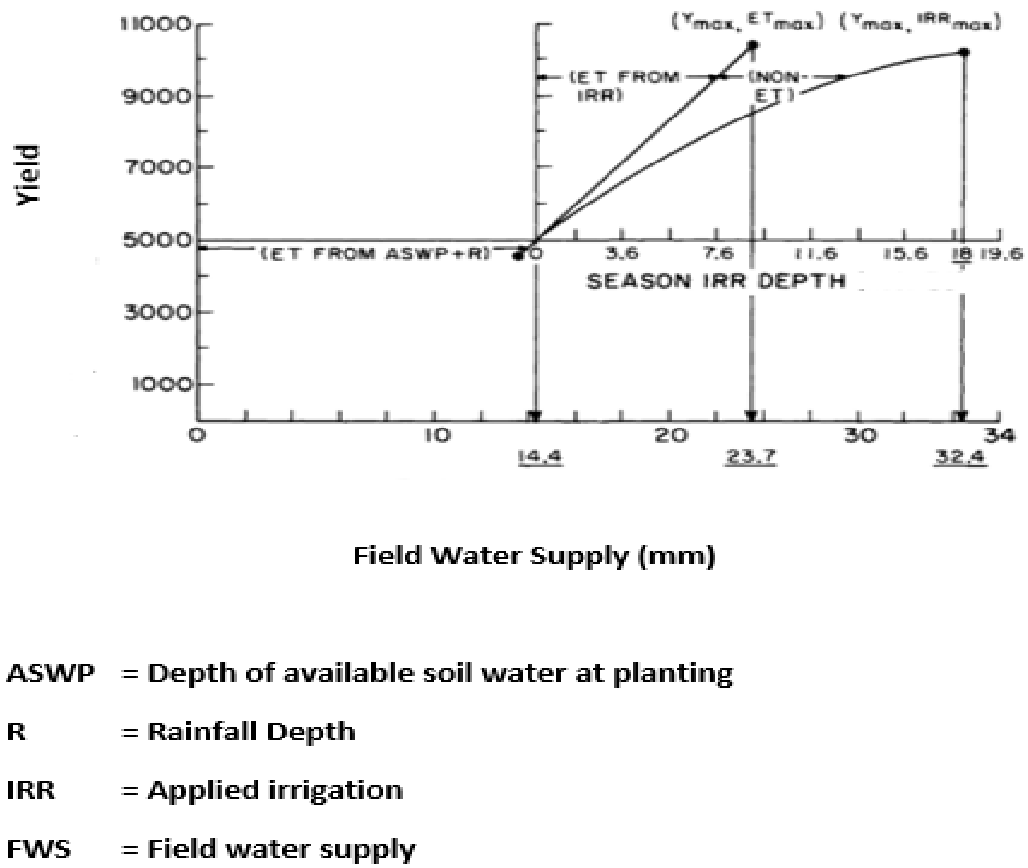
2.3.3 Discussion and Conclusion

There are two approaches for water availability assessment, historical and probabilistic approach. The probabilistic approach to water availability assessment is a widely accepted approach to water resource planning and operational management in South Africa. The models that are mostly used in this approach are WRYM and/or WRPM, while in this project the model that is adopted is MIKE Hydro Basin. MIKE Hydro Basin is a water allocation model in the same manner as WRYM/WRPM, the difference in the models is the engine used to optimise the water allocation among the nodes in the system.

2.3 CROP WATER PRODUCTION FUNCTION VS WATER PRODUCTION FUNCTION

De Juan Terjuelo, Valiente and Garcia (1996) defined a production function as it is applied to any relationship that characterizes a crops response to different inputs (such as water, fertilizers, energy). Therefore, it is important to differentiate between the production factors used to define a specific production function. A crop water production function defines a linear relationship between crop yield and evapotranspiration (Stewart and Hagan, 1973; Barrett and Skogerboe, 1980; Vaux and Pruitt, 1983). However, a water production function is a non-linear relationship between crop yield and irrigation water applied (Li, 1998).

Figure 2.5 shows crop yield as a function of respectively evapotranspiration and applied irrigation water, all set within the yield and field water supply relationship.



Source: Stewart and Hagan (1973)

Figure 2.5: Relationship between the crop yield, evapotranspiration and applied irrigation functions, all set within yield and field water supply (includes stored soil water, rainfall and applied irrigation).

Figure 2.5 is used in the subsequent sections to highlight the differences between a crop water production function and a water production function as well as the factors affecting these relationships.

2.3.1 CROP WATER RELATIONSHIP

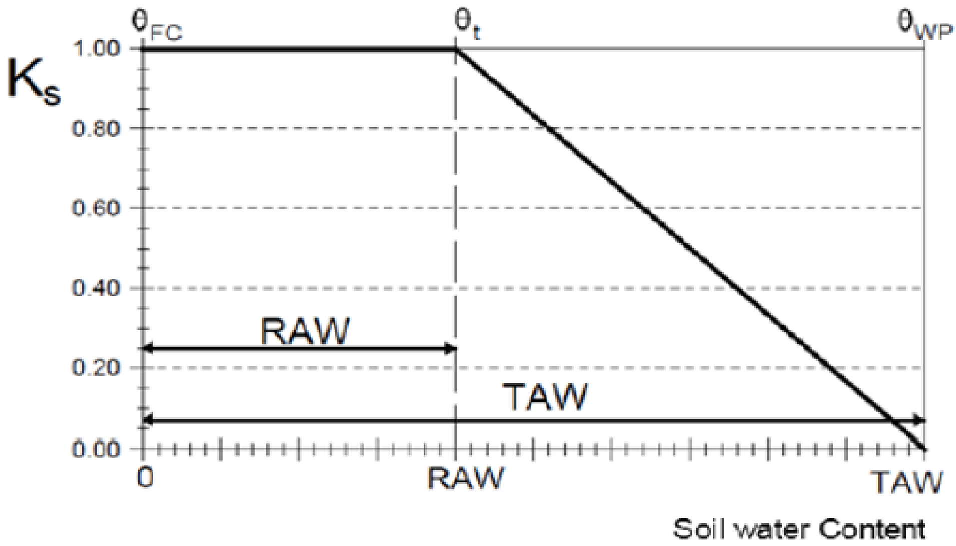
Figure 2.5 shows the relationship between yield vs evapotranspiration (ET) and yield vs applied irrigation (IRR) functions, both set within yield (Y) vs field water supply (FWS). The yield vs ET function shows a linear relationship whereby evapotranspiration is a combination of two separate processes namely transpiration from plant and evaporation from the soil. Irmak (2017) argued that in field measurements the two processes are usually considered together as it is difficult to separate them from each other as they occur simultaneously. Therefore, evapotranspiration refers to water that is lost from soil surface by evaporation and water lost from the crop by transpiration (Allen, Pereira, Raes and Smith, 1998).

Evapotranspiration process is affected by crop, soil and management factors. However, it is primarily driven by climatic conditions such as radiation, air temperature, humidity and wind speed (Irmak, 2017; Allen *et al.*, 1998). As the plants transpire water and evaporation occurs from the soil and plant surface, water moves to the surrounding atmosphere in the form of small water vapour particles. The field movement of this water vapour within, from or to field is mainly determined by wind speed and direction although other climatic factors can play a role. Evapotranspiration increases with increasing air temperature and solar radiation as these two factors are the primary drivers of ET (Irmak, 2017).

ET originates from water stored in the root zone which is augmented (increased/improved) by applied irrigation, rainfall and where possible capillary rise from shallow water tables. Hence, field water supply represents these sources. Field water supply is the result of a continuous process of supplying irrigation water into the soil in order to refill the stored soil water or soil profile so that the crop doesn't experience water stress.

Allen *et al.* (1998) argue that under non-standard conditions when root zone depletion (Dr) exceeds readily available water (RAW) (Dr>RAW) crop moisture stress is induced. Readily available water is the fraction of total available water (TAW) that a crop can extract from the root zone without suffering water stress. Water stress conditions limit the amount of water lost from the root zone through evapotranspiration resulting in actual evapotranspiration (ETa) reducing below maximum evapotranspiration levels (ETm). Crop water stress can hence be quantified by evaluating the extent by which ETa falls short of ETm (Rao, Sarma and Chander, 1988; Kallitsari, Georgiou and Babajimopoulos, 2011). A crop stress coefficient (Ks) represents the reduction of ETa under water stress conditions. Figure 2.6 indicates the effect of soil water content on ETa as quantified by the water stress coefficient, Ks according to Allen *et al.* (1998).

Figure 2.6 shows that a crop does not experience water stress as long as the water content of the soil (θ) is between field capacity (θ_{FC}) and the threshold soil water content (θ_t) at which water stress sets in. θt defines the portion of TAW that is readily available for transpiration or RAW. Above θ_t the value of Ks is equal to one which indicates that no ET deficits occur. As water is extracted from the soil beyond θ_t, ETa decreases below ETm as represented by a proportional reduction of Ks until wilting point (Allen *et al.*, 1998). Crop water stress is a function of soil water content θ where applied water is used to increase θ. Crop water stress can influence crop growth and yield (Kallitsari *et al.*, 2011). Thus, to avoid crop stress it is important to manage the field water supply through irrigation water applications in such a way that θ stays above θ_t. Keeping track of θ through a daily water budget routine is therefore of the utmost importance when managing the amount and timing of irrigation water applications as irrigation applications do not prevent crop water stress per se but the θ in relation to θ_t does.



Where;

TAW = Total available water in the root zone (mm)

RAW = Readily available water in the root zone (mm)

FC = Field capacity soil water content (mm)

WP = Wilting point soil water content (mm)

Θ = Soil water content

Source: Allen *et al.*, 1998

Figure 2.6: Effects of soil water stress on actual crop evapotranspiration (ETa) as represented by a crop stress coefficient Ks.

Although evapotranspiration is the field level water parameter associated with yield, irrigation represents water applied which is a concern to planners, including economists and irrigators (Stewart and Hagan, 1973). Next, the relationship between applied irrigation water and crop yield is discussed.

2.3.2 WATER PRODUCTION FUNCTION

Water production function is a relationship between applied irrigation water and yield which is significant for management decisions (Stewart and Hagan, 1973; Rao *et al.*, 1988; Li, 1988; Kipkorir, Raes and Massawe, 2002; Fereres and Soriano, 2007). Farmers hence control FWS through water applied rather than ET in order to manage the soil water content to meet crop water requirements. Irrigation events are scheduled to compensate water losses to avoid crop water stress when rainfall is not sufficient (Irmak, 2017).

The curvilinear form of the Y vs IRR in Figure 2.5 shows a decrease in irrigation efficiency due to applied water that is not consumed (water losses). Water losses are represented as non-evapotranspiration (NON-ET) which are deep percolation, runoff, and wind drift. Deep percolation and run-off is a direct result of irrigation management and irrigation technology use. Inefficiencies results if water is applied non-uniformly since part of the field will be over irrigated and the other part under irrigated (Stewart and Hagan, 1973; Vaux and Pruitt, 1983, Li, 1988).

2.3.3 DISCUSSION AND CONCLUSION

A clear distinction between yield as a function of ET and yield as a function of applied irrigation water is necessary when developing economic decision-making models of crop water use as the former relationship ignores inefficiencies. Furthermore, it is important to note that applied irrigation water per se does not relate to crop growth but root water content in relation to the threshold root water content where water stress sets in. However, irrigation is the most important variable controlled by the farmer to ensure that the water content of the root zone is depleted beyond θ_t. Changes in soil water content is a dynamic which requires daily water budget calculations to track the onset and magnitude of water stress.

2.4 ALTERNATIVE CROP WATER PRODUCTION FUNCTIONS

Figure 2.5 suggests that the relationship between crop yield and ET is linear. However, research has shown that the sensitivity of a crop to water stress in different crop growth stages are different. Furthermore, a combination of water stress in different crop growth stages may alter the rate at which ET is reduced. Several methods exist to relate ET deficits in specific crop growth stages to actual crop yield which are reviewed in this section.

2.4.1 DOORENBOS AND KASSAM (1979) FUNCTION

Doorenbos and Kassam (1979) presented the following linear relationship between relative yield decrease and relative evapotranspiration deficit:

$$\frac{Ya}{Ym} = 1 - Ky\left(1 - \frac{ETa}{ETm}\right) \tag{2.1}$$

Ya = actual yield
Ym = maximum yield
Ky = yield response factor
ETa = actual evapotranspiration
ETm = maximum evapotranspiration

The yield response factor (Ky) of this function represents the slope of the linear relationship between the relative evapotranspiration and relative yield over the season. Consequently, the function cannot be used to evaluate the effect of water stress during the different growth stages on crop yield. Doorenbos and Kassam (1979) do provide Ky factors indicating the impact of water stress on crop yield in different crop growth stages. However, no guidance is given on how to apply the Ky factors when water shortage occurs cumulatively during different stages of crop growth (Saleh, Abdulaziz and Wardlaw, 2009).

2.4.2 EXPONENTIAL FUNCTION

Jensen (1968) proposed an exponential function (Cobb-Douglas) to determine the increasing effect of water shortage in different stages of crop growth. The function used is of the form:

$$\frac{Ya}{Ym} = \prod_{i=1}^{n} \left(\frac{ETai}{ETmi}\right)^{\lambda i} \qquad (2.2)$$

The Jensen function uses the crop sensitivity factor (λi) for each stage of growth and the sensitivity factor is used as an indicator for the effect of water stress on crop yield. The higher the value of (λi), the greater the effect of water stress. A problem with the application of the function is in determining appropriate values of (λi). The multiplicative nature of the function requires that some level of ET must be satisfied in each stage otherwise Y will equal to 0.

2.4.3 MULTIPLICATIVE FUNCTION

The Minhas *et al.* (1974) developed an economic production function taking relative evapotranspiration in various crop growth stages for prediction of crop yield.

$$\frac{Ya}{Ym} = \prod_{i=1}^{n} \left(1.0 - \left(1.0 - \frac{ETa}{ETm}\right)_i^2\right)^{\delta i} \qquad (2.3)$$

Where (δi) refers to the sensitivity of crop stage (i) for moisture availability and the other parameters are the same as defined by Equation 2.1. This function was developed to evaluate the interdependence between the rate of water used by plants and available moisture in the soil. Igbadun, Tarimo, Salim and Mahoo (2007) who compared the accuracy of different crop water production functions to predict irrigated maize crop yield found that the Minhas model was the least preferred model.

The Stewart, Danielson, Hanks, Jackson, Hagon, Pruit, Franklin and Riley (1977) function uses the Ky-factors reported by Doorenbos and Kassam (1979) for each growth stage to relate relative yield to relative evapotranspiration deficits in each stage.

$$\frac{Ya}{Ym} = \prod_{i=1}^{n} \left[1 - Kyi\left(1 - \left(\frac{ETai}{ETmi}\right)\right)\right] \qquad (2.4)$$

The multiplicative nature of the function ensures that a more than proportional reduction of crop yield is modelled if water stress occurs in more than one crop growth stage. The function assumes that lack of evapotranspiration at any crop growth stage may not necessarily lead to total crop failure but could have severe impact on yield performance (Kallitsari *et al.*, 2011).

2.4.4 ADDITIVE FUNCTION

$$\frac{Ya}{Ym} = 1 - \sum_{i=1}^{n} Kyi\left[1 - \left(\frac{ETai}{ETmi}\right)\right] \qquad (2.5)$$

Stewart *et al.* (1977) also introduced an additive version of the Stewart *et al.* (1977) multiplicative function. The additive function assumes that the effect of water stress in each of the crop growth stages is independent of each other (Rao *et al.*, 1988). A potential advantage of the function is the fact that it is linear.

2.4.5 DISCUSSION AND CONCLUSIONS

The Stewart *et al.* (1977) multiplicative function is a widely applied function to model the economics of deficit irrigation systems (Grové, 2019; Grové & Oosthuizen, 2010; Montazar, Riazi and Behbahani, 2010; Garg and Dadhich, 2014; Montazar, 2013; Domínguez, de Juan, Tarjuelo, Martínez, and Martínez-Romero, 2012) because it models a more than proportional relative yield decrease if water deficits occur in more than one crop growth stage. The function is easy to apply because the Ky-factors for the function is published in Doorenbos and Kassam (1979). The only factor influencing crop yield is evapotranspiration deficits. Consequently the crop water production functions need to be integrated with some type of inefficiency model to relate applied irrigation water to crop yield.

2.5 DEVELOPING WATER PRODUCTION FUNCTIONS

The main purpose of this section is to review the different methods used by researchers to model the relationship between applied irrigation water and crop yield while using a relative evapotranspiration function to quantify the impact of ET deficits on crop yield. Incorporating inefficiencies into the modelling framework is inherently difficult because irrigation efficiency is a function of the timing of water applications, amount of water applied, soil water status at the time of irrigation and the uniformity of irrigation applications (Grové & Oosthuizen, 2010). Typically, efficiencies increases nonlinearly with increasing levels of ET deficits.

2.5.1 CONSTANT EFFICIENCIES

Constant efficiencies implies that the efficiency with which water is applied does not change with increasing levels of ET deficits. Such a condition is modelled by substituting the actual to potential evapotranspiration ratio (ETai/ETmi) with the ratio of actual water irrigated to potential water irrigated under non-stressed water conditions (Wai/Wpi) as is illustrated by Equation 2.6 for the Stewart multiplicative crop water production function (Montazar & Rahimikob, 2008; Montazar *et al.*, 2010):

$$\frac{Ya}{Ym} = \prod_{i=1}^{n}\left[1 - Kyi\left(1 - \left(\frac{Wai}{Wpi}\right)\right)\right] \tag{2.6}$$

English *et al.* (2002) argue that the economic benefits of deficit might be under-estimated if increasing efficiencies under deficit irrigation is ignored. Consequently assuming constant efficiencies is not realistic.

2.5.2 LINEAR INCREASES

Grové & Oosthuizen (2002) optimized agricultural water use while quantifying economic environmental trade-offs of maintaining instream flow requirements. The impact of water deficits on crop yield were optimised using the Stewart multiplicative crop water production function while irrigation efficiencies were assumed to increase linearly between maximum water application and a given maximum allowed deficit. The procedures developed by Willis (1993) were used to model increasing efficiencies through synthetic irrigation activities. Results indicated that it is profitable to practice deficit irrigation while spreading available water over larger irrigation areas. Though increasing irrigation efficiencies were modelled as the crop was deficit irrigated in each growth stage, no link exists between the water budgets in different crop growth stages. Consequently, the time step of the model is a growth stage.

Homayounfar, Lai, Zomorodian, Sepaskhah and Ganji (2014) also developed a procedure to increase irrigation efficiency linearly with increasing levels of deficit irrigation. In essence the procedure is similar to the constant efficiency approach described in the previous section. However, the Wai/Wpi ratio is adjusted with a factor (α) based on the proportion of total water requirement that is supplied (x). The value of α was calculated using different assumed values of x using the following equations:

$$\begin{cases} \alpha = 1 & x = 0 \qquad (i) \\ \alpha = 1 + 2\left(\frac{1}{\eta f} - 1\right) & 0 < x < 0.5 \qquad (ii) \\ \alpha = \frac{1}{\eta f} & x \geq 0.5 \qquad (iii) \end{cases} \tag{2.7}$$

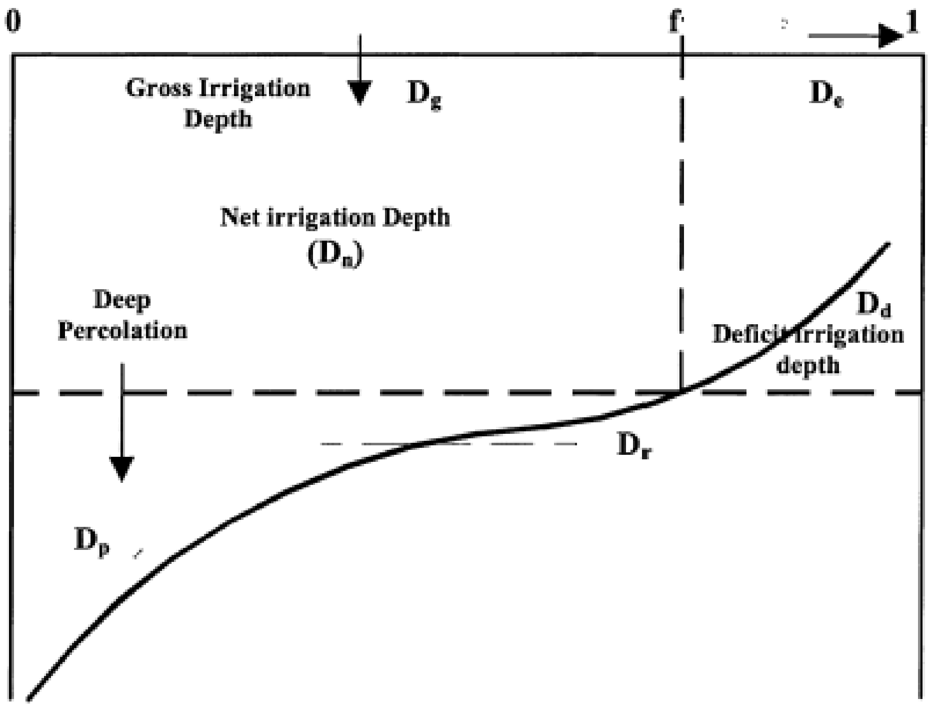
ηf defines the irrigation efficiency under conditions of no water stress. A shortcoming of the procedure is the fact that α is determined exogenously. Thus, the level of water deficit must be known ahead of the analysis.

2.5.3 NON-LINEAR INCREASES

Non-uniform water applications may result in inefficiencies because some portion of the irrigation field will be over irrigated and some portion will be under irrigated. The chance of deep percolation increases significantly when trying to ensure that the portion that is under irrigated receives the right amount of irrigation water, thereby decreasing the irrigation efficiency. The uniformity with which an irrigation system applies water is measured with the Christiansen's coefficient of uniformity (CU). Two procedures have developed in literature to account for the CU of an irrigation system. The first approach is based on a seasonal relationship while the second explicitly models non-uniform water application using multiple water budgets. These two approaches are discussed in more detail below.

2.5.3.1 CU – production functions

The first method assumes a statistical distribution to simulate non-uniform water distribution for irrigation systems based on the Christiansen's Uniformity coefficient (De Juan *et al.*, 1996; Reca, Roldan, Alcaide, Lopez and Camacho, 2001). Normally distributed irrigation depths are shown in Figure 2.7.



Source: Li (1998)

Figure 2.7: Distribution of irrigation depths assuming a normal distribution

Figure 2.7 shows that only a portion (f) of the field will receive the net irrigation amount of Dn. Non-uniform water applications result in percolation of Dp amount of water while satisfying the Dn on f while the other portion of the field (1-f) receives too little water as indicated by Dd. The effect of non-uniform irrigation applications can be incorporated into relative crop water production functions by relating the relative evapotranspiration deficit (1-ETa/ETm) to the relative deficit coefficient (1-Dd/Dn). The CU of an irrigation system is directly related to the dispersion of irrigation applications. A water production function could be derived for any level of CU for an assumed statistical distribution. The normal and uniform distributions are mostly used in literature to model the impact of non-uniform irrigation applications (Mantovani, Villa Lobos, Orgaz and Fereres, 1995; Li, 1998; De Juan *et al.*, 1996; Reca *et al.*, 2001; Barragan, Cots, Monserrat, Lopez and Wu, 2010).

2.5.3.2 Multiple water budgets

The second method uses soil water budget simulation models to explicitly simulate the impact of non-uniform water applications (Lecler, 2003; Lecler & Griffith, 2003; Junman & Lecler, 2010; Grové & Oosthuizen, 2010; Venter & Grové, 2016). Venter and Grové (2016) investigated the gains associated with increasing the number of water budgets to represent non-uniformity and found that three water budgets were sufficient to capture the non-uniformity of irrigation water applications while assuming a uniform distribution of water between a given maximum and minimum water application. The amount of water that is allocated to each water budget is derived from the CU of the irrigation system in conjunction with the assumed statistical distribution of irrigation water for the system. Lecler (2003) found that the different levels of irrigation system performances had a significant impact on the yields obtained thereby accentuating the importance of modelling inefficiencies.

The only factor that contributes to inefficiencies in the research described above is differences in irrigation water applications between the different water budget simulation models. Several researchers (Hamilton, Green and Holland, 1999; Lopez-Mata *et al.*, 2010) have recognised the spatial variability in soil depths, infiltration characteristics, water holding capacities and distribution of applied water as a source of inefficiency. Hamilton *et al.* (1999) integrated the Cropping Systems Simulation Model (CropSyst) and the Irrigation Efficiency Model (IEM) to produce crop water production functions to simulate how changes in water use have an effect on crop yields as well as quantity and quality of excess applied water. CropSyst is a multilayer, multi-crop, daily time-step growth simulation model that examines effects of crop-systems management on crop productivity and the environment. The model has an irrigation management module, though it does not differentiate between irrigation technologies and assumes constant irrigation uniformity. The IEM models inefficiencies by dividing the irrigation field into sectors with randomly assigned values by using Monte Carlo simulation. The integration of both models allows the researchers to model inefficiencies associated with application of irrigation water.

2.5.4 DISCUSSION AND CONCLUSION

Various methods are available to model the impact of inefficiencies on crop yield. The methods vary in terms of sophistication with the majority of the models not relating inefficient water applications to the soil water status on a daily basis. Inefficiencies is the direct consequence of over irrigation of certain parts of the soil due to non-uniform water application. Losses that result in inefficiencies only occur if enough water is applied to wet the soil beyond field capacity. Non-uniform water applications might be efficient if the soil is relatively dry. Consequently the best way to model inefficiencies is to keep track of the soil water status which requires daily water budget calculations.

2.6 SOLVING COMPLEX WATER ALLOCATION PROBLEMS

Managing soil water moisture through irrigation applications is critical to ensure that enough water is available to sustain crop transpiration needs necessary to achieve potential crop yields. Irrigation management becomes even more important under conditions of limited water supply since the reduction in crop yield due to water deficits is differentiated by crop growth stage (Doorenbos and Kassam, 1979). A further complicating factor is that the onset and duration of crop water stress is determined by the root water content in relation to a critical root water content. Therefore, the only way to determine the onset and duration of crop water stress is to keep track of the root water content. Including soil water budget calculations into standard mathematical programming approaches to optimise irrigation water allocation problems is troublesome due to the necessity of using discontinuous nonsmooth functions that might be nonlinear to represent the system (Grové, 2019). In many cases the water allocation problem becomes too complex to represent within a mathematical programming matrix or the available solvers are unable to achieve global optimal solutions (Schütze *et al.*, 2012).

Several optimisation techniques have developed recently to overcome the problems of standard linear and non-linear optimisation algorithms to achieve global optimal solutions of complex optimisation problems. Evolutionary algorithms provide an alternative optimisation procedure to mathematical programming approaches to optimise complex real world problems (Maier, Kapelan, Kasprzyk, Kollat, Mattot, Cunha, Dandy, Gibbs, Keedwell, Marchi, Ostfeld, Savic, Solomtatine, Vrugt, Zecchin, Minsker, Barbour, Kuczera, Pasha, Castelleti, Giuliani and Reed, 2014; Cobo, Camacho, Montesinos and Diaz, 2014; Garica, Diaz, Camacho, and Montesinos, 2013; Nicklow, Reed, Dessalegne, Harrell, Chan-Hitton, Karamouz, Minsker, Ostfeld, Sign, Zechman, 2010; Spall, 2003). Various evolutionary algorithms have developed over time, however, genetic algorithms (GAs) are the most popular of the evolutionary algorithms (Louati, Benabdallah, Lebdi,Milutin., 2011; Nicklow *et al.*, 2010; Sivanandam and Deepa, 2008; Spall, 2003) with several applications within water resource related problems (Johns,KeedWell and Savic, 2014; Schütze *et al.*, 2012; Haq, Anwar, and Clarke, 2008; Michalewicz, 1996). Another interesting approach is to combine genetic algorithms with mathematical programming optimisation methods in what is called a hybrid approach to optimise complex systems (Rani and Srivastava, 2016; Cai, McKinney and Lasdon, 2001). Next the working op GAs and the hybrid approach is discussed in more detail.

2.6.1 GENETIC ALGORITHMS

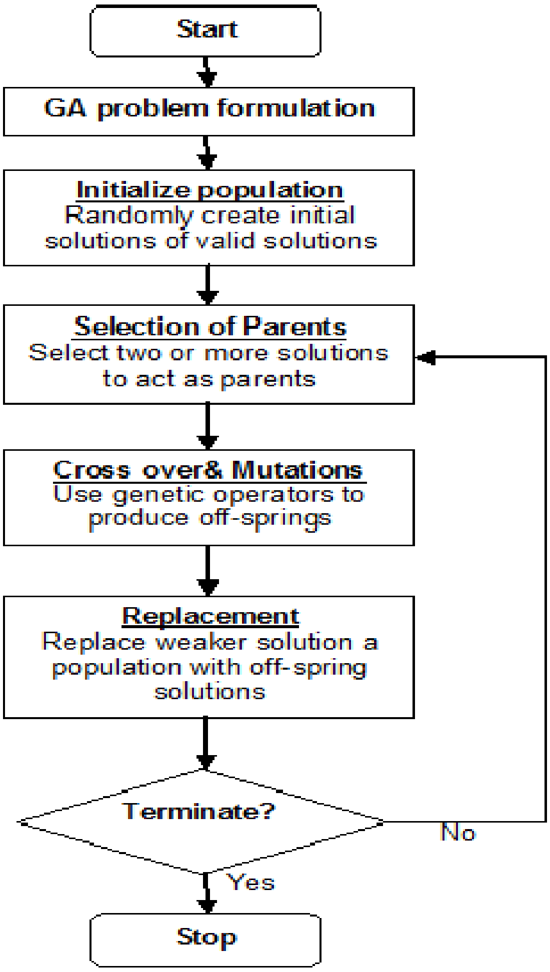
GAs are stochastic optimization procedures, which are inspired by the theories of natural selection and the mechanics of biological evolution (Akhbari and Grigg, 2014; Rana, Khan and Rahimi, 2008).

Methodologically a GA aims at improving the initial population of candidate solutions through the processes of crossover and mutation (Elsayed, Sarker and Essam., 2014; Anwar and Haq, 2013; Louati *et al.*, 2011; Karamouz, Zahraie, Kerachian and Eslami., 2010; Van Vuuren, Van Rooyen, Van Zyl and Van Dijk, 2005). The evolution process also requires other processes like the choice of candidate solutions (parents) that need to be altered through crossover and mutation to produce new candidate solutions (offspring) and the replacement of unfit candidate solution. Selection is done based on the quality of the answer of the candidate solution which is referred to as the fitness of the solution.

Figure 2.8 provides a flowchart of the processes necessary to employ a genetic algorithm. Formulating the optimisation problem provides the first step in applying a GA as it defines the number of variables as well as the search space of the variables that need to be optimised. The next step is to randomly generate a population of possible feasible solutions to the optimisation problem. The number of solutions that need to be specified is problem specific (Anwar and Haq, 2013). The most important consideration when generating the initial population is that the magnitude of the variables must be as diverse as possible between candidate solutions irrespective of the problem and initialising method (Rajkumar and Thompson, 2002).

Once the population of candidate solutions are generated, the levels of the variables are used in the mathematical accounting equations representing the system being modelled to determine the outcome or fitness. The mathematical specification of the system does not matter as the levels of the decision variables in the model are determined exogenously. Consequently, the mathematical representation of the system being modelled is some form of complex simulation model conditionally on the calculation of an outcome variable that determines the quality of the solution. The outcome variable is synonymous with the objective function value of a mathematical programming model and is referred to as the “fitness” within GA applications. The fitness of the solutions provides a means to determine the quality of the candidate solutions in the population. The initial population evolves through the processes of crossover and mutation which requires the selection of parents.

Parents of offspring are selected given a large chance of selecting a parent with a high fitness value (Maier *et al.*, 2014; Van Vuuren *et al.*, 2005). Therefore, the genes of the best individuals (higher fitness) are transferred to the next generation (Anwar and Haq, 2013; Haq and Anwar, 2010). Popular selection methods used in literature are the roulette wheel, uniform random and tournament selection methods (Van Vuuren *et al.*, 2005). Nicklow *et al.* (2010) provides a detailed discussion of the alternative methods. The selected parents are subject to crossover in order to produce new offspring.



Source: Van Vuuren *et al.* (2005)

Figure 2.8: Flow chart of the basic methodology of genetic algorithms.

The main purpose of crossover is to produce offspring that share some of the characteristics of the parent solutions with higher fitness values (Elsayed *et al.*, 2014; Anwar and Haq, 2013). During the crossover process, the genes of a selected parent's chromosome are allowed to be swapped with the other parent's genes according to certain criteria (Spall, 2003; Mitchell, 1996). The specific criteria that is used to govern the crossover process define different methods of crossovers such as single point or multiple crossover, uniform random crossover and arithmetic crossover. A detail description of employing the criteria to facilitate crossover is given by Van Vuuren *et al.* (2005) and Sivanandam and Deepa (2008). According to Kerachian and Karamouz (2005) literature does not provide guidance regarding the superiority of the different crossover methods.

Crossover alone may result in loss of diversity which may cause early convergence at a local optima. Mutation provides a powerful means to restore the diversity of the population, thereby decreasing the chance of early convergence or getting trapped in at a local optima (Elsayed *et al.*, 2014). During mutation the genes of the offspring are allowed to be changed which results in a chromosome that is different from the parents. By implication, mutation allows for a larger search space to be explored.

After determining the fitness of the newly produced offspring, the next step is to determine which members of the population the offspring must replace. Replacing the weakest individual (candidate solution) might not be advisable as this might lead to quick convergence (Spall, 2003). Careful consideration should therefore be given to the replacement strategy (Van Vuuren *et al.*, 2005).

The initial population is evolved through the processes of crossover and mutation until an appropriate termination criteria is attained (Anwar and Haq, 2013; Haq and Anwar, 2010; Van Vuuren *et al.*, 2005; Spall, 2003). According to Maier *et al.* (2014) the best termination criteria for terminating real-world irrigation problems is yet to be determined. Termination criteria could be very simple such as terminating the GA after a predefined number of iterations or specified fitness value is achieved (Anwar and Haq, 2013). Other termination criteria are based on the convergence of the model such as terminating the GA when no significant increase in the fitness is observed (Anwar and Haq, 2013). Determining the right termination criteria requires knowledge of the problem and experimentation (Nicklow *et al.*, 2010; Van Vuuren *et al.*; 2005).

2.6.1.1 Constraint handling

Satisfying constraints within a model that is solved with a GA is troublesome if the constraint relates to a variable that is a function of a variable that is controlled by the GA. In such cases there are no easy means of generating variable levels that will satisfy the constraint that is related to the dependant variable. A popular method to handle this type of constraint is to add the violations of the constraint as a penalty in the fitness function of the model where the penalty term measures the solution's distance from feasibility (Elsayed *et al.*, 2014; Johns *et al.*, 2014; Anwar and Haq, 2013; Nicklow *et al.*, 2010). Elsayed *et al.* (2014) distinguish two types of penalty function namely as static and dynamic. Static penalty is usually used and it is a penalty factor remains constant throughout the EAs. Another method commonly used in irrigation water allocation problems is proportional scaling. Grové and Du Plessis (2019) used proportional scaling to ensure that the sum of irrigation amounts over the season is less or equal to the allocated water quota for one crop.

2.6.2 HYBRID METHODS

In the context of this research, hybrid methods refer to the use of a combination of different methods to solve complex representations of water allocation problems. Of particular interest is the method developed by Cai, McKinney and Lasdon (2001) in which a GA is used in combination with linear programming (LP) to solve a multiperiod reservoir operation model. The method begins by identifying complicating variables in the original model. Fixing the complicating variables in a model to predetermined levels allows standard solvers to solve the complex model to optimality. With the hybrid solution procedure, the GA is used to fix the complicating variables and a linear programming solver is used to solve the linear program to optimality. In essence, the LP model is used to determine the fitness of the population of candidate solutions generated by the GA. The added benefit is that the variables that are optimised with the LP need not to be part of the GA as these variables will be optimal for each GA generation.

Results from the analysis show that the GA-LP approach was able to find a better solution when compared to a mathematical programming approach of the complex system. However, it is important to note that the nonlinear programming solver was able to improve the GA solution when the GA solution was provided as a starting point.

2.6.3 Discussion and Conclusion

Realistic representations of agricultural water use in economic decision-making models require daily water budget calculations which are extremely difficult to incorporate into mathematical programming models that use gradient-based solvers. Consequently, alternative means of solving complex systems need to be explored. The Hybrid GA-LP approach seems to be a good alternative, as it is able to handle nonsmooth nonlinear functions while the LP handles the constraint set. Such an approach requires development of a tailor made algorithm.

CHAPTER 3: INTEGRATED HYDRO-ECONOMIC MODEL DEVELOPMENT

The conceptual framework that guided the development of methods, procedures and models to model the hydro-economic impact of water curtailments is discussed first. Subsequently, the different components of the framework are discussed in more detail followed by the model configuration.

3.1 CONCEPTUAL FRAMEWORK FOR ANALYSING WATER CURTAILMENTS

The conceptual model of analysis uses the hybrid optimisation method to optimise the irrigation water use over the short-run and long-run to determine the hydro-economic impact of water curtailments. The conceptual linking of the hydrology, crop water use and economic models to optimise the long-run responses of irrigation farmers to water curtailments with the hybrid solution methodology is presented in Figure 3.1.

The GA controls the optimisation because the algorithm determines the data flow between different models. The GA firstly generates a population of candidate irrigation schedules based on the type of crops, soils and uniformity of the irrigation systems. The FAO 56 type irrigation model that is integrated into MIKE HYDRO BASIN is used to simulate the crop water use and crop yield of a specific crop, soil and irrigation system uniformity combination using a daily soil water budget in conjunction with the Stewart *et al.* (1977) evapotranspiration crop yield function. The irrigation schedules from the GA and the crop yields from the irrigation model is used to calculate the gross margins associated with each irrigation schedule. The gross margins and irrigation schedules are then used in a long-run dynamic mixed integer nonlinear programming (DMINLP) model to determine crop areas and irrigation technology adoption decisions in the long-run.

The hydrology component of the MIKE HYDRO BASIN model is used to provide the DMINLP model with the water availability for a specific water curtailment scenario. A unique capability of using a mathematical programming model to optimise the choice of irrigation technology and crop areas in the long-run is the fact that it is easy to ensure that the modelled response is within the financial, resource and water availability constraints of the farm.

Applying the conceptual framework to evaluate the hydro-economic impact of water curtailments entails the development of the GA, DMINLP model and linking of the different components.

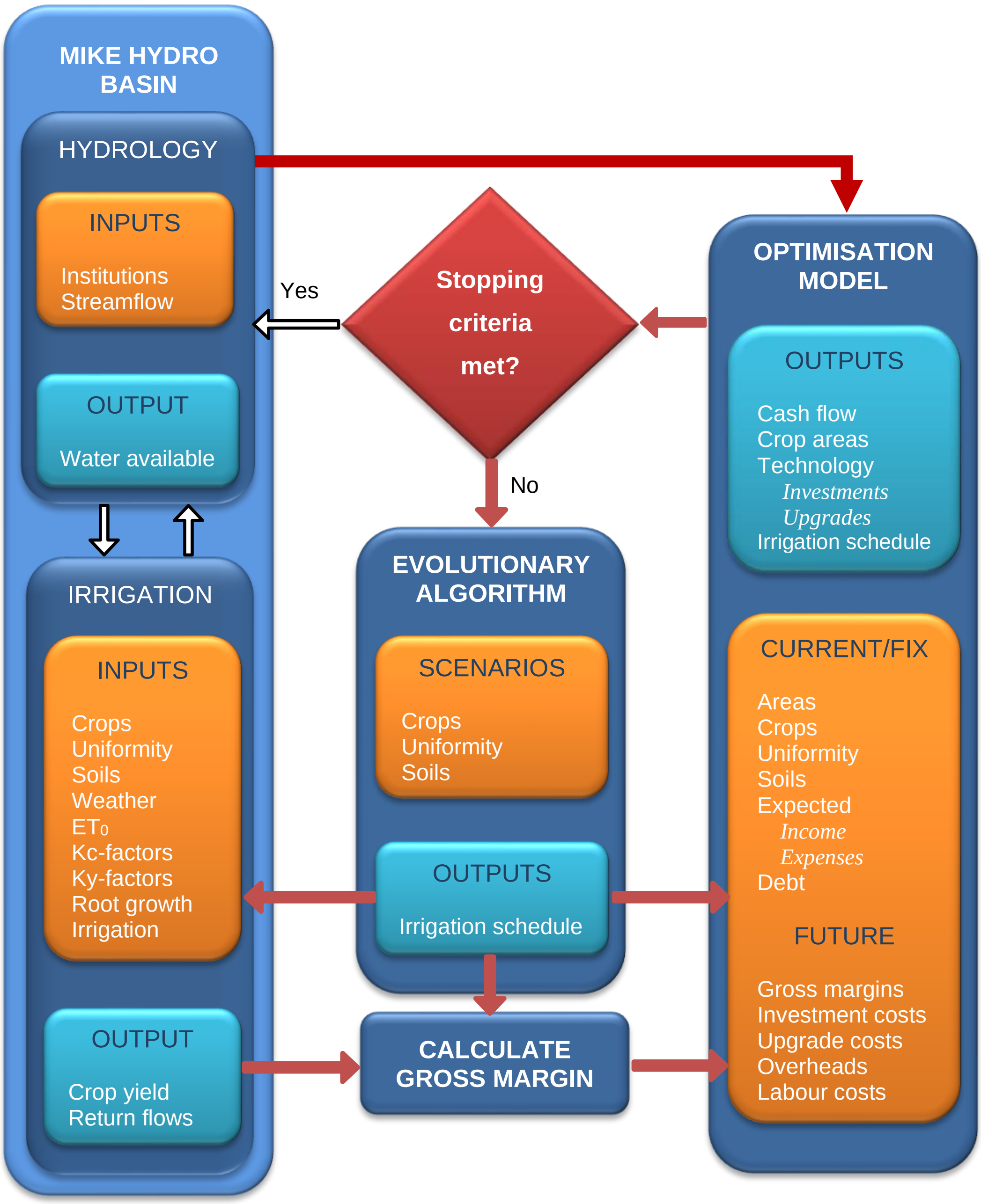


Figure 3.1: Conceptual hydro-economic model linking for evaluating water curtailments

3.2 CATCHMENT SCALE HYDROLOGY

The Catchment scale water resource assessment is discussed in section 2. The preferred model in for the catchment scale hydrology is MIKE Hydro Basin. Mike Hydro Basin is referred to as a node-and-channel network model. There are different types of nodes, including nodes to represent catchments, dams and water users. The channel represents rivers, pipes, canals and other links between water sources and water users. The MIKE Hydro Basin model requires time series of river flows, details of dams (e.g. the height, volume area relationship of dams), and a time series of water demands by water users.

In this study, a MIKE Operation platform was adopted to register the catchment scale, farm scale Irrigation model and the Excel economic model in to the platform. The models that are registered into the MIKE Operation Platform were linked in series to run and produce the Gross Margin on the Farms that are modelled. The MIKE Operation platform has an optimisation module built-in, which was used in this study to optimised or maximise the Gross Margins.

MIKE Operation is developed by DHI for developing client specific solutions and decision support systems. The MIKE Operation is a platform to develop customised solutions or decision support system customised to client needs. One of the main or important characteristics of the MIKE Operation software is that it is open to any user to add functionality to the platform without access to the main source code. In this study, the scenario manager of the MIKE Operation platform is utilised to register the Hydrology and economic model into the platform. In the platform, the Hydrology and economic models are coupled to run in series and the optimisation in the MIKE Operation is configured to run the coupled model and optimise the Gross Margins.

3.3 SHORT-RUN IRRIGATION WATER USE OPTIMISATION MODEL

The manner in which the temporal crop water use is represented in an irrigation water use optimisation model is key to the prediction an irrigator's optimal response to water curtailments. Under limited water supplies the allocation of water between crops becomes complex because water stress and resulting crop yield reductions set in when root water content falls below a critical point. The problem is that the onset and duration of water stress can only be determined by tracking the root water content. Consequently, daily water budget calculations become key to the prediction of an irrigator's optimal response to water curtailments. Next, a description of the short-run irrigation water use optimisation model that includes daily water budget calculations is given. The convention is followed whereby capital letters are used to represent variables in the model.

3.3.1 OBJECTIVE FUNCTION

The objective of the short-run irrigation water use optimisation model is to maximise certainty equivalent of the distribution of total gross margin of the farm:

$$MAX\ CE = \frac{\ln\left(-\sum_r^R \frac{1}{R}\left(-e^{-ara(TGM_r)}\right)\right)}{-ara} \tag{3.1}$$

CE	absolute risk aversion certainty equivalent (R)
TGM_r	total gross margin for state of nature r (R)
ara	absolute risk aversion coefficient

The certainty equivalent represents the sure amount which makes a decision-maker indifferent between the sure amount and the risk. The certainty equivalent is calculated suing a negative exponential utility function using a constant absolute risk aversion coefficient. The *ara* levels were chosen such that the risk premium conforms to be less than 1.25 standard deviations for a normally distributed risk.

3.3.2 VARIABLE CALCULATIONS

3.3.2.1 Distribution of total gross margin

The following equation is used to calculate the distribution of total gross margin of the farm:

$$\begin{aligned} TGM_r = & \sum_i \sum_s \sum_{cu} \sum_c \sum_w HA_{i,s,cu,c} p_{r,c} Y_{r,w,s,cu,c}/3 \\ & - \sum_i \sum_s \sum_{cu} \sum_c \sum_w HA_{i,s,cu,c} oy_{r,i,s,cu,c}\, Y_{r,w,s,cu,c}/(3card(r)) \\ & - \sum_i \sum_s \sum_{cu} \sum_c \sum_w HA_{i,s,cu,c} of_{i,s,cu,c}\, Y_{r,w,s,cu,c}/3 \\ & - \sum_i \sum_s \sum_{cu} \sum_c HA_{i,s,cu,c} oa_c \\ & - \sum_i \sum_s \sum_d \sum_{tou} HOUR_{i,s,d,tou} rep_i \\ & - \sum_i \sum_s \sum_d \sum_{tou} HOUR_{i,s,d,tou} aetar_{d,tou} kw_i \\ & - \sum_i \sum_s \sum_d \sum_{tou} HOUR_{i,s,d,tou} retar_{d,tou} kvar_i \\ & - \sum_i \sum_s \sum_d \sum_{tou} HOUR_{i,s,d,tou} flow_i wtar \end{aligned} \tag{3.2}$$

$HA_{i,s,cu,c}$	hectares of crop *c* planted on soil *s* using irrigation system *i* with a CU of *cu* (ha)
$p_{r,c}$	randomly generated price *r* for crop *c* (R/ton)
$Y_{r,w,s,cu,c}$	crop yield of crop *c* for water budget *w* on soil *s* using irrigation system *i* with a CU of *cu* for random weather *r* (t/ha)
$oy_{i,s,cu,c}$	yield dependant harvesting costs of crop *c* planted on soil *s* using irrigation system *i* with a CU of *cu* (R/ton)
$of_{i,s,cu,c}$	yield dependant fertiliser costs of crop *c* planted on soil *s* using irrigation system *i* with a CU of *cu* (R/ton)
oa_c	area dependant costs of crop *c* (R/ha)

$HOUR_{i,s,d,tou}$	irrigation hours for day d within time-of-use timeslot tou using irrigation system i on soil s (hours)
rep_i	repair and maintenance cost of irrigation system i (R/hour)
$aetar_{d,tou}$	active energy charge on day d within time-of-use timeslot tou (R/kWh)
kw_i	kilowatt of irrigation system i (kW)
$retar_{d,tou}$	reactive energy charge on day d within time-of-use timeslot tou (R/kVarh)
$kvar_i$	kilovar of irrigation system i (kVar)
$flow_i$	flow rate of irrigation system i (m^3/h)
$wtar$	water tariff (R/m^3)

The endogenously calculated crop yield that is used to calculate production income is the average yield across the three water budgets that is used to model the effect of non-uniform irrigation applications in the irrigation model. Variable costs are differentiated based on whether the specific cost component is dependant on harvested crop yield, expected yield to calculate fertiliser costs, area planted or the hours needed to pump irrigation water. Electricity, repair and maintenance costs are dependant on the use of the irrigation system. The model allows for the incorporation of the Ruraflex time-of-use electricity tariff which is the most commonly used electricity tariff in the research area. The water tariff is charged based on the total amount of water pumped.

3.3.2.2 Crop yield and water budget calculations

The model uses the Stewart *et al.* (1977) relative ET function to relate crop yield to relative ET deficits using the Ky-factors reported by Doorenbos and Kassam (1979):

$$Y_{r,w,s,cu,c} = yp_{r,c} \prod_{g} \left[1 - ky_{g,c} \left(1 - \frac{\sum_{d \in g} ETa_{r,w,s,cu,c,d}}{\sum_{d \in g} etm_{r,w,s,cu,c,d}} \right) \right] \tag{3.3}$$

$yp_{r,c}$	yield potential of crop c for random weather r (t/ha)
$ky_{g,c}$	crop yield response factor of crop c in crop growth stage g (fraction)
$ETa_{r,w,s,cu,c,d}$	actual evapotranspiration of crop c on day d for water budget w, soil s, CU cu and random weather r (mm)
$etm_{r,w,s,cu,c,d}$	maximum potential evapotranspiration of crop c on day d in year t for water budget w, soil s, CU cu and random weather r (mm)

Crop yield is calculated by relating relative evapotranspiration deficits to relative yield differences using a multiplicative calculation which results in a more than proportional yield decrease if the crop is deficit irrigated in more than one crop growth stage. The yield function is highly nonlinear due to the multiplicative calculation which makes the model more difficult to solve with standard nonlinear programming solvers. Yield potential is a function of the weather and is therefore random. ETa values are calculated using daily water budget calculations which are discussed next.

Water budget calculations are done based on the basic methodology proposed in FAO-56 (Allen *et al.*, 1998) to calculate ETa based on a single crop coefficient. However, the water budget calculations are more refined as it include a potential root zone. The significance of the potential root zone is that water that drains below the roots during early development of the crop is not lost as the crop may use the water when the roots develop deeper. The following equations are used to calculate ETa from the water budget calculations:

$$ETa_{r,w,s,cu,c,d} = min \left| \begin{matrix} etm_{r,c,d} \\ etm_{r,c,d} \left(\frac{RWC_{r,w,s,cu,c,d}}{trwc_{r,s,c,d}} \right) \end{matrix} \right. \tag{3.4}$$

$$RWC_{r,w,s,cu,c,d} = min \left| \begin{matrix} rwcap_{s,c,d} \\ RWC_{r,w,s,cu,c,d-1} - ETa_{r,w,s,cu,c,d} + R_{r,d-1} + IR_{s,cu,c,d-1} sf_{w,cu} + TR_{r,w,s,cu,c,d} \end{matrix} \right. \tag{3.5}$$

$$BRWC_{r,w,s,cu,c,d} = \\ min \left| \begin{matrix} brwcap_{s,c,d} \\ BRWC_{r,w,s,cu,c,d-1} - ETa_{r,w,s,cu,c,d} + R_{r,d-1} + IR_{s,cu,c,d-1} sf_{w,cu} + TR_{r,w,s,cu,c,d} - rwcap_{s,c,d} \end{matrix} \right. \tag{3.6}$$

$$BR_{r,w,s,cu,c,d} = max \left| \begin{matrix} 0 \\ rwc_{r,w,s,cu,c,d-1} - ETa_{r,w,s,cu,c,d} + R_{r,d-1} + IR_{s,cu,c,d-1} sf_{w,cu} + TR_{r,w,s,cu,c,d} - rwcap_{s,c,d} \end{matrix} \right. \tag{3.7}$$

$$TR_{r,w,s,cu,c,d,t} = rg_{c,d} BRWC_{r,w,s,cu,c,d,t} \tag{3.8}$$

$RWC_{r,w,s,cu,c,d}$	root water content of water budget w on day d for soil s, CU cu, crop c and random weather r (mm)
$trwc_{r,s,c,d}$	threshold root water content of soil s on day d for crop c and random weather r (mm)
$rwcap_{s,c,d}$	total amount of water that could be stored on day d in the root zone of crop c planted on soil s (mm)
$R_{r,d}$	rainfall on day d for random weather r (mm)
$IR_{s,cu,c,d}$	irrigation of crop c on soil s using a CU of cu on day d for random weather r (mm)
$sf_{w,cu}$	scaling factor to adjust irrigation water of water budget w based on a CU of cu (fraction)
$TR_{r,w,s,cu,c,d}$	amount of water that move to the root zone on day d for water budget w, soil s, CU of cu and crop c
$BRWC_{r,w,s,cu,c,d}$	root water content below the roots of water budget w on day d for soil s, CU cu, crop c and random weather r (mm)
$brwcap_{s,c,d}$	total amount of water that could be stored on day d in the potential root zone of crop c planted on soil s (mm)
$BR_{r,w,s,cu,c,d}$	amount of water that drain below the root zone on day d for water budget w, soil s, CU of cu, crop c and random weather r
$rg_{c,d}$	root growth of crop c on day d

The ETa calculation shows that ETa is reduced once the RWC of the root zone falls below a certain threshold water content. RWC is a function of ETa, net irrigation (IR), rainfall (R) and water that drain below the root zone (BR) and any additions to RWC due to root growth (TR). The min function that is used to calculate RWC places an upper limit on the level of the RWC. Consequently, it is not necessary to include BR explicitly in the calculation of RWC. The water content of the zone below the roots ($BRWC$) on any day is a function of the previous days water content, the amount of water that drains into the zone as well as the amount of water that is lost due to root growth.

Non-uniform water applications cause some portion of the field to receive less and some portion to receive more water. Three distinct water budgets were included in the model to model non-uniformity with which an irrigation system applies water. The procedure proposed by Li (1998) is used to adjust the irrigation amounts of the water budget that receives more water and the one that receives less water while assuming that the non-uniform applications are uniformly distributed between a certain maximum and minimum level. Important to note is that variable IR is the average amount of water applied to the whole field.

3.3.2.3 Pumping hours

The calculation of pumping hours provides the means to link the extent with which an irrigation system is used to cost components that are dependant on the use of the irrigation system. Specifically the hours necessary to pump irrigation water is calculated as follows:

$$\sum_{tou} HOUR_{i,s,d,tou} = \sum_{cu} \sum_{c} \left(\frac{HA_{i,s,cu,c} IR_{s,cu,c,d}}{eff} \right) 10/flow_i \tag{3.9}$$

eff irrigation water application efficiency (fraction)

The water budget calculations use net irrigation amounts. Consequently, IR must be adjusted to account for wind drift losses. An application efficiency coefficient is used to take wind drift losses into account. The hours pumped within a specific time-of-use timeslot is restricted to be less than or equal to the hours available within a specific timeslot using the following constraint:

$$HOUR_{i,s,d,tou} \leq touhour_{d,tou} inum_{i,s,cu}$$

$touhour_{d,tou}$ hours available on day d within time-of-use timeslot tou (h)

$inum_{i,s,cu}$ quantity of irrigation system i with a CU of cu irrigating soil s (integer)

3.3.3 CONSTRAINTS

3.3.3.1 Irrigation area

The area irrigated is restricted to be less than the area available using the following constraint:

$$\sum_{c} HA_{i,s,cu,c} \leq inum_{i,s,cu} size_i \tag{3.10}$$

$size_i$ area of irrigation system i (ha)

3.3.3.2 Water quota

The annual water use is restricted to be less than or equal to the annual water quota using the following constraint:

$$\sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{d} \frac{HA_{i,s,cu,c} IR_{s,cu,c,d}}{eff} \leq alloc_r \tag{3.11}$$

$alloc_r$ annual water allocation in state of nature r (mm.ha)

3.3.3.3 Contracts

Production of popcorn is highly profitable in the research area conditional on the availability of a production contract. The area planted with popcorn is restricted using the following constraint.

$$\sum_{i}\sum_{s}\sum_{cu}\sum_{c=popcorn} HA_{i,s,cu,c} \leq contract \tag{3.12}$$

$contract$ maximum area allocated to popcorn production (ha)

3.4 LONG-RUN FARM-LEVEL FINANCIAL WATER USE OPTIMISATION MODEL

The short-run irrigation water use optimisation model provides a detailed representation of irrigation water use on a farm for a production year. The short-run model is further developed into a mixed integer dynamic nonlinear programming model which allows for the optimisation of short-run responses within the long-run structure of the model.

A disequilibrium known life type of dynamic linear programming model specification (McCarl and Spreen, 2003) is used to develop the model. Known life means that resources and fund flows are committed for a fixed period of time, whereas disequilibrium implies that the same activity does not need to follow the previous activity and can be replaced with another activity. The model allows for investments in new more efficient irrigation technology as well as upgrading inefficient irrigation technology.

A more detailed description of the complete model follows, with capital letters representing variables. All the input parameters were discounted to present values before entering the optimisation model, and therefore no discounting is shown when the model is specified. Discounting is, however, necessary if cash flows generated in one period is transferred to the next period.

3.4.1 OBJECTIVE FUNCTION

The objective function of the long-run model is to maximise the certainty equivalent of the distribution of cash flows in the last year of the planning horizon plus terminal values associated with crop production, irrigation system investments and borrowed money:

$$MAX\ CE = \left(\sum_{r}^{R}\frac{1}{R}\left(CF_{r,15} + \sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{it} HA_{i,s,cu,c,it}\, ctv_{i,s,cu,c,it} + \sum_{i}\sum_{s}\sum_{it} IB_{i,s,it} itv_{i,it} + \sum_{it} ILOAN_{it} ltv_{it}\right)^{1-rra}\right)^{\frac{1}{(1-rra)}} \tag{3.13}$$

$CF_{r,t}$	cash flow in year t for random realisation r (R)
$HA_{i,s,cu,c,it}$	hectares of crop c planted in year it on soil s using irrigation system i with a CU of cu (ha)
$IB_{i,s,it}$	variable for buying new irrigation system i in year it on soil s (integer)
$ILOAN_{it}$	amount of money borrowed to finance irrigation system purchases in year t (R)
$ctv_{i,s,cu,c,it}$	terminal value of crop c planted in year it on soil s using irrigation system i with a CU of cu (R)
$itv_{i,it}$	terminal value of irrigation system i bought in year it (R)
ltv_{it}	terminal value of money borrowed to finance irrigation system purchases in year it
rra	relative risk aversion

3.4.2 VARIABLE CALCULATIONS

3.4.2.1 Distribution of cash flow

Cash flow calculations may seem straight forward. However, the calculations are complicated by the fact that the financial year and the production season of the crops do not coincide. Special care was taken to ensure that all costs that are included could be allocated to a specific financial year.

Consequently, the cash flow of the first year includes a constant amount of money to reflect costs and income of crops already planted in the previous financial year.

A cash flow is calculated for each random realisation in each year as follows:

$$
\begin{aligned}
CF_{r,t} = \; & bo_t \\
& + \frac{CF_{r,t-1}}{1+dr} + ICF_{r,t} \\
& + \sum_i \sum_s \sum_{cu} \sum_c \sum_{it} \sum_w HA_{i,s,cu,c,it} p_{r,c,t} Y_{r,w,s,cu,c,t,it}/3 \\
& + \sum_i \sum_s \sum_{cu} \sum_c \sum_w cha_{i,s,cu,c} p_{r,c,t} cy_{r,w,s,cu,c,t}/3 \\
& + \sum_i \sum_s \sum_{cu} \sum_c IB_{i,s,it} sal_{i,t,it} + inum_{i,s,cu} csal_{i,t,it} \\
& - household - overheads - cpay_t \\
& - \sum_i \sum_s \sum_{d=1}^{272} \sum_{tou} HOUR_{i,s,d,tou,t} rep_i - \sum_i \sum_s \sum_{d=273}^{365} \sum_{tou} HOUR_{i,s,d,tou,t} rep_i \\
& - \sum_i \sum_s \sum_{d=1}^{272} \sum_{tou} HOUR_{i,s,d,tou,t} aetar_{d,tou} kW_i - \sum_i \sum_s \sum_{d=273}^{365} \sum_{tou} HOUR_{i,s,d,tou,t} aetar_{d,tou} kW_i \\
& - \sum_i \sum_s \sum_{d=1}^{272} \sum_{tou} HOUR_{i,s,d,tou,t} retar_{d,tou} kvar_i - \sum_i \sum_s \sum_{d=273}^{365} \sum_{tou} HOUR_{i,s,d,tou,t} retar_{d,tou} kvar_i \\
& - \sum_i \sum_s IU_{i,s,t}\, uc_i - BC_t - \sum_{it} ILOAN_{it}\, ipay_{t,it} \\
& - \sum_c OC_{r,c,t,\text{"1"}} - \sum_c \frac{OC_{r,c,t-1,\text{"2"}}}{1+dr} - \sum_{c,it} OLOAN_{c,it}\, opay_{c,t,it} \\
& - \sum_i \sum_s \sum_{d=1}^{272} \sum_{tou} HOUR_{i,s,d,tou,t} flow_i wtar - \sum_i \sum_s \sum_{d=273}^{365} \sum_{tou} HOUR_{i,s,d,tou,t} flow_i wtar \\
& - TI_{r,t} tax
\end{aligned}
\tag{3.14}
$$

bo_t	cash flow opening balance in year one (R)
$ICF_{r,t}$	interest earned or paid on random cash flow *r* in year *t* (R)
dr	discount rate (fraction)
$p_{r,c,t}$	randomly generated price *r* for crop *c* in year *t* (R/ton)
$Y_{r,w,s,cu,c,t}$	crop yield of crop *c* in year *t* for water budget *w* planted on soil *s* using irrigation system *i* with a CU of *cu* for random weather *r* in year *it* (t/ha)
$cha_{i,s,cu,c}$	hectares of crop *c* currently planted on soil *s* using irrigation system *i* with a CU of *cu* (ha)
$cy_{r,w,s,cu,c,t}$	crop yield of crop *c* for water budget *w* of crop *c* currently planted on soil *s* using irrigation system *i* with a CU of *cu* for random weather *r* (t/ha)
$sal_{i,t,it}$	salvage value of irrigation system *i* in year *t* bought in year *it* (R/system)

$csal_{i,t,it}$	salvage value of irrigation system i in year t currently on the soil (R/system)
$inum_{i,s,cu}$	number of irrigation system i on soil s with CU of cu (integer)
$household$	household expenses (R)
$overheads$	overheads (R)
$cpay_t$	total current payment in year t (R)
$HOUR_{i,s,d,tou,t}$	irrigation hours for day d within time-of-use timeslot tou in year t using irrigation system i on soil s (hours)
rep_i	repair and maintenance cost of irrigation system i (R/hour)
$aetar_{d,tou}$	active energy charge on day d within time-of-use timeslot tou (R/kWh)
kW_i	kilowatt of irrigation system i (kW)
$retar_{d,tou}$	reactive energy charge on day d within time-of-use timeslot tou (R/kVarh)
$kvar_i$	kilovar of irrigation system i (kVar)
$IU_{i,s,t}$	variable for upgrading irrigation system i in year t on soil s (integer)
uc_i	cost of upgrading irrigation system i (R/system)
BC_t	amount of money allocated to cash purchases of irrigation systems in year t (R)
$ipay_{t,it}$	payment in year t on borrowed money in year it to finance irrigation system investments (R)
$OC_{r,c,t,"1"}$	operating cost of crop c in year t allocated to year t for random realisation r (R)
$OC_{r,c,t,"2"}$	operating cost of crop c in year t allocated to year $t+1$ for random realisation r (R)
$OLOAN_{c,it}$	operating loan to finance operating cost of crop c in year it (R)
$opay_{c,t,it}$	payment in year t on borrowed money in year it to finance operating cost of crop c (R)
$flow_i$	flowrate of irrigation system i (m^3/h)
$wtar$	water tariff (R/m^3)
$TI_{r,t}$	taxable income in year t for random realisation r (R)
tax	marginal tax rate (fraction)

The cash flow variable (CF) and interest variable (ICF) are defined as a “free variable” which imply that the level of these variables could either be positive or negative. A positive cash flow is generated if the income generated is greater than the cash expenses. Income is generated through the production of crops, interest earned on a positive bank balance and any salvage income. A constant amount of income is added to the cash flow to account for crop areas that are already planted and salvage income that is not a function of new investments. Expenses pertain to overheads, loan repayments, household expenses, electricity costs for pumping water, water use charges, operating cost and cash used to upgrade irrigation systems to increase their efficiencies.

3.4.2.2 Taxable income

Taxable income is calculated while allowing for deductions of interest paid and tax depreciation of capital goods. A special feature of the taxable income calculation is that it can defer tax payments to future years if a negative taxable income is generated using variable $TT_{r,t}$.

The following calculation of taxable income is included in the model:

$$\begin{aligned}
TI_{r,t} = \ & ICF_{r,t} \\
& + \sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{it}\sum_{w} HA_{i,s,cu,c,it} p_{r,c,t} Y_{r,w,s,cu,c,t,it}/3 \\
& + \sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{w} cha_{i,s,cu,c} p_{r,c,t} cy_{r,w,s,cu,c,t}/3 \\
& + \sum_{i}\sum_{s}\sum_{cu}\sum_{c} IB_{i,s,it} sal_{i,t,it} + inum_{i,s,cu} csal_{i,t,it} \\
& - overheads - cpayi_t \\
& - \sum_{i}\sum_{s}\sum_{d=1}^{272}\sum_{tou} HOUR_{i,s,d,tou,t} rep_i - \sum_{i}\sum_{s}\sum_{d=273}^{365}\sum_{tou} HOUR_{i,s,d,tou,t} rep_i \\
& - \sum_{i}\sum_{s}\sum_{d=1}^{272}\sum_{tou} HOUR_{i,s,d,tou,t} aetar_{d,tou} kW_i - \sum_{i}\sum_{s}\sum_{d=273}^{365}\sum_{tou} HOUR_{i,s,d,tou,t} aetar_{d,tou} kW_i \\
& - \sum_{i}\sum_{s}\sum_{d=1}^{272}\sum_{tou} HOUR_{i,s,d,tou,t} retar_{d,tou} kvar_i - \sum_{i}\sum_{s}\sum_{d=273}^{365}\sum_{tou} HOUR_{i,s,d,tou,t} retar_{d,tou} kvar_i \\
& - \sum_{i}\sum_{s} IU_{i,s,t}\, uc_i - BC_t - \sum_{it} ILOAN_{it} ipayi_{t,it} - \sum_{i}\sum_{s}\sum_{it} IB_{i,s,t,it} taxdep_{t,it} \\
& - \sum_{c} OC_{r,c,t,"1"} - \sum_{c} \frac{OC_{r,c,t-1,"2"}}{1+dr} - \sum_{c,it} OLOAN_{c,it} opay_{c,t,it} \\
& - \sum_{i}\sum_{s}\sum_{d=1}^{272}\sum_{tou} HOUR_{i,s,d,tou,t} flow_i wtar - \sum_{i}\sum_{s}\sum_{d=273}^{365}\sum_{tou} HOUR_{i,s,d,tou,t} flow_i wtar \\
& + TT_{r,t} - \frac{TT_{r,t-1}}{1+dr}
\end{aligned} \tag{3.15}$$

$ipayi_{t,it}$	interest portion of payment *t* on borrowed money to finance irrigation system investments in year *it* (R)
$taxdep_{t,it}$	tax deduction in year *t* for capital investments in year *it* (fraction)
$TT_{r,t}$	taxable income transferred in year *t* due to a negative taxable income for random realisation *r* (R)

3.4.2.3 Interest earned or paid

A piecewise linear function is used to model the amount of interest earned or paid conditionally on respectively a positive or negative cash flow balance. Consequently $ICF_{r,t}$ is a free variable that could take on a positive or negative value depending on whether interest is earned or paid. The following equations are used to model $ICF_{r,t}$ as a piecewise linear function of $CF_{r,t}$:

$$ICF_{r,t} = \sum_{bp} W_{r,t,bp} pwl_{bp,\text{"}y-axis\text{"}} \tag{3.16}$$

$$\frac{CF_{r,t-1}}{1+dr} = \sum_{bp} W_{r,t,bp} pwl_{bp,\text{"}x-axis\text{"}} \tag{3.17}$$

$$W_{r,t,\text{"}neg\text{"}} \leq 1 - BIN_{r,t} \tag{3.18}$$

$$W_{r,t,\text{"}pos\text{"}} \leq BIN_{r,t} \tag{3.19}$$

$W_{r,t,bp}$	weight of breakpoint *bp* in year *t* for random realisation *r*
$pwl_{bp,xy}$	values of coordinate *xy* for breakpoint *bp*
$BIN_{r,t}$	binary variable indicating whether positive or negative breakpoint values must be used in year *t* for random realisation *r*

Only two breakpoints were included since the transition from paying interest to earning interest is at coordinate (0,0). Multiplying with a zero makes the transition breakpoint redundant.

3.4.2.4 Operating costs

The following equation shows that operating costs could be paid cash or financed with a production loan:

$$\begin{aligned} OC_{r,c,t,dt} + OLOAN_{c,t} = & \sum_{i}\sum_{s}\sum_{cu}\sum_{it}\sum_{w} HA_{i,s,cu,c,it}\, oy_{r,i,s,cu,c,t,dt,it}\, Y_{r,w,s,cu,c,t,it}/3 \\ & + \sum_{i}\sum_{s}\sum_{cu}\sum_{it}\sum_{w} HA_{i,s,cu,c,it}\, of_{i,s,cu,c,t,dt,it}\, Y_{r,w,s,cu,c,t,it}/(3card(r)) \\ & + \sum_{i}\sum_{s}\sum_{cu}\sum_{it} HA_{i,s,cu,c,it}\, oa_{c,t,dt,it} \\ & + \sum_{i}\sum_{s}\sum_{cu}\sum_{w} cha_{i,s,cu,c}\, coy_{i,s,cu,c,t}\, Y_{r,w,s,cu,c,t}/3 \\ & + \sum_{i}\sum_{s}\sum_{cu}\sum_{w} cha_{i,s,cu,c}\, cof_{i,s,cu,c,t}\, Y_{r,w,s,cu,c,t}/(3card(r)) \\ & + \sum_{i}\sum_{s}\sum_{cu}\sum_{w} cha_{i,s,cu,c}\, coa_{i,s,cu,c,t} \end{aligned} \tag{3.20}$$

$oy_{r,i,s,cu,c,t,dt,it}$ yield dependant harvesting costs in year t distributed to year dt of crop c planted in year it on soil s using irrigation system i with a CU of cu (R/ton)

$of_{i,s,cu,c,t,dt,it}$ yield dependant fertiliser costs in year t distributed to year dt of crop c planted in year it on soil s using irrigation system i with a CU of cu (R/ton)

$oa_{c,t,dt,it}$ area dependant costs in year t distributed to year dt of crop c planted in year it (R/ha)

$coy_{i,s,cu,c,t}$ yield dependant harvesting costs in year t of crop c currently occupying soil s using irrigation system i with a CU of cu (R/ton)

$cof_{i,s,cu,c,t}$ yield dependant fertiliser costs in year t of crop c currently occupying soil s using irrigation system i with a CU of cu (R/ton)

$coa_{i,s,cu,c,t}$ area dependant costs in year t of crop c currently occupying soil s using irrigation system i with a CU of cu (R/ha)

Operating costs include the production costs of crops planted before the start of the planning horizon of the model. Electricity costs are not included as operating costs because electricity costs could not be financed through a production loan. Consequently electricity costs are paid from the cash flow.

3.4.2.5 Irrigation system investment costs

The following equation shows that irrigation system investment costs could be paid cash or financed with a loan:

$$BC_t + ILOAN_t = \sum_i \sum_s IB_{i,s,t}\, ic_i \tag{3.21}$$

ic_i investment costs of irrigation system i (R)

3.4.2.6 Crop yield and water budget calculations

The same detail with which the crop yields are calculated in the short-run model is incorporated into the long-run model. However the calculations need to be repeated for each year of the planning horizon. The parameters and variables are therefore also indexed over year t. The equations necessary to calculate crop yield using the daily water budget calculations of each year is shown below for completeness.

Crop yield calculation:

$$Y_{r,w,s,cu,c,t,it} = yp_{r,c,t,it} \prod_g \left[1 - ky_{g,c}\left(1 - \frac{\sum_{d\in g} ETa_{r,w,s,cu,c,d,t}}{\sum_{d\in g} etm_{r,w,s,cu,c,d,t}}\right)\right] \tag{3.22}$$

$yp_{r,c,t,it}$	yield potential of crop c in year t for random weather r planted in year it (t/ha)
$ky_{g,c}$	crop yield response factor of crop c in crop growth stage g (fraction)
$ETa_{r,w,s,cu,c,d,t}$	actual evapotranspiration of crop c on day d in year t for water budget w, soil s, CU cu and random weather r (mm)
$etm_{r,w,s,cu,c,d,t}$	maximum potential evapotranspiration of crop c on day d in year t for water budget w, soil s, CU cu and random weather r (mm)

Water budget calculations:

$$ETa_{r,w,s,cu,c,d,t} = min \left| \begin{array}{l} etm_{r,c,d,t} \\ etm_{r,c,d,t} \left(\frac{RWC_{r,w,s,cu,c,d,t}}{trwc_{r,s,c,d,t}} \right) \end{array} \right. \tag{3.23}$$

$$RWC_{r,w,s,cu,c,d,t} = min \left| \begin{array}{l} rwcap_{s,c,d} \\ RWC_{r,w,s,cu,c,d-1,t} - ETa_{r,w,s,cu,c,d,t} + R_{r,d,t-1} + IR_{s,cu,c,d-1,t} sf_{w,cu} + TR_{r,w,s,cu,c,d,t} \end{array} \right. \tag{3.24}$$

$$BRWC_{r,w,s,cu,c,d,t} = min \left| \begin{array}{l} brwcap_{s,c,d} \\ BRWC_{r,w,s,cu,c,d-1,t} - ETa_{r,w,s,cu,c,d,t} + R_{r,d,t-1} + IR_{s,cu,c,d-1,t} sf_{w,cu} + TR_{r,w,s,cu,c,d,t} - rwcap_{s,c,d} \end{array} \right. \tag{3.25}$$

$$BR_{r,w,s,cu,c,d,t} = max \left| \begin{array}{l} 0 \\ rwc_{r,w,s,cu,c,d-1,t} - ETa_{r,w,s,cu,c,d,t} + R_{r,d,t-1} + IR_{s,cu,c,d-1,t} sf_{w,cu} + TR_{r,w,s,cu,c,d,t} - rwcap_{s,c,d} \end{array} \right. \tag{3.26}$$

$$TR_{r,w,s,cu,c,d,t} = rg_{c,d} BRWC_{r,w,s,cu,c,d,t} \tag{3.27}$$

$RWC_{r,w,s,cu,c,d,t}$	root water content of water budget w on day d in year t for soil s, CU cu, crop c and random weather r (mm)
$trwc_{r,s,c,d,t}$	threshold root water content of soil s on day d in year t for crop c and random weather r (mm)
$rwcap_{s,c,d}$	total amount of water that could be stored on day d in the root zone of crop c planted on soil s (mm)
$R_{r,d,t}$	rainfall on day d in year t for random weather r (mm)
$IR_{s,cu,c,d,t}$	irrigation of crop c on soil s using a CU of cu on day d in year t for random weather r (mm)
$sf_{w,cu}$	scaling factor to adjust irrigation water of water budget w based on a CU of cu (fraction)
$TR_{r,w,s,cu,c,d,t}$	amount of water that move to the root zone on day d in year t for water budget w, soil s, CU of cu and crop c
$BRWC_{r,w,s,cu,c,d,t}$	root water content below the roots of water budget w on day d in year t for soil s, CU cu, crop c and random weather r (mm)

$brwcap_{s,c,d}$	total amount of water that could be stored on day d in the potential root zone of crop c planted on soil s (mm)
$BR_{r,w,s,cu,c,d,t}$	amount of water that drain below the root zone on day d in year t for water budget w, soil s, CU of cu, crop c and random weather r
$rg_{c,d}$	root growth of crop c on day d

3.4.2.7 Pumping hours

Pumping hours is a function of the amount of water pumped, the application efficiency and the flow rate of the irrigation system. The calculation of total pumping hours takes cognisance of crops that are already established.

$$\sum_{tou} HOUR_{i,s,d,tou,t} = \sum_{cu}\sum_{c}\sum_{it}\left(\frac{HA_{i,s,cu,c,it}coc_{c,t,it}IR_{s,cu,c,d,t}}{eff}\right)10/flow_i + \sum_{cu}\sum_{c}\left(\frac{cha_{i,s,cu,c}ccoc_{i,s,cu,c,t,"1"}IR_{s,cu,c,d,t}}{eff}\right)10/flow_i \tag{3.28}$$

The assumption is made that all the pivots could irrigate simultaneously. Consequently the pumping hours are restricted to the available hours within a time-of-use timeslot multiplied with the number of pivots with the following equation:

$$HOUR_{i,s,d,tou,t} \leq touhour_{d,tou}\left(inum_{i,s,cu}cioc_{i,t,"1"} + \sum_{it} IB_{i,s,it}\, ioc_{i,t,it}\right) \tag{3.29}$$

$coc_{c,t,it}$	parameter indicating land occupation in year t by crop c planted in year it
$ccoc_{i,s,cu,c,t,"1"}$	parameter indicating land occupation in year t by crop c that is currently established on soil s using irrigation system i with a CU of cu .
$cioc_{i,t,"1"}$	parameter indicating whether irrigation system i that is currently operating will still be operating in year t
$ioc_{i,t,it}$	parameter indicating whether irrigation system i installed in year it will still be operating in year t

3.4.3 CONSTRAINTS

3.4.3.1 Borrowing capacity

The following equation is used to restrict the amount of money borrowed to finance production costs or irrigation system investments to be less than a specified borrowing capacity minus currently outstanding payments:

$$\sum_{c} OLOAN_{c,t} + \sum_{it} ILOAN_{it} ipayo_{t,it} + cpayo_t \leq bc \tag{3.30}$$

$ipayo_{t,it}$ outstanding capital in year t of money borrowed in year it to finance irrigation system investments (R)

$cpayo_t$ currently outstanding capital payments in year t

bc borrowing capacity

3.4.3.2 Irrigation technology use, upgrades and expansions

In the long-run irrigators are allowed to increase the uniformity of their irrigation systems to improve water use efficiencies through the installation of improved nozzle packages. Consequently the model needs to keep track of the area irrigated with irrigation systems with low CUs and irrigation systems with high CUs using the following equations:

$$\sum_{c}\sum_{it} HA_{i,s,"low",c,it}\, coc_{c,t,it} \leq inum_{i,s,"low"} size_i cioc_{i,t,"1"} - \sum_{it} IU_{i,s,it} size_i cuoc_{i,t,it} - \sum_{c} cha_{i,s,"low",c} ccoc_{i,s,"low",c,t,"1"} \tag{3.31}$$

$$\sum_{c}\sum_{it} HA_{i,s,"high",c,it}\, coc_{c,t,it} \leq inum_{i,s,"high"} size_i cioc_{i,t,"1"} + \sum_{it} IU_{i,s,it} size_i cuoc_{i,t,it} + \sum_{it} IB_{i,s,it} size_i ioc_{i,t,it} \tag{3.32}$$

$$\sum_{it} IU_{i,s,it} size_i cuoc_{i,t,it} \leq inum_{i,s,"low"} size_i cioc_{i,t,"1"} \tag{3.33}$$

$$\sum_{i}\sum_{cu} inum_{i,s,cu} size_i cioc_{i,t,"1"} + \sum_{i}\sum_{it} IB_{i,s,it} size_i ioc_{i,t,it} \leq \sum_{i}\sum_{cu} inum_{i,s,cu} size_i \tag{3.34}$$

$cuoc_{i,t,it}$ parameter indicating whether irrigation system i that is upgraded in year it will still be operating in year t

The assumption is made that an old irrigation system can only be replaced with an irrigation system with a high uniformity. By implication, only the irrigation systems that are currently in operation could the upgraded with new nozzle packages.

3.4.3.3 Water quota

The water use of the crop areas currently irrigated and the optimised water use is restricted to be less or equal to the annual water allocation using the following constraint:

$$\sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{d}\sum_{it}\frac{HA_{i,s,cu,c,it}coc_{c,t,it}IR_{s,cu,c,d,t}}{eff} - \sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{d}\frac{cha_{i,s,cu,c}ccoc_{i,s,cu,c,t,"1"}IR_{s,cu,c,d,t}}{eff} \leq alloc_r \tag{3.35}$$

3.4.3.4 Contracts

A Prerequisite for popcorn production is the availability of a production contract. The following equation restrict popcorn production to be less than or equal to the size of the contract:

$$\sum_{i}\sum_{s}\sum_{cu}\sum_{c}\sum_{it} HA_{i,s,cu,"popcorn",it}\, coc_{"popcorn",t,it} \leq contract_{it} \tag{3.36}$$

$contract_{it}$ maximum area allocated to popcorn production in year it (ha)

3.5 OPTIMISATION USING HYBRID OPTIMISATION TECHNIQUE

Both the short-run and the long-run irrigation water use optimisation models are highly non-linear while discontinuity enters the water budget calculations through the use of the min() and max() functions which make it impossible to solve the models using standard procedures. The hybrid optimisation technique developed by Cai *et al.* (2001) is used to solve the hydro-economic optimisation models. The solution technique relies on the use of an evolutionary algorithm to fix the complicating variables in the optimisation models. Fixing the complicating variables linearise the mathematical programming model which makes it easier to solve.

The General Algebraic Modeling System (GAMS) is used to develop and integrate the different components necessary to solve the model using the hybrid optimisation technique. The GAMS code for solving the hydro-economic optimisation model through the application of the hybrid GA-LP solution methods is shown in Appendix A. Figure 3.2 shows the flow data and the procedures necessary to solve the HE optimisation model.

Knowing the irrigation schedule to be applied to each crop result in a straight forward exogenous calculation of the water budget components and the resulting crop yield. Thus, choosing the irrigation schedule as the complicating variable allows for the exclusion of the water budget calculations in the mathematical programming model. Furthermore, crop yields could be calculated which means that the crop yields could also be fixed in the mathematical programming model. The GA is then used to optimise the irrigation schedule through an iterative procedure whereby the GA suggest an irrigation schedule and the DMINLP model optimises the timing of irrigation technology adoption decisions.

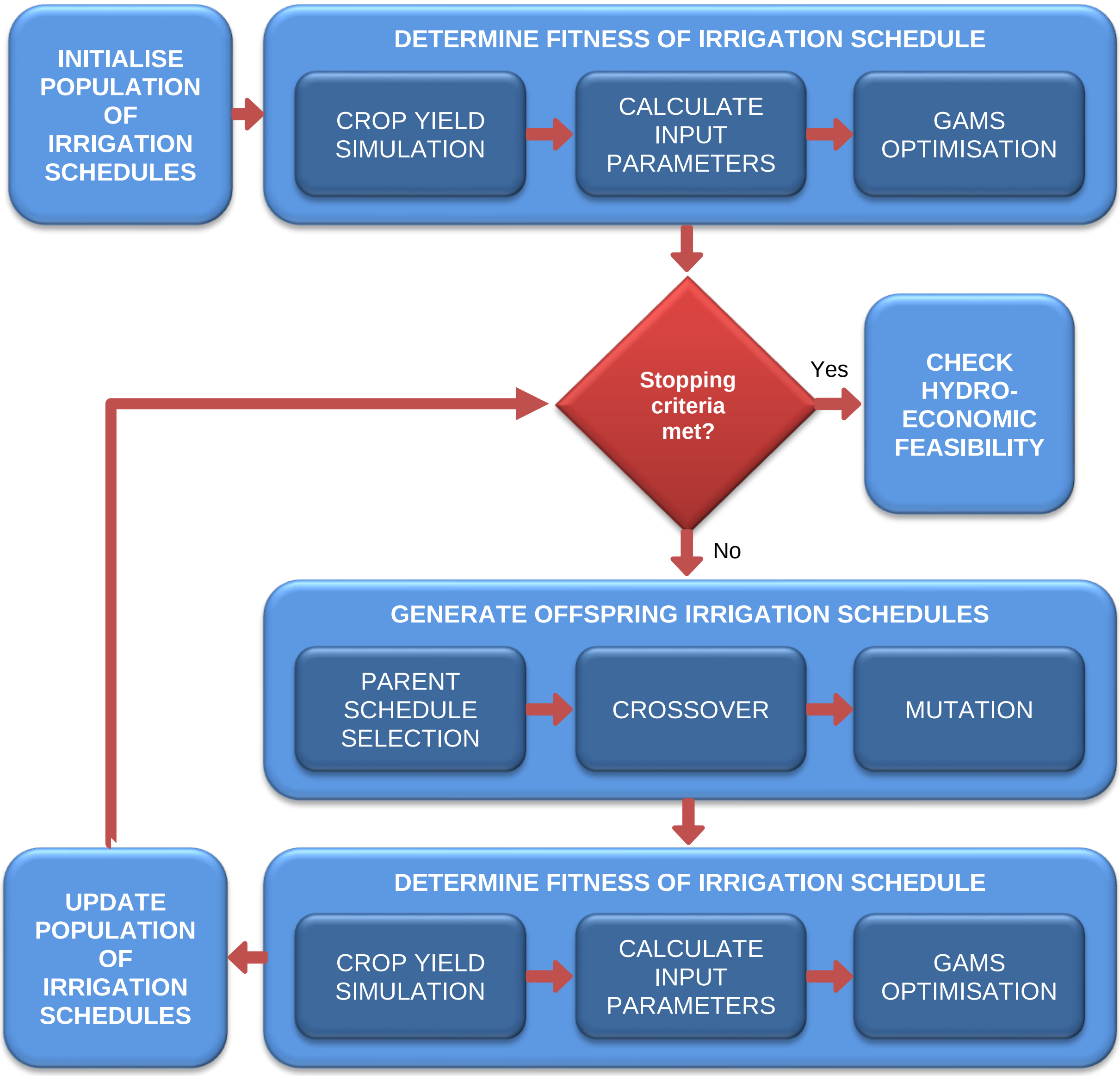


Figure 3.2: Flow chart of the hybrid solution technique

The first step in applying the GA to optimise the irrigation schedules is to generate a population pool of candidate solutions. The "fitness" of each of the candidate solutions is determined by firstly simulating the crop yield associated with each irrigation schedule for each crop, soil and irrigation technology combination. Much time is lost when models are loosely coupled. Models are loosely coupled when an independent model needs to generate an output file that needs to be transferred to another independent model. In an effort to reduce solution time the water budget and crop water budget calculations included in MIKE HYDRO BASIN were programmed in GAMS (see Appendix B) to facilitate a quicker and more effective information transfer between the model components. The irrigation schedules and associated crop yields are used to calculate the necessary input parameters for the optimisation model which is used to evaluate the "fitness" of the irrigation schedule. Using an optimisation model to determine the "fitness" of each generated irrigation schedule, distinguishes the hybrid solution procedure from normal applications of GA. A benefit of using the an optimisation model to determine the “fitness” of the irrigation schedules is that all the other variables are solved to optimality given the constraint set that applies.

Next, the population evolves through a series of loops whereby offspring is generated from selected parents through the process of crossover and mutation. The “fitness” of the offspring irrigation schedules is determined using the same procedure to determine the “fitness” of the initial population. Selected individuals from the initial population is replaced by the offspring and the process is repeated until the GA stopping criteria is met. The final step is to check the final solution for hydro-economic soundness.

3.6 HYDRO-ECONOMIC MODEL CONFIGARATION

3.6.1 CATCHMENT SCALE HYDROLOGY MODEL CONFIGURATION

In this Section, the model setup that is configured for the Upper Orange Catchment on catchment scale is discussed. In the catchment scale, the Upper Orange Catchment is configured in the MIKE Hydro BASIN model, where the operating rules for the catchment are implemented. The Orange River is heavily dependent of the flow that is generated in the catchment above the Gariep dam, where the two dams are built to supply the Integrated Vaal Catchment, hence the flow that flows to the Gariep dam is dependent of the water demand and development of the Integrated Vaal catchment. The following sections will discuss how the operating rules and the MIKE Hydro Basin setup is conceptualised for the Upper Orange catchment for the purpose of this project.

3.6.1.1 Upper Orange MIKE HYDRO BASIN Setup

The Upper Orange MIKE HYDRO Basin Setup was conceptualized from the WRYM/WRPM setup of the Orange River that was implemented/setup for the purpose of developing the Annual Operating Rules of the Orange River system. The Upper Orange MIKE Hydro Basin simplified as per the following:

- The flow to the Gariep dam was sourced from the WRPM model that was setup for the development of the Annual operating rules for the Orange River. The flow is derived from the base line study with the present development and the model was run with historical mode for the period of 1920 to 2004.
- The flow from the Vaal catchment to the Orange River at the confluence is influenced by the water availability of the Vaal catchment. The Observed recorded flow at the gauge C9H025 is in the Upper Orange MIKE hydro Basin setup to represent the flow from the Vaal catchment to the Orange River.
- All information on the reservoirs, specific catchment flows and water use demands are sourced from the WRYM/WRPM setups that are setup for the development of the Annual Operating Rules of the Orange system.
- The Operating Rules of the Gariep and Vandekloof dams are implemented as per the operating objects of the system

3.6.1.2 Operating Objectives of Gariep and Vanderkloof Dam

Gariep and Vanderkloof dams are operated in tandem with a storage control curve to avoid spillage from the two dams and maximise generation of hydro-electric power through the turbines. Refinement of the hydro power operating rules was done in 1995 by BKS for Eskom. Most of the components of the 1995 recommended operating rules/objectives are currently still applied in the annual operating analysis carried out for the Orange River System with some modifications.

“The hydro power operating rule for Gariep and Vanderkloof dam, currently in use as based on the rules developed in 1995 consist of the following components.”

a) Allocation curve (allocation of surplus water in the system)
b) Mirror pattern for releases from Gariep Dam
c) Release rule for releases from Gariep Dam
d) Flood control curve when dams are full or close to full supply level
e) Vanderkloof Dam operating level
f) Energy demand pattern

“The objective of the refinement of Hydro Power Operating Rules in 1995 was to enhance and refine the operating rule used for Gariep and Vanderkloof dams and to prove the applicability of the rule. This was performed in line with the following objectives or criteria stating how the system should behave in comparison with results of the previous study at the time. a) Increase energy generation at high assurance levels. b) Implement a control curve that allows releases from Gariep Dam to decrease gradually as the level in the dam drops. This rule substitutes the 20% live storage cut-off level rule, prior to 1995. The purpose of this and the initial rule is to protect Gariep Dam against failure in the water supply to the Eastern Cape while Vanderkloof might still have more than sufficient water to supply all the downstream demands. c) Comply with the restricition criteria defined for the consumptive users, which has the purpose of maintaining the reliability of supply to those users. d) Evaluate the influence of flood control on energy and water supply reliability.”

The Operating rules/objectives of the Gariep and Vanderkloof dam is guided by the hydro power operating rules and control curve rule for the dams to restrict users based on the water availability in the dams. In this research project, the hydro power operating rules and control curve rule for the dams are implemented in the MIKE Hydro Basin setup.

3.6.1.3 Catchment Scale Hydrological Management Indicators

Catchment indicators are indicators that explain the status of water availability in the catchment. Catchment indicators can show water managers if a catchment is stressed, over-allocated or if the present infrastructures in the catchment are adequate; or if additional water infrastructure is required to support the current and future water demands. The Catchment indicators used in this project are listed below:

- Reservoir Storage trajectories – storage trajectories are time varying water levels/ water storage of reservoirs
  - Historical reservoir storage trajectory – it a reservoir storage for a time period for which historical streamflow data is available
  - Stochastic reservoir storage trajectories – statistical probable reservoir storage trajectories
- Expected annual water supply reliability to users – it is the expected water supply reliability for coming year to water users.
  - It is assumed the current water demand and distribution is implemented for the coming year
  - Operating rules for the existing dams are developed on the assumption the distribution and the water demand amount is similar to the current water request
- Short term yield curves – short term yield curves are maximum water availability from a catchment/dam at different assurance of supply.
  - Short term yield curves are produced from a stochastic run of the catchment models and no restriction is imposed on the existing dams
  - It indicates whether the resources in the catchment sustainably support the current water request

3.6.2 FARM-LEVEL IRRIGATION WATER USE OPTIMISATION MODEL

3.6.2.1 Farm size and irrigation technology

The long-run optimisation model is concerned with irrigation technology adoption. Investing in an irrigation system is a lumpy decision. Farms were defined not only in terms of area but in terms of number of piovts of a given size to more realistically represent the technology use of the farms. To determine the representative farm size and the pivot size, a k-means cluster analysis was used to group the farms in groups with relatively similar fixed resource scenario. Data on irrigation areas, irrigation technology, water allocation and crop rotation was obtained for 75 water users along the Orange River below Vanderkloof dam.

The Statistical Package for the Social Sciences (SPSS) Statistic software developed by IBM was used to perform a k-means cluster analysis. The k-means clustering algorithm was firstly developed by Macqueen (1967) and then further developed by Hartigan and Wong (1979) (cited in Al Bashish, Braik and Bani-Ahmad, 2011). The algorithm allows the grouping of the farms with minimum Euclidean distances. However, a k-means cluster is sensitive towards outliers and one need to specify the amount of clusters prior to the analysis (Bernhardt, Allen and Helmers, 1996). To overcome the sensitivity of outliers the data was first tested for outliers with SPSS. SPSS define points as outliers if they extend more than 1.5 box-lengths from the edge of the box (Pallant, 2007). Outliers were then removed. The Elbow method was utilized to determine the optimal amount of clusters (k). A k-value equal to two is used in the beginning of the analysis, and it's increased up until ten. The mean square errors are then graphed versus the amount of clusters (k). At a certain cluster one will see the mean square errors drop dramatically and after that it reaches a plateau as k increase. The optimal number of cluster occur where this dramatic drop can be observed.

The first k-means cluster analysis was done to determine the most common farm sizes with two variables namely the amount of hectares under pivot irrigation and the amount of hectares under other irrigation methods. When the total irrigated area was determined, the next question was how these hectares were divided up into pivots and what pivot sizes were the most common. Another k-means cluster analysis was then preformed to first determine the pivot size for the farm sizes that were determined in the first k-means cluster analysis. The final question that needed to be answered was how many of each the pivot size will be present in each farm size.

A MOTAD type model was used to allocate pivots sizes to the representative irrigation areas. The MOTAD model minimized the square differences between the observed pivot sizes within a cluster and the assigned (Q_p) pivot sizes that correspond to cluster centres while ensuring that the total area of the farm larger or equal to the farm size determined by the cluster analysis on farm size. The specification of the model follows:

$$Min = \sum_{ip} \left(TArea_{ip} - \left(Q_p \times Cen_p \right) \right)^2 \qquad (3.37)$$

$TArea_{ip}$ Total area (ha) that is irrigated of each irrigator (i) within each cluster (p)

Q_p Amount of pivot within each cluster (p)

Cen_p The cluster center of each cluster (p) (ha)

Subject to:

$$Q_p \times Cen_p \leq TFarmSize \qquad (3.38)$$

$TFarmSize$ The total farm size (ha)

The optimal number of clusters for the total irrigation area is when k is equal to four. Table 3.1 shows that 78%t of the water users fall in cluster one and four with more or less an equal share in each. The two farm sizes that were used are 61 ha and 225 ha

Table 3.1: Summary of cluster analysis for farm size.

Cluster	Range		Cluster centres		Percentage of water users
	Pivot (ha)	Other irrigation technology (ha)	Pivot (ha)	Other irrigation technology (ha)	
1	6-128	0-30	61	3.00	38
2	410-739	0-13	579	2	12
3	853-1029	0-12	910	2	10
4	149-361		225	5	40

With respect to farm size one, four outliers were identified and removed from the data set hence 82 pivots were included in the analysis. The Elbow method showed that the optimal number of clusters is when k is equal to three. Table 3.2 shows the cluster centres for the pivot sizes to be included in farm size one as 4 ha, 15 ha and 30 ha. The MOTAD model determined that the distribution of the pivot sizes will be as follows: four pivots of 4 ha, one pivot of 15 ha and one pivot of 30 ha.

Table 3.2: Summary of Cluster analysis for the pivot size of representative farm size 1.

Cluster	Range of pivot sizes (ha)	Cluster centres (ha)	Percentage water users (%)
1	2-10	4	54
2	22-40	30	19
3	10-21	15	27

Six outliers were identified and removed from the data set and 192 pivots were included in the analysis of the pivot sizes for farm size two. The Elbow method showed that the optimal number of clusters is when k is equal to four. Table 3.3 shows the cluster centres for the pivots that were included for the large farm. The MOTAD model determined that the distribution of the pivot sizes will be as follows; four pivots of 4 ha, two pivots of 13 ha, three pivots of 30 ha and two pivots of 47 ha.

Table 3.3: Summary of Cluster analysis for the pivot size of representative farm size 2.

Cluster	Range of pivot size (ha)	Cluster centres (ha)	Percentage of water users (%)
1	1-8	4	30
2	23-36	30	33
3	9-20	13	21
4	40-50	47	16

For practical reasons the pivot sizes were standardised to 47 ha, 30 ha, 15 ha and 4 ha to correspond to the irrigation system design data that was available.

3.6.2.2 Crops

The dataset on the water users included hectares planted to different crops. Figure 3.3 shows the area distribution of the most important crops that are grown in the area.

The most dominant crops grown in the area are maize, wheat, popcorn, lucern and soya beans. In total these crops represent 90% of the crops grown in the area. Farmers that produce popcorn are doing so conditionally on the availability of a contract.

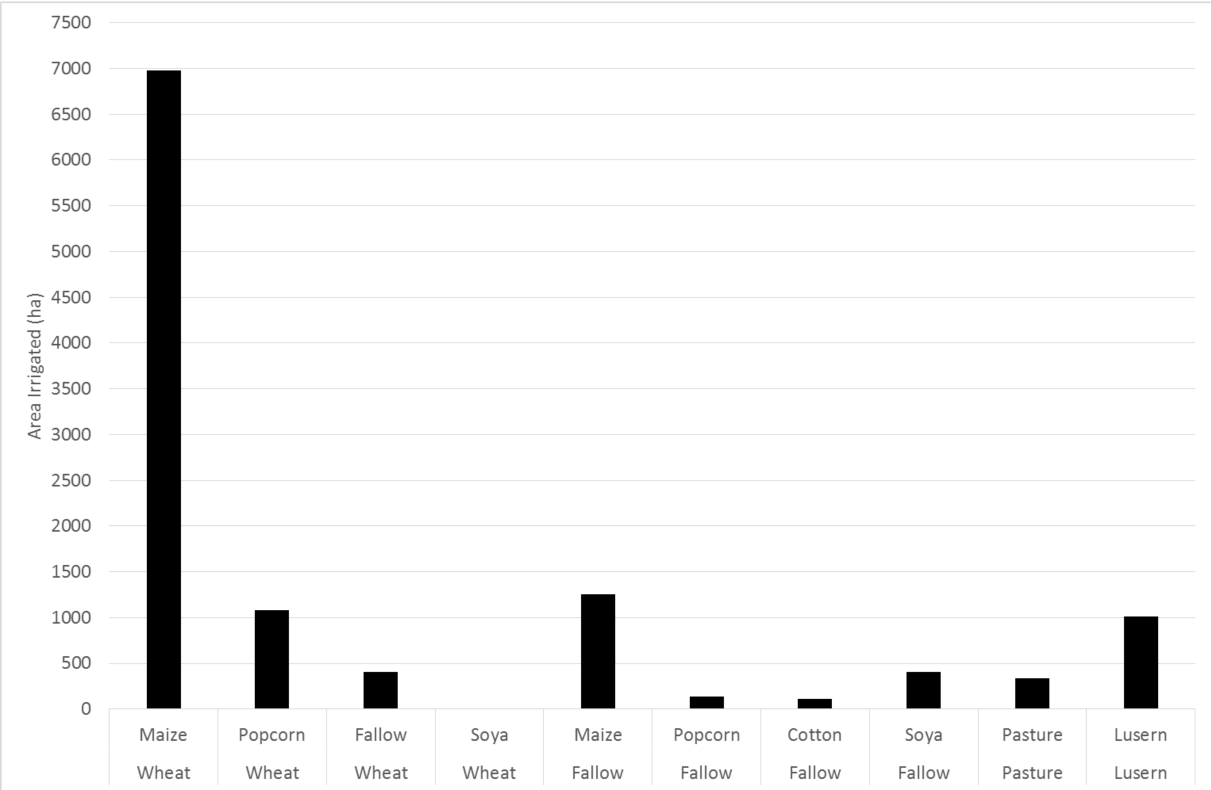


Figure 3.3: Crop rotation for 2015-2016 production season

An enterprise budget for each crop was compiled by using information from Griekwaland-Wes Korporatief (2018) input cost guide as the basis. The information was verified with irrigation farmers in the research area.

3.6.2.3 Gross income variability

Risk enters the optimisation models through price risk, potential yield risk and yield reductions that are dependant on the irrigation strategy. Potential crop yield variability for each crop were derived from the potential evapotranspiration of each crop. Price risk of each crop was simulated for the long-run model using the procedures developed by Richardson, Klose and Gray (2000) to simulate multivariate empirical distributions. Figure 3.4 to Figure 3.6 respectively, show the quantile plots of gross income variability for maize, popcorn and wheat over a period of 15 years using the potential yields of the crops. The y-axis of the plots are scaled the same for a better visualisation of the risk.

The plots show that maize has the highest risk since the quantile lines are the furthest apart. However, maize also has the best chance of generating higher gross income when compared to the other crops. Wheat has the lowest risk and the lowest potential to generate gross income that is higher than the other crops. The minimum gross income that could be generated with wheat is however, larger than the other two crops.

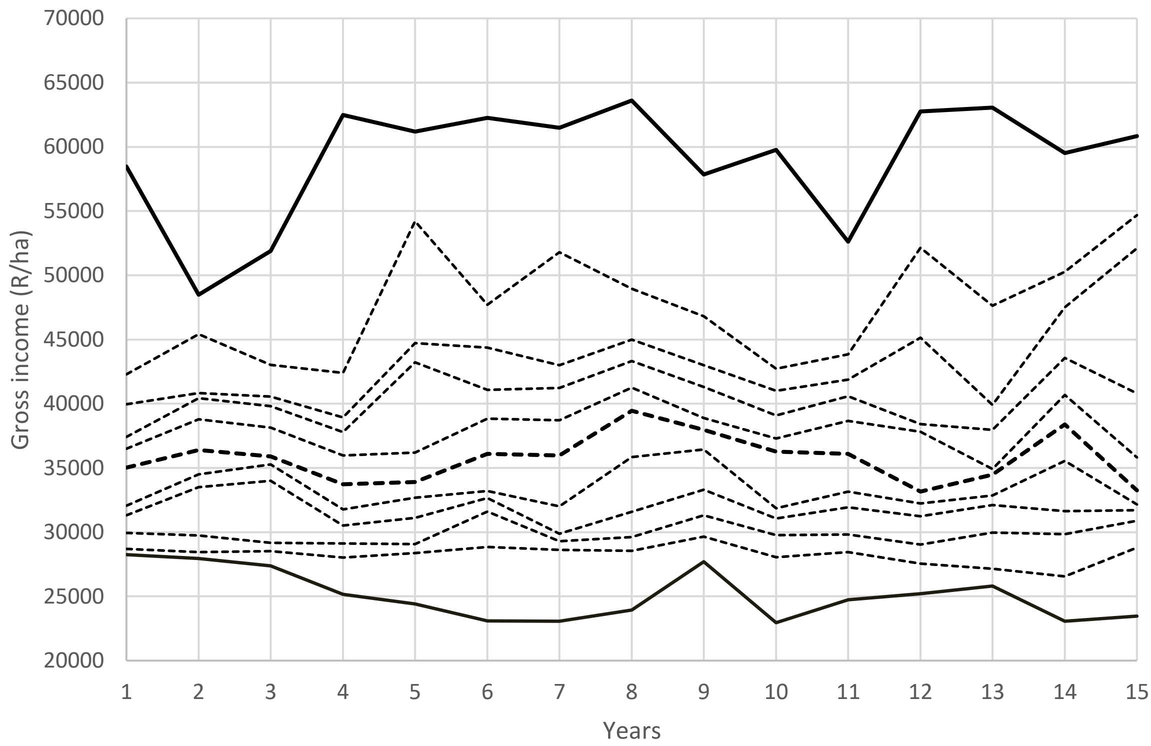


Figure 3.4 Quantile plot of maize gross income under conditions of no water stress (2018)

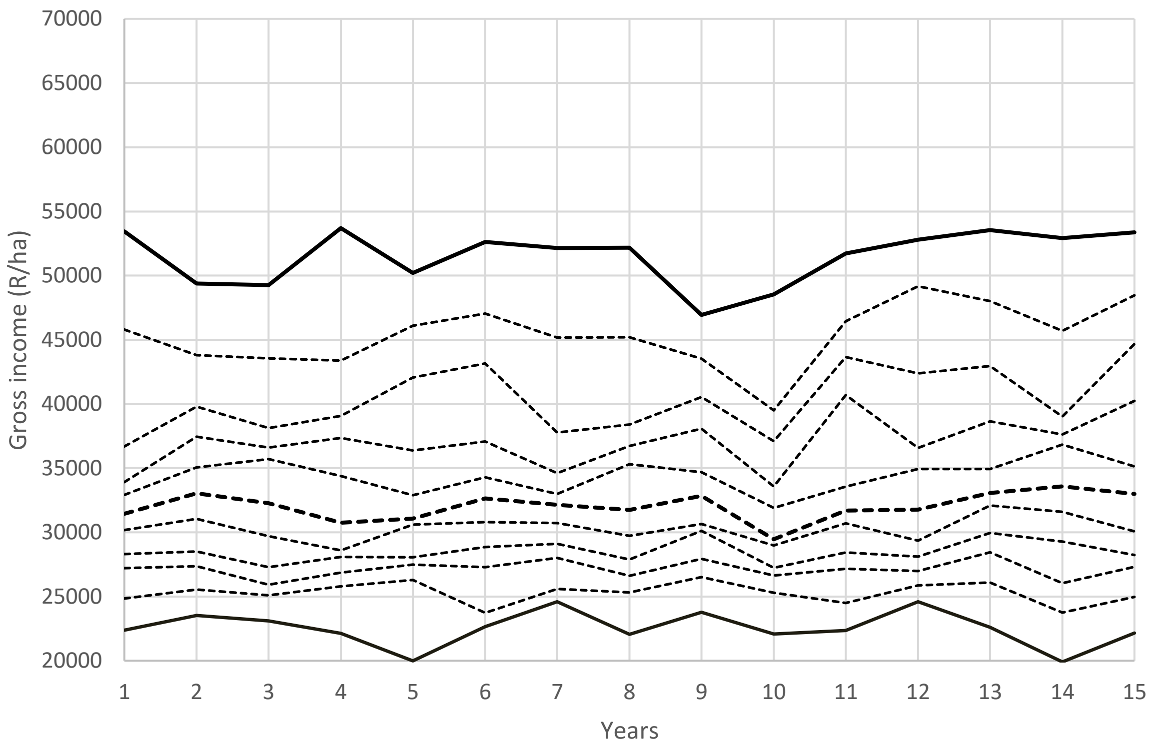


Figure 3.5 Quantile plot of popcorn gross income under conditions of no water stress (2018)

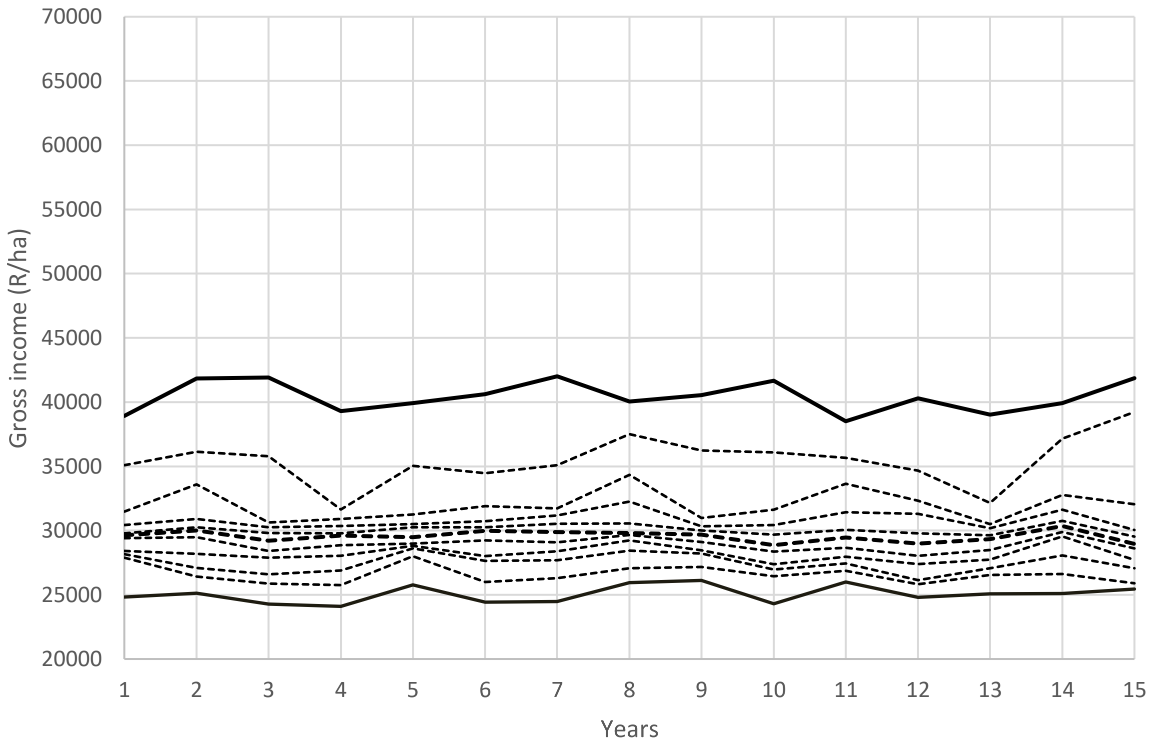


Figure 3.6 Quantile plot of wheat gross income under conditions of no water stress (2018)

3.6.2.4 Cost data

Historical data on production costs were gathered from Griekwaland-Wes Cooperative. The irrigation costs included in the enterprise budgets were recalculated for each pivot using the Ruraflex electricity tariff structures for 2017/2018 season. Information regarding the irrigation systems was obtained from a local irrigation system designer. The overheads of the irrigation farms were acquired from personal interviews with several irrigation farmers in the area.

CHAPTER 4:
HYDRO-ECONOMIC MODELING RESULTS

4.1 INTRODUCTION

The results are discussed in three sections. The first is concerning the assessment of irrigation water availability. The last two sections are used to discuss the short-run and long-run responses of irrigators to a water curtailment of 15%. The applicability of the models and procedures that were developed in this research is demonstrated by reporting the results for the 233 ha farm.

4.2 ASSESSMENT OF WATER AVAILABILITY

Water resource assessments of a catchment or system are performed to determine a yield. The yield is the volume of water that can be abstracted with a certain level of acceptable failure for a given specific demand over a specified period of time. Water resource planners use two approaches, namely historical analysis and stochastic analysis to determine the yield. Historical analysis is performed using one sequence of historical flows to gain an understanding of the historical yield potential of a catchment with the current water resource infrastructure and water uses in place.

Three types of yield are assessed per catchment / system. These are firm, average and base yields. The firm yield is the maximum water demand that can be abstracted from a catchment without any failures. The average yield is the average amount of water that can be abstracted from the catchment with a given demand profile. The base yield is the minimum amount of water that can be abstracted from the catchment for a given demand. If the demand is less than the firm yield, then all three yield calculations will give the same result. However, as the demand increases and the available water is not able to supply demands, then the average and base yields become lower than the demand.

Figure 4.1 shows that the historical and stochastic total yield of all water users including the losses of the Orange River catchment given the simplified model that was setup for this study. Figure 4.2 shows the stochastic base yield (minimum yield) for all water users including the losses of the Orange River catchment. Figure 4.3 and Figure 4.4 show that the long term stochastic total yield and based yield for Irrigation water users in the Orange River catchment. Table 4.1 summarises the long term base yield or minimum water supply reliability for irrigation and all water users in the Orange River catchment at 1:50 and 1:10 years risk level. The water supply reliability or assurance of supply that are determined in this study are reasonably realistic to the estimates that are done in other studies. The overall assurance of water supply for water users in Orange River catchment is modelled as part of the Integrated Vaal system and it is dependent of the river flow from Lesotho highlands and the development in other catchment of the Integrated Vaal system. The Orange River catchment MIKE Hydro Basin setup is simplified to meet the requirement of the study.

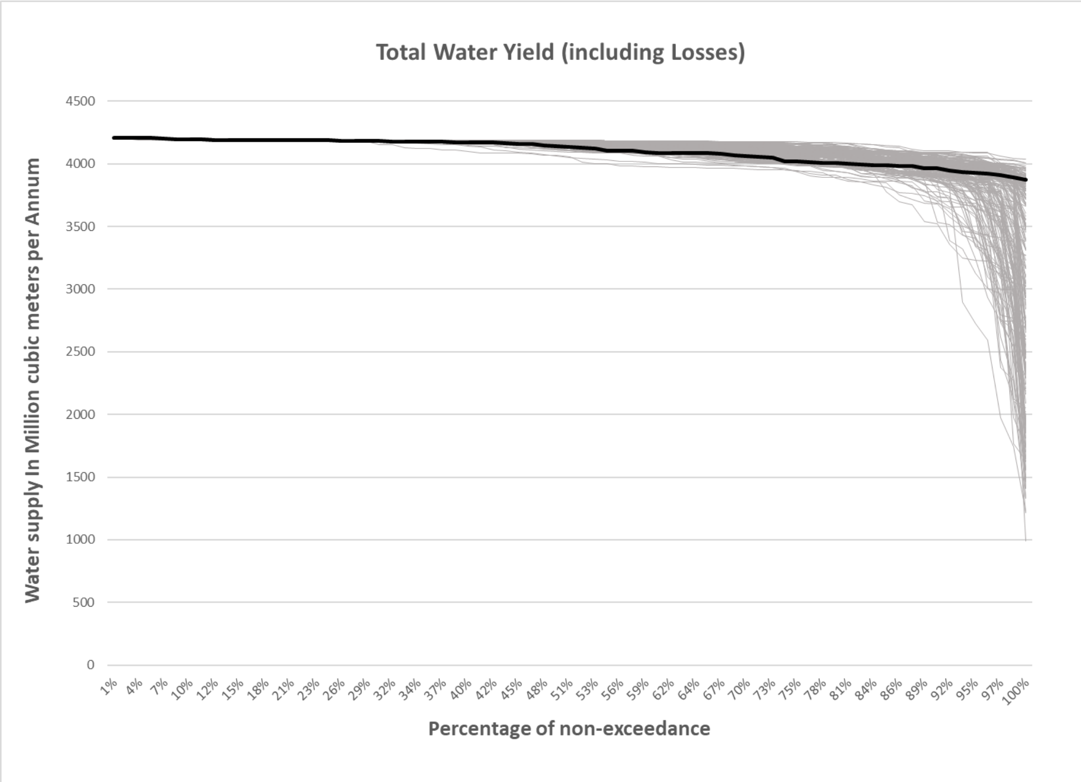


Figure 4.1: Total yield for all water users including loses from Orange River catchment

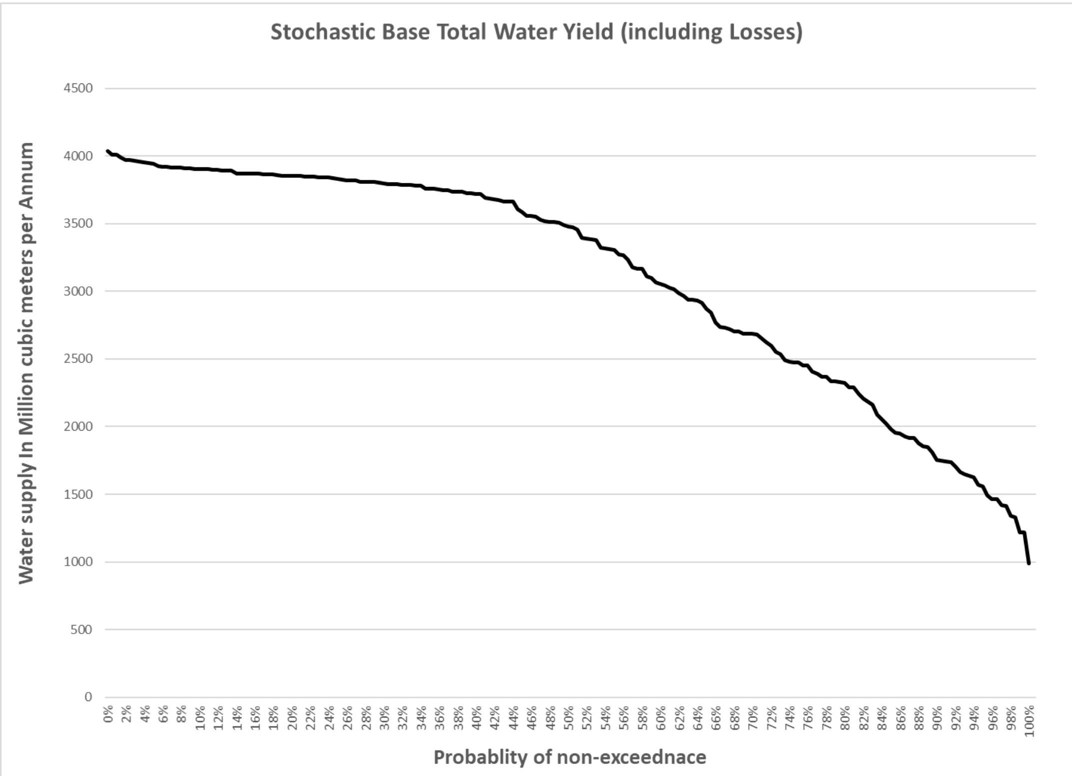


Figure 4.2: Stochastic base yield for all water users including loses from Orange River catchment

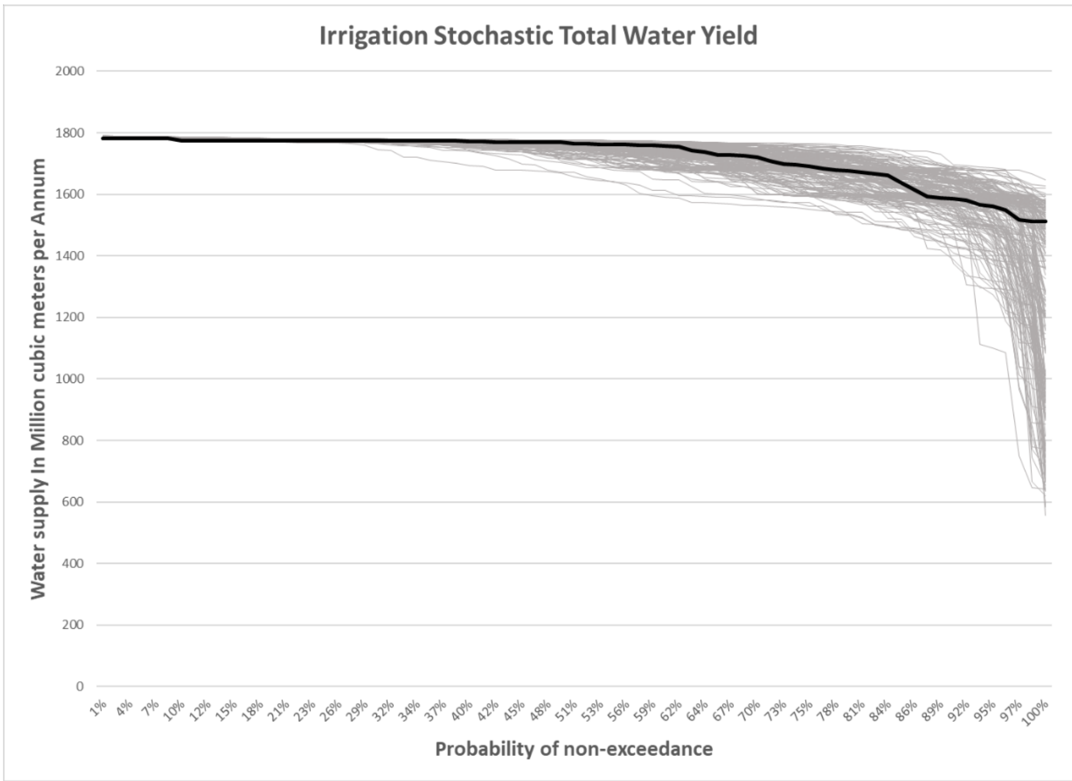


Figure 4.3: Total yield for all Irrigation water users from Orange River catchment

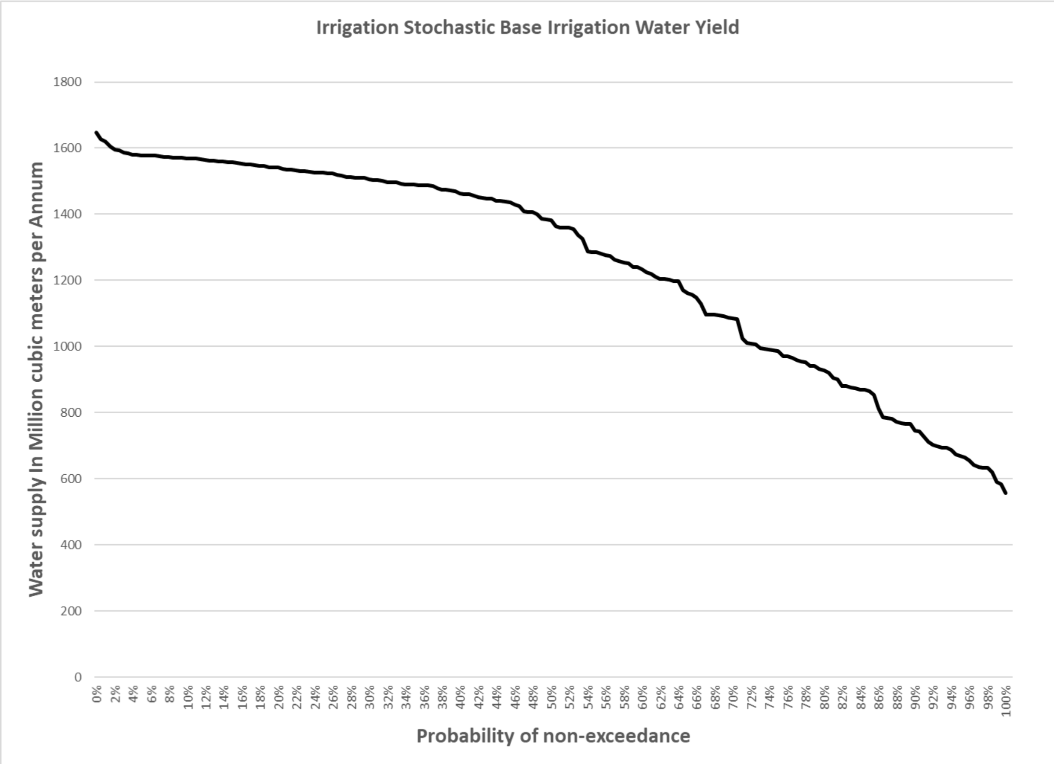


Figure 4.4: Stochastic base yield for Irrigation water users from Orange River catchment

Table 4.1: Long term stochastic base yield or minimum water assurance of supply of reliability for the Orange River catchment water users are at 1:50 and 1:10 years risk level

	Volume (Million m^3/annum)		Percentage relative to total allocation (%)	
	1:50 years	1:10 years	1:10 years	1:50
Irrigation Sector	1572.88	1646.79	88.62	92.79
All users including losses	3844.93	4009.42	91.82	95.74

4.3 SHORT-RUN REPSONSES TO WATER CURTAILMENTS

The short-run response to a water curtailment was optimised using the integrated GA-LP solution procedure that was developed as part of the research for an irrigation farm with 233 ha of centre pivot irrigation growing maize, wheat and popcorn. The near optimal solutions of the GA-LP model were used as starting points for a non-linear programming model that incorporates the water budget calculations to check whether it was possible to improve the near optimal solutions with the programming model. Results showed that it is possible to improve the near optimal solutions with the programming model. The results of the programming model are discussed in two sections. The first section is concerned with the financial impact and the second evaluate the management responses.

4.3.1 PROFITABILITY

The short-run financial impact of a water curtailment was determined for risk neutral and risk averse decision-makers by evaluating the optimised certainty equivalents and the utility weighted risk premiums to move from a less preferred scenario to a preferred scenario. A certainty equivalent quantifies the sure amount of money that will make the decision-maker indifferent between accepting the risk or the sure amount. The difference between the certainty equivalents of two scenarios provides an estimate of the benefit for a decision-maker to move form the less preferred scenario to the preferred scenario.

4.3.1.1 Total gross margin certainty equivalents

The stochastic efficiency with respect to a negative exponential utility function results are shown in Figure 4.5 for a full and curtailed water allocation scenario under the assumption that the irrigation farm has either irrigation systems with a low or high irrigation water application uniformity. A higher uniformity implies that irrigation water applications will be more uniform.

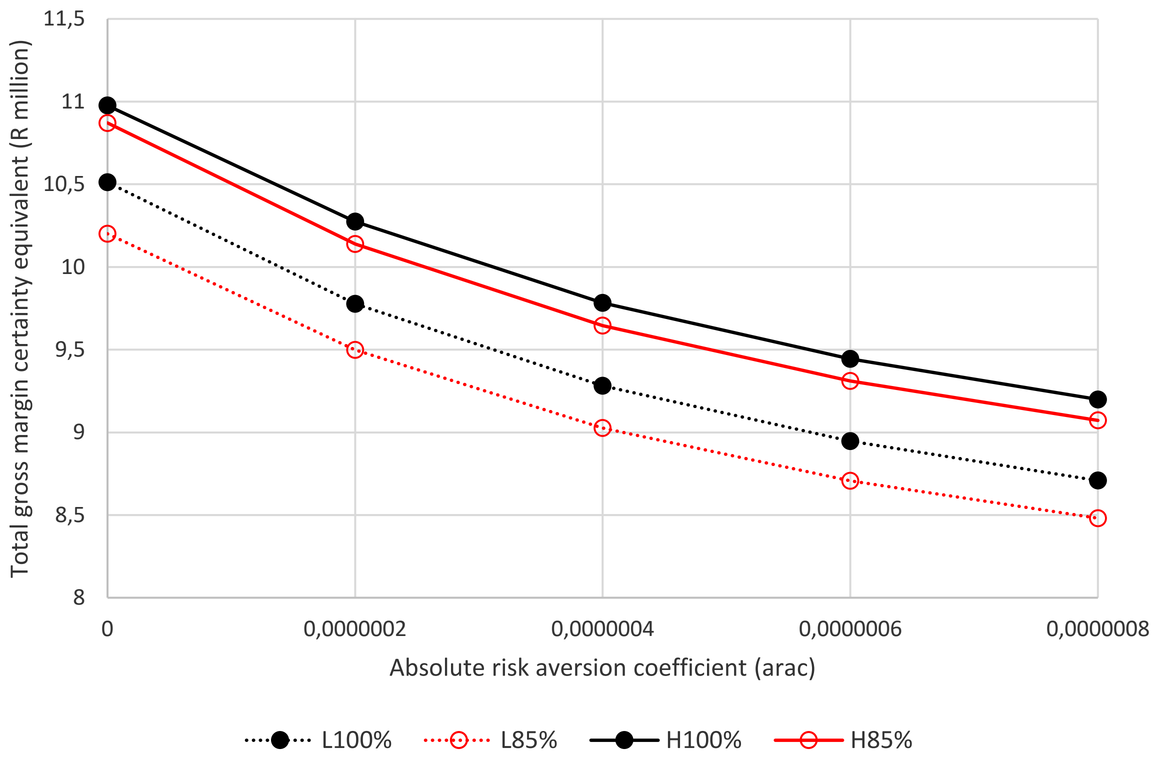


Figure 4.5: Optimised short-run gross margin certainty equivalents under a full (100%) and curtailed (85%) water allocation for a low (L) and high (H) uniformity scenario (2018).

An increase in risk aversion is indicated by increasing levels of absolute risk aversion while a value of zero indicates risk neutrality. As expected, the certainty equivalents (CE) were reduced when enforcing a water curtailment of 15%. For a risk neutral decision-maker the reduction in CE amounts to about R310 000 or R1 334/ha. The CEs of the high uniformity scenario is higher when compared with the low uniformity scenario even when the water allocation is curtailed. The result shows that large gains are possible through increases in irrigation efficiency. Interestingly, the impact of a water curtailment is less severe when considering the high uniformity scenario. Increasing the uniformity of the irrigation systems resulted in higher irrigation efficiencies and consequently 2.5% less than the full water quota is used which shows that area is limiting production. Therefore, the difference between the a full water allocation scenario and a curtailed water allocation scenario is less.

The impact of risk aversion is severe. The level of absolute risk aversion were chosen such that the extreme level of risk aversion will not exceed a lower confidence interval of 90% (Barry *et al.*, 2009). An extremely risk averse (arac=0.0000008) decision-maker will accept a CE which is on average about R1.77 million less than that of a risk neutral decision-maker.

4.3.1.2 Total gross margin utility weighted risk premium

Figure 4.6 shows the negative exponential utility weighted risk premiums which is an indication of the benefit a decision-maker will derive when changing from a less preferred alternative to a more preferred alternative.

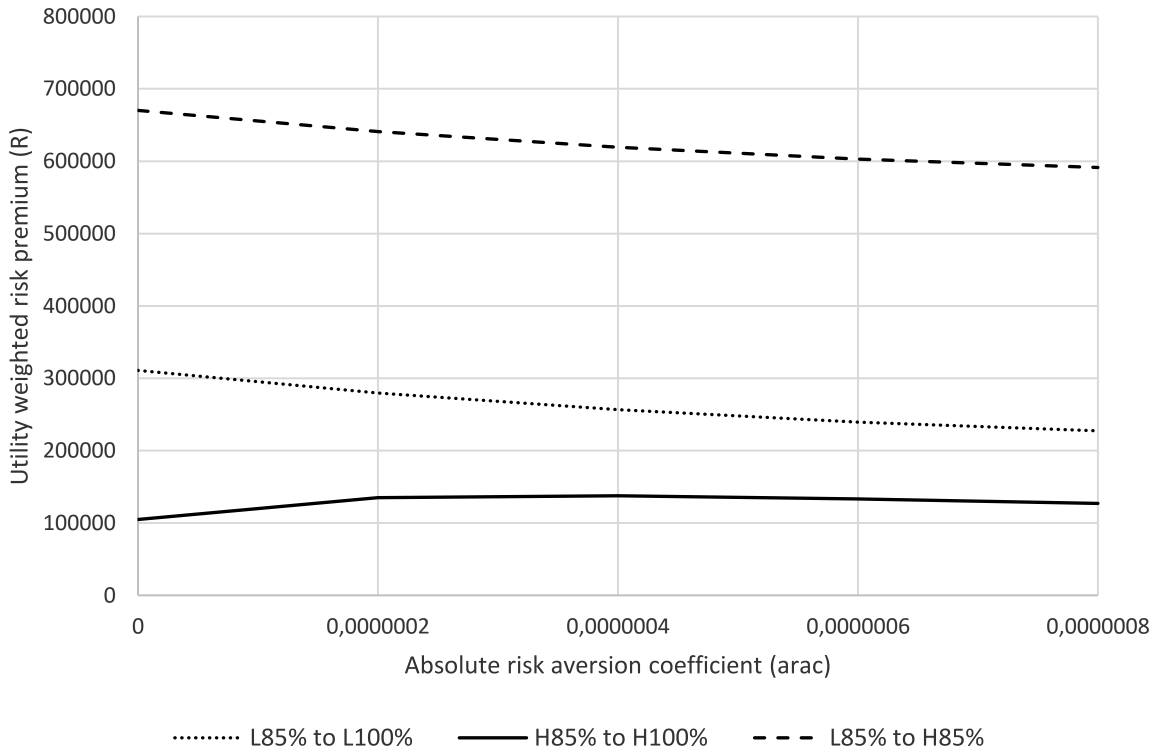


Figure 4.6: Optimised short-run gross margin utility weighted risk premiums to move from less preferred to a prefered alternative under a full (100%) and curtailed (85%) water allocation for a low (L) and high (H) uniformity scenario (2018).

The results show that the benefits of having a full water allocation when respectively concerning a low uniformity and a high uniformity scenario are R310 000 and R104 000. The small benefit for the high uniformity scenario is again attributed to the fact that the higher uniformities caused a reduction in water use of 2.5%. The benefit of improving irrigation water use efficiency through increasing irrigation system uniformity as a strategy to combat water curtailments amounts to R670 000 which is large. The results clearly show that it is highly profitable to improve irrigation water use efficiency to combat water curtailments.

The impact of risk aversion on the calculated risk premiums is less profound and with mixed results. The benefit of having a full water allocation increases with increasing levels of risk aversion for the high uniformity scenario while the benefit decreases with increasing risk aversion for the other scenarios considered. Whether the benefits increase or decrease is the direct result of changes in the distribution of gross margins that where used to calculate the CEs of the scenarios.

Next the impact of a 15% water curtailment on irrigation water use is discussed.

4.3.2 IRRIGATION WATER USE IMPLICATIONS

The optimised short-run water use results for a full and curtailed water allocation scenario under the assumption that the irrigation farm has either irrigation systems with a low or high irrigation water application uniformity are shown for a risk neutral and extremely risk averse decision-maker in Table 4.2.

Irrigation farmers could respond to water curtailments on the extensive margin through changes in crop mix and area irrigated or on the intensive margin through better irrigation management. The results show that responses at the intensive margin dominate the response of irrigation farmers to a water curtailment of 15% since the crop mix and irrigated areas are constant irrespective of the irrigation uniformity scenario considered. Recall that the high uniformity scenario does not use all the irrigation water that is allocated to the farm. Consequently, this scenario represents economically optimal water use under unlimited water supply. Results from the scenario show that average crop yields are very close to potential crop yields which is indicative of crop prices being much higher than the marginal factor cost of irrigation water.

In essence, irrigation famers are operating in the land limiting phase (Grové, 2019) where it is profitable to irrigate all the available land at irrigation rates that are determined the availability of water or the micro-economic conditions for optimal water use under unlimited water supply. Irrigated area will only be reduced once water becomes so limited that production per hectare is not profitable, and the only way to make production profitable is to reduce the irrigated area to increase production levels per hectare. The movement along the irrigation water production function in the area limiting phase is evident when considering curtailed water allocations of the two uniformity scenarios. In both cases the crop yields of the curtailed water allocation are lower when compared with the full water allocation.

Cognisance should be taken of the potential hydrological impact of changing irrigation efficiency. Increasing irrigation efficiency through increased water application uniformity will result in a decrease in potential return flows since the amount of water that percolates below the root zone of the crops decreases while the crops consumptive water use (evapotranspiraion) increases. Let’s consider the curtailed water availability scenario when increasing the uniformity of the irrigation system. In both scenarios the total water abstraction for the farm amounts to 218 510 mm.ha for the production year. However, changes in the irrigation water allocation between crops are made to increase the contribution of rainfall to evapotranspiration losses. Consequently the actual crop evapotranspiration of all the crops are higher when compared to the lower uniformity scenario. Higher evapotranspiration also results in crop yields being consistently higher for the high uniformity scenario.

Table 4.2: Optimised short-run response by a risk neutral and risk averse irrigation farmer to a 15% water curtailment for a low and high uniformity scenario

	NEUTRAL											
	LOW UNIFORMITY						HIGH UNIFORMITY					
	Full allocation			Curtailed allocation			Full allocation			Curtailed allocation		
	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn
Area (ha)	234	204	30	234	204	30	234	204	30	234	204	30
Crop yield (ha)	7.72	15.95	6.38	7.48	15.41	6.25	7.97	16.47	6.48	7.84	16.16	6.41
Potential crop yield (ton/ha)	8	16.5	6.5	8	16.5	6.5	8	16.5	6.5	8	16.5	6.5
Net irrigation (mm)	454	549	446	382	470	387	453	530	399	393	462	357
Deep percolation (mm)	89	125	121	46	75	82	60	83	74	29	35	44
Actual evapotranspiration (mm)	406	486	438	387	467	428	424	506	446	415	495	440
Potential evapotranspiration (mm)	426	507	448	426	507	448	426	507	448	426	507	448
	RISK AVERSE (ARAC* = 0.0000008)											
	LOW UNIFORMITY						HIGH UNIFORMITY					
	Full allocation			Curtailed allocation			Full allocation			Curtailed allocation		
	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn	Wheat	Maize	Popcorn
Area (ha)	234	204	30	234	204	30	234	204	30	234	204	30
Crop yield (ha)	7.74	15.91	6.24	7.51	15.33	6.12	7.98	16.43	6.42	7.85	16.13	6.25
Potential crop yield (ton/ha)	8	16.5	6.5	8	16.5	6.5	8	16.5	6.5	8	16.5	6.5
Net irrigation (mm)	460	551	392	391	464	359	458	516	371	398	461	323
Deep percolation (mm)	90	122	99	49	67	85	64	71	63	30	31	37
Actual evapotranspiration (mm)	406	484	425	391	467	417	425	505	442	415	492	428
Potential evapotranspiration (mm)	426	507	448	426	507	448	426	507	448	426	507	448

* Absolute risk aversion coefficient

A risk averse decision-maker is expected to increase water applications given the assumption that irrigation water applications are considered a risk reducing input. Evaluating the impact of risk aversion using the average results in Table 4.2 is troublesome since the utility function of a risk averse decision-makers emphasises improvement of the lower tail of the distribution rather than the average. Comparing risk neutrality to the case that considers risk aversion shows that the net irrigation water applications of wheat crop yields are consistently higher when risk aversion is considered albeit marginally so on average. On the contrary, the irrigation water applications and crop yields of maize and popcorn is consistently lower when risk aversion is considered. The changes are however, marginal.

Next, the results from the long-run analyses are discussed to determine the financial feasibility of imposing a 15% water curtailment.

4.4 LONG-RUN RESPONSES TO WATER CURTAILMENTS

The main purpose of modelling the long-run response of irrigation farmers to water curtailments is to determine it is profitable and financially feasible to adopt more efficient irrigation systems. The near optimal long-run responses of risk neutral and risk averse irrigation farmers to a water curtailment of 15% are presented in this section. A power utility function is used model relative risk averseness because the outcome variable relates more closely to changes in wealth. The assumption was made that the farm only has irrigation technology with low uniformity.

4.4.1 PROFITABILITY

The short-run financial impact of a water curtailment was determined for risk neutral and risk averse decision-makers by evaluating the optimised certainty equivalents and the utility weighted risk premiums to move from a less preferred scenario to a preferred scenario. Certainty equivalents are again used to evaluate the profitability of irrigation farming when enforcing a water curtailment of 15% while the cost of imposing a water curtailment is determined with the power utility weighted risk premiums.

4.4.1.1 Net present value certainty equivalents

The net present value (NPV) certainty equivalents of the full water allocation and curtailed (85%) water allocation scenarios are shown in Figure 4.7. The distribution of NPVs that were optimised to calculate the certainty equivalents include the cumulative yearend cash flows plus terminal values for cropping activities that uses irrigation technology that extends pass the planning horizon of 15 years.

The near optimal solutions indicate that the average NPV (rrac = 0) for the full water allocation is about R18.33 million while imposing a water curtailment will reduce the average NPV to R16.72 million. Relative risk aversion reduces the certainty equivalent of the full water allocation to R17.37 million while the NPV is reduced to R16.63 million when imposing the water curtailment of 15%.

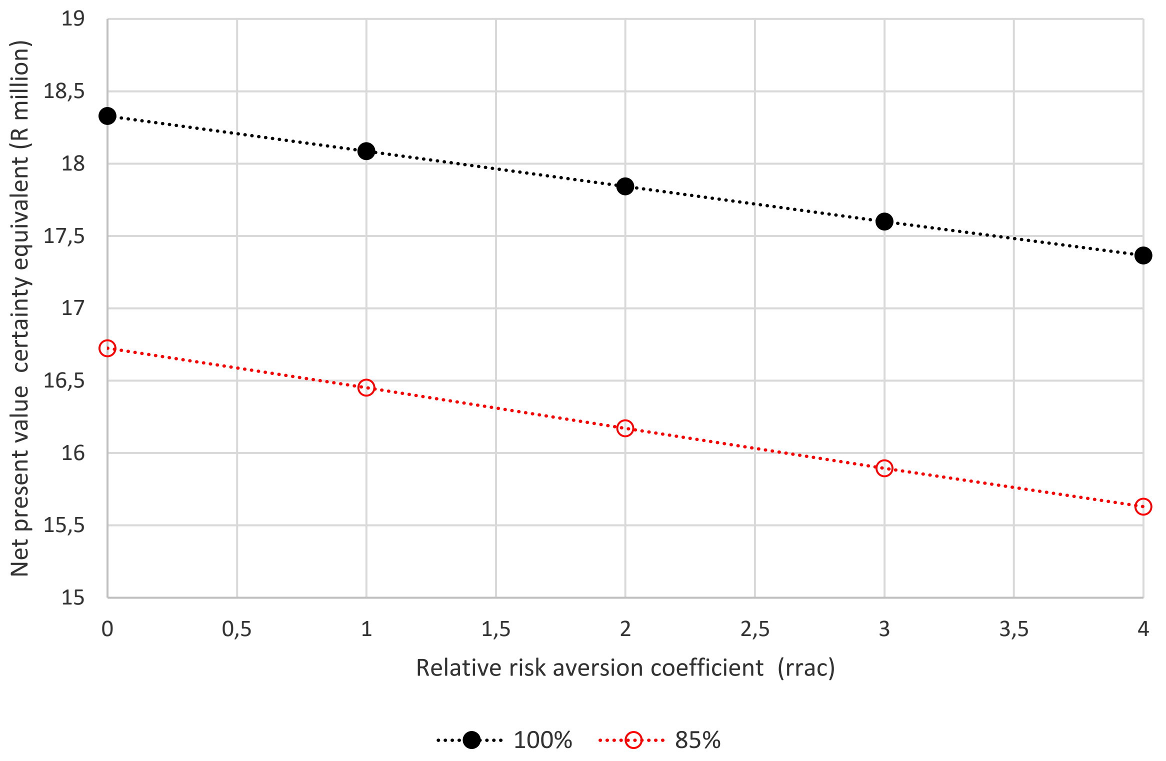


Figure 4.7: Optimised long-run net present value certainty equivalents under a full (100%) and curtailed (85%) water allocation scenario (2018).

Next the power utility weighted risk premium are discussed to determine the monetary cost to decision-makers with varying levels of risk aversions when imposing a water curtailment of 15%.

4.4.1.2 Net present value utility weighted risk premium

The power utility weighted risk premiums to move from a curtailed water allocation scenario to a full water allocation scenario are shown in Figure 4.8 for decision-makers with varying degrees of risk aversion.

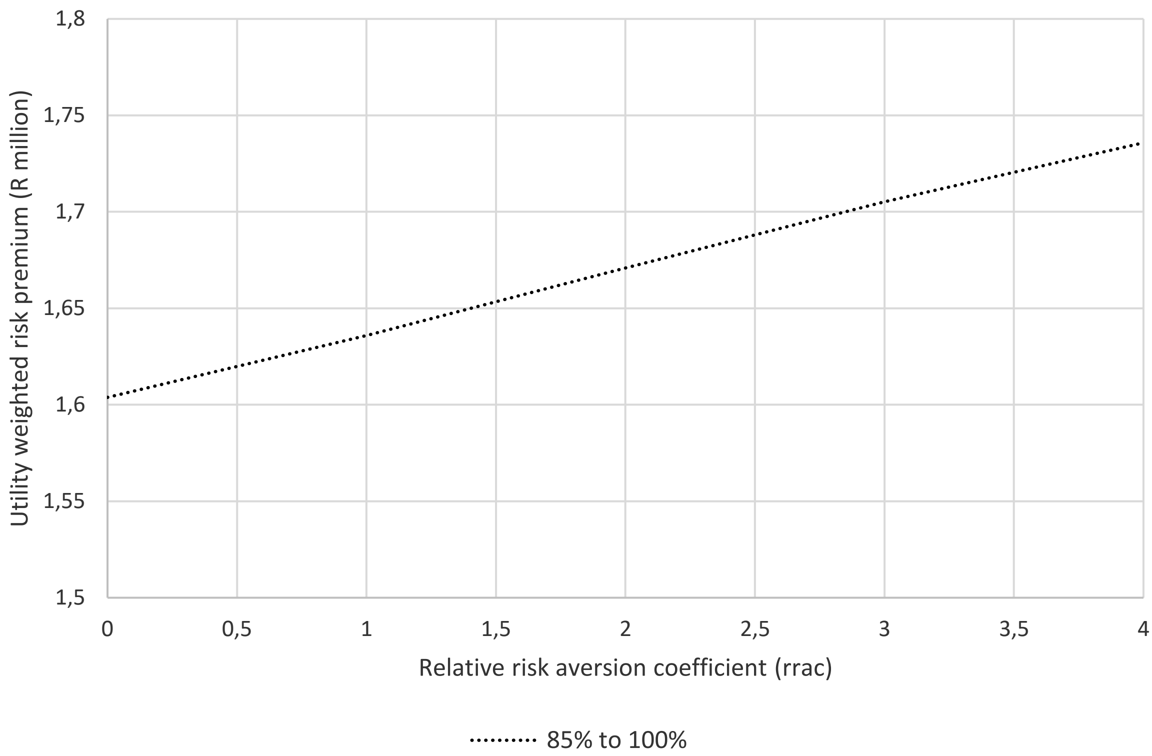


Figure 4.8: Optimised long-run net present value utility weighted risk premiums to move from a curtailed (85%) water allocation to a full (100%)water allocation scenario (2018).

The results from the analysis shows that the benefit to a risk neutral (rrac=0) irrigator when moving from a curtailed water allocation to a full water allocation is about R1.6 million. The benefit increases steadily to a maximum of about R1.74 million under extreme relative risk aversion. The increase in the risk premium is directly related to the fact that the impact of risk aversion was more when the curtailed water allocation scenario is considered.

4.4.2 IRRIGATION TECHNOLOGY USE

The short-run analyses indicated that the adoption of irrigation technology with higher uniformities that result in higher water use efficiencies will play an important role in combatting the negative effects of water curtailments on irrigation farming profitability. The long-run responses with respect to improving irrigation technology include upgrading existing irrigation technology with low uniformity by installing improved irrigation nozzle packages and replacing old technology with new technology. The long-run optimisation model has a disequilibrium known life type of structure which implies that existing irrigation technology will remain until the end of their economic life after which the technology could be replaced by any other technology. The optimised irrigation technology use by a risk neutral and risk averse irrigation farmer under the full water allocation scenario are respectively given in Table 4.3 and Table 4.4.

The results show that 233.7 ha of pivot irrigation is available for production irrespective of the level of risk aversion. The risk neutral irrigator upgraded one 47 ha pivot, all three the 30.1 ha pivots and one 15 ha pivot. The 30.1 ha pivots are upgraded even though only two years of their economic life is remaining. Interestingly, only two of the small (4.5 ha) pivots are replaced by the same size pivot at the beginning of Year 2 irrespective of risk aversion level. Consequently, only 224.7 ha is available for production in Year 2. The 30.1 ha pivots reaches the end of their economic life at the end of Year 2. The 30.1 ha pivots are replaced with two 47.7 ha pivots which resulted in an increase in production area availability to 229.8 ha. The area available for production stays constant until Year 8 when the tow 15 ha pivots are replaced with two 30.1 ha pivots. Consequently total irrigation area availability increases to 229.9 ha. The two old 47.7 ha pivots are replaced in Year 12 with two 4.5 ha pivots. The total available irrigation area therefore reduced to only 134.5 ha. The massive reduction available irrigation area causes the irrigation farmer not to utilise his full water quota. In the years to follow investments in new 4.5 ha pivots and 15 ha pivots are made to gradually increase the total availability of irrigation area to a maximum of 233.5 ha. In none of the last four years did the irrigation farm utilise its full water quota. The risk averse irrigator followed exactly the same investment path as the risk neutral irrigator with the exception that only two of the three 30.1 ha pivots were upgraded during the first year.

The irrigation system investment decisions made by a risk neutral and risk averse irrigator under the curtailed water allocation scenario are respectively shown in Table 4.5. and Table 4.6. The investment decisions of the risk neutral irrigator confronted with a curtailed water allocation are similar to that of his full water allocation counterpart. The only difference is that none of the small (4.5 ha) pivots are replaced until Year 12. Consequently the total area available for production is slightly less when compared to the full water allocation scenario. Like before, all the water that is available for production is not used in Year 4 as well as the last four years of analyses. Interestingly, the risk averse irrigator decides to invest in more irrigation systems to increase the availability of irrigation area. Specifically, one of the four 4.5 ha pivots that reached the end of its economic life at the end of Year 1 is replaced by a pivot of the same size. Consequently, the total area available for irrigation is larger when compared to the risk neutral case for a curtailed water allocation but smaller than the total area available under the full water allocation. The slightly larger area resulted in all the available water being used in Year 4.

Table 4.3: Optimised long-run irrigation technology use (ha) by a risk neutral (rrac=0) irrigation farmer under a full water allocation scenario.

	47.7 ha Pivot Uniformity			30.1 ha Pivot Uniformity			15 ha Pivot Uniformity			4.5 ha Pivot Uniformity			
	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	TOTAL*
Year 1	47.7	47.7			90.3		15	15		18			233.7
Year 2	47.7	47.7			90.3		15	15				9	224.7
Year 3	47.7	47.7	95.4				15	15				9	229.8
Year 4	47.7	47.7	95.4				15	15				9	**229.8**
Year 5	47.7	47.7	95.4				15	15				9	229.8
Year 6	47.7	47.7	95.4				15	15				9	229.8
Year 7	47.7	47.7	95.4				15	15				9	229.8
Year 8	47.7	47.7	95.4			30.1						9	229.9
Year 9	47.7	47.7	95.4			30.1						9	229.9
Year 10	47.7	47.7	95.4			30.1						9	229.9
Year 11	47.7	47.7	95.4			30.1						9	229.9
Year 12	**		95.4			30.1						9	**134.5**
Year 13			95.4			30.1						27	**152.5**
Year 14			95.4			30.1			15			45	**185.5**
Year 15			95.4			30.1			45			63	**233.5**

* Bold values indicate years when less than the allotted water allocation were used

** A blank space means that the specific technology is not available

Table 4.4: Optimised long-run irrigation technology use (ha) by a risk averse (rrac =4) irrigation farmer under a full water allocation scenario.

	47.7 ha Pivot Uniformity			30.1 ha Pivot Uniformity			15 ha Pivot Uniformity			4.5 ha Pivot Uniformity			
	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	TOTAL*
Year 1	47.7	47.7		30.1	60.2		15	15		18			233.7
Year 2	47.7	47.7		30.1	60.2		15	15				9	224.7
Year 3	47.7	47.7	95.4				15	15				9	229.8
Year 4	47.7	47.7	95.4				15	15				9	**229.8**
Year 5	47.7	47.7	95.4				15	15				9	229.8
Year 6	47.7	47.7	95.4				15	15				9	229.8
Year 7	47.7	47.7	95.4				15	15				9	229.8
Year 8	47.7	47.7	95.4			30.1						9	229.9
Year 9	47.7	47.7	95.4			30.1						9	229.9
Year 10	47.7	47.7	95.4			30.1						9	229.9
Year 11	47.7	47.7	95.4			30.1						9	229.9
Year 12	**		95.4			30.1						9	**134.5**
Year 13			95.4			30.1						27	**152.5**
Year 14			95.4			30.1			15			45	**185.5**
Year 15			95.4			30.1			45			63	**233.5**

* Bold values indicate years when less than the allotted water allocation were used

** A blank space means that the specific technology is not available

Table 4.5: Optimised long-run irrigation technology use (ha) by a risk neutral (rrac=0) irrigation farmer under a curtailed (85%) water allocation scenario.

	47.7 ha Pivot Uniformity			30.1 ha Pivot Uniformity			15 ha Pivot Uniformity			4.5 ha Pivot Uniformity			
	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	TOTAL*
Year 1	47.7	47.7		90.3			15	15		18			233.7
Year 2	47.7	47.7		90.3			15	15					215.7
Year 3	47.7	47.7	95.4				15	15					220.8
Year 4	47.7	47.7	95.4				15	15					**220.8**
Year 5	47.7	47.7	95.4				15	15					220.8
Year 6	47.7	47.7	95.4				15	15					220.8
Year 7	47.7	47.7	95.4				15	15					220.8
Year 8	47.7	47.7	95.4			30.1							220.9
Year 9	47.7	47.7	95.4			30.1							220.9
Year 10	47.7	47.7	95.4			30.1							220.9
Year 11	47.7	47.7	95.4			30.1							220.9
Year 12	**		95.4			30.1						9	**134.5**
Year 13			95.4			30.1						27	**152.5**
Year 14			95.4			30.1			15			45	**185.5**
Year 15			95.4			30.1			45			63	**233.5**

* Bold values indicate years when less than the allotted water allocation were used

** A blank space means that the specific technology is not available

Table 4.6: Optimised long-run irrigation technology use (ha) by a risk averse (rrac=4) irrigation farmer under a curtailed (85%) water allocation scenario.

	47.7 ha Pivot Uniformity			30.1 ha Pivot Uniformity			15 ha Pivot Uniformity			4.5 ha Pivot Uniformity			
	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	Low Current	High Upgrade	High Invest	TOTAL*
Year 1	47.7	47.7		90.3			15	15		18			233.7
Year 2	47.7	47.7		90.3			15	15				4.5	215.7
Year 3	47.7	47.7	95.4				15	15				4.5	225.3
Year 4	47.7	47.7	95.4				15	15				4.5	225.3
Year 5	47.7	47.7	95.4				15	15				4.5	225.3
Year 6	47.7	47.7	95.4				15	15				4.5	225.3
Year 7	47.7	47.7	95.4				15	15				4.5	225.3
Year 8	47.7	47.7	95.4			30.1						4.5	225.4
Year 9	47.7	47.7	95.4			30.1						4.5	225.4
Year 10	47.7	47.7	95.4			30.1						4.5	225.4
Year 11	47.7	47.7	95.4			30.1						4.5	225.4
Year 12	**		95.4			30.1						9	**134.5**
Year 13			95.4			30.1						27	**152.5**
Year 14			95.4			30.1			15			45	**185.5**
Year 15			95.4			30.1			45			63	**233.5**

* Bold values indicate years when less than the allotted water allocation were used

** A blank space means that the specific technology is not available

4.4.3 FINANCIAL FEASIBILITY

The profitability analyses indicated that irrigation farming will be profitable since the certainty equivalents based on the net present values were all positive. The purpose of the financial feasibility analyses is to determine whether the irrigation farm is producing enough cash to cover its costs. For this purpose the cash flow ratio is used. The cash flow ratio is the ratio of all income generated to all expenses paid excluding income tax. The threshold used to evaluate the feasibility is 1.15% which imply that total income must exceed expenses by 15%. The optimised statistical moments for the cash flow ratio during each year is given in Table 4.7 for a risk neutral and risk averse decision-maker under a full and curtailed water allocation scenario.

The results showed that on average the optimised cash flow ratios for the full water allocation scenario are above the threshold of 1.15 with the exception of Year 5 for a risk averse decision-maker as well as the last three years of the analyses. The expansion of irrigation area during the last three years put severe pressure on the cash flow ratio and during the last two years income is not enough to cover expenses. The overall impact of risk aversion on the cash flow ratio is minimal with the most noteworthy differences corresponding to changes in irrigation system upgrades and investments in new systems.

In general a water curtailment of 15% reduces the magnitude of the cash flow ratio and increases the probability that the cash flow ratio will be below threshold level of 1.15. On average the ration is about 2.3 percentage points with an average increase of shortfall probability of 1.2 percentage points. These changes are less that expected. However, they are conditional on the ability of the decision-maker optimally respond to the reduction in water availability.

Table 4.7: Cash flow ratio statistical moments for a full water allocation and curtailed (85%) water allocation scenario.

	Year1	Year2	Year3	Year4	Year5	Year6	Year7	Year8	Year9	Year10	Year11	Year12	Year13	Year14	Year15
	Risk neutral (rrac = 0) full water allocation														
Minimum	1.09	1.10	0.96	1.07	1.02	1.14	1.06	1.01	0.99	1.03	0.99	1.24	0.91	0.89	0.81
Median	1.15	1.27	1.17	1.17	1.12	1.20	1.23	1.21	1.11	1.15	1.30	1.44	1.02	0.98	0.99
Maximum	1.32	1.65	1.56	1.51	1.32	1.38	1.54	1.74	1.58	1.39	1.35	1.96	1.37	1.08	1.08
Average	1.19	1.28	1.19	1.20	1.15	1.22	1.23	1.24	1.21	1.20	1.26	1.53	1.05	0.97	0.99
Probability < 1.15	0.54	0.21	0.44	0.22	0.58	0.07	0.41	0.28	0.53	0.49	0.12	0.00	0.89	1.00	1.00
	Risk averse (rrac = 4) full water allocation														
Minimum	1.11	1.09	0.96	1.07	1.02	1.15	1.06	1.06	0.95	1.04	0.99	1.24	0.91	0.89	0.81
Median	1.17	1.25	1.17	1.17	1.12	1.20	1.23	1.26	1.07	1.15	1.30	1.44	1.02	0.98	0.99
Maximum	1.35	1.62	1.54	1.51	1.32	1.39	1.54	1.81	1.50	1.39	1.35	1.96	1.37	1.08	1.08
Average	1.21	1.26	1.18	1.20	1.14	1.23	1.23	1.30	1.16	1.20	1.26	1.53	1.05	0.97	0.99
Probability < 1.15	0.38	0.38	0.45	0.22	0.59	0.05	0.41	0.19	0.57	0.49	0.12	0.00	0.89	1.00	1.00
	Risk neutral (rrac = 0) curtailed (85%) water allocation														
Minimum	1.13	1.10	0.96	1.06	1.08	1.07	1.10	0.95	0.93	1.02	1.02	1.17	0.85	0.85	0.82
Median	1.19	1.24	1.12	1.17	1.18	1.18	1.26	1.19	1.03	1.12	1.29	1.34	0.99	0.94	1.00
Maximum	1.39	1.61	1.41	1.51	1.40	1.27	1.57	1.74	1.51	1.36	1.35	1.75	1.28	1.08	1.11
Average	1.23	1.27	1.12	1.20	1.20	1.17	1.26	1.21	1.13	1.17	1.25	1.42	1.00	0.93	1.01
Probability < 1.15	0.09	0.24	0.61	0.23	0.44	0.42	0.32	0.42	0.62	0.53	0.11	0.00	0.92	1.00	1.00
	Risk averse (rrac = 4) curtailed (85%) water allocation														
Minimum	1.13	1.09	0.96	1.07	1.06	1.06	1.10	1.01	0.89	1.01	1.02	1.20	0.86	0.85	0.83
Median	1.19	1.22	1.11	1.17	1.17	1.17	1.27	1.26	0.99	1.11	1.29	1.37	1.00	0.95	1.01
Maximum	1.39	1.59	1.39	1.52	1.38	1.27	1.58	1.83	1.43	1.35	1.35	1.79	1.29	1.09	1.12
Average	1.23	1.25	1.11	1.20	1.19	1.16	1.27	1.27	1.07	1.16	1.25	1.45	1.01	0.94	1.02
Probability < 1.15	0.09	0.35	0.77	0.22	0.47	0.45	0.18	0.30	0.73	0.54	0.11	0.00	0.92	1.00	1.00

CHAPTER 5:
CONCLUSIONS AND RECOMMENDATIONS

5.1 BACKGROUND

Currently the pressure on irrigated agriculture to use irrigation water more efficiently is mounting in order to provide the necessary water resources to develop an integrated rural economy. The lack of an integrated hydro-economic modeling system that is able to integrate institutional changes on water availability, irrigators operational and investment responses to changes and the resulting impact on the hydrology of the catchment hampers the evaluation of alternative measures to combat the farm-level financial impact and feasibility of policy changes with respect to water allocation. Available hydro-economic modelling frameworks in South Africa do not allow for the necessary detail for meaningful integration of irrigation decisions and technology use at the farm-level and their interactions with water institutions and the hydrology to provide decision-support.

The overall objective of this research is to develop and apply a long-run hydro-economic risk simulation and optimisation modeling framework to quantify the hydro-economic impact of water curtailments. The framework consists of MIKE HYDRO BASIN that is used to simulate the hydrological water availability and impact of farmer response on hydrology. Daily crop water budget calculations are used in the short-run economic irrigation water use optimisation model to optimise the water allocation between different irrigation fields of the farm. The impact of non-uniform water applications on irrigation efficiency was explicitly modelled through the inclusion of multiple water budgets for the same irrigation field, which allows for the calculation of percolation losses at the field scale while optimising the allocation of water between multiple crops. The short-run model is integrated in totality within the long-run optimisation model to determine the hydro-economic feasibility of irrigation technology use decisions to combat water curtailments. The integrated long-run dynamic optimisation model is extremely large, highly nonlinear, discontinuous and therefore very difficult to solve. A tailor made algorithm that utilises a genetic algorithm to optimise the impact of irrigation decisions on crop yield in conjunction with standard mathematical programming to balance resource use over the short-run and long-run is developed to solve the model.

The integrated hydro-economic model is applied to determine the hydro-economic feasibility of a 15% water curtailment for a representative farm below the Vanderkloof dam.

5.2 SUMMARY OF RESULTS AND CONCLUSIONS

The short-run modelling results on profitability showed that irrigation farming will still be profitable when water allocations are curtailed by 15%. Increasing irrigation application uniformity proves to be an important strategy to combat water curtailments. The impact of absolute risk aversion on the reduction in certainty equivalents was large. However, the reduction should be evaluated taking cognisance of the fact that the extreme level of absolute risk aversion corresponds to a lower confidence interval of 90%.

Evaluating the optimised responses to a water curtailment of 15% showed that management responses at the intensive margin dominates extensive margin responses. Intensive margin responses are associated with changes in irrigation scheduling while extensive margin responses are associated with changes in area irrigated to different crops. Cognisance should be taken of the fact that optimal intensive margin responses are conditional on knowing the soil-water status, the impact of water deficits on crop growth and yield, the impact of non-uniform water applications on crop yield, the distributional characterisation of future weather variables and the managerial ability to integrate all the information into an optimal decision. Results further showed that improving water use efficiency through optimal irrigation scheduling and more uniform water application rates, may potentially have a negative impact on the hydrology through changes in return flows and increases in evapotranspiration. The impact of risk aversion on the intensive margin responses to a water curtailment seems negligible small. The last mentioned may be the direct result of the small changes in crop yields modelled for economically rational water application amounts in the area limiting phase of production where the production function is fairly flat. The conclusion is that the results should be interpreted with caution as the economic impact of a water curtailment is based on optimal response that may not be feasible within the managerial ability of irrigation farmers.

The long-run results showed that Irrigation farming might still be profitable over the long-run because the NPV for a water curtailment scenario is positive. The reduction in NPV due to a water curtailment is not proportional to the water curtailment which imply that irrigation farmers are able to reduce the impact a water curtailment through management responses in the form of intensive margin management and irrigation technology adoption decisions. The long-run technology use decisions for the full water allocation showed that it is profitable to upgrade the nozzle packages of irrigation systems even though the systems might be close to the end of their economic lives. Interestingly pivots are not replaced to sustain and irrigation area of 233.7 ha and the irrigation area is slightly less for a large part of the planning horizon. Risk aversion had no impact on the replacement strategy that was followed, however, less of the total irrigation area was upgraded to systems with higher irrigation application uniformities. The irrigation system replacement strategy followed under the curtailed water allocation scenario of the risk neutral decision makers is similar tot that of the full water allocation. The only difference being that less pivots are replaced which causes the total irrigation area to less when compared to the full water allocation scenario. Risk aversion causes a slight increase in irrigation area when considering the curtailed water allocation scenario.

The financial feasibility of irrigation technology replacements were evaluated using the cash flow ratio. Results showed the average cash flow ratio for the full water allocation scenario to be higher than the threshold used by commercial banks to loan money. A water curtailment reduced the magnitude of the ratio while increasing the probability of falling below the threshold of 1.15. Overall the impact of risk aversion was relatively small.

The importance of intensive margin management responses to combat water curtailments in the short- and long-run emphasises the need to understand the inefficiencies associated with agricultural water use better. Very few analyses are done in South Africa that explicitly take cognisance of the factors driving inefficiencies.

5.3 RECOMMENDATIONS FOR FUTURE RESEARCH

The research has made a significant contribution towards representing the biophysical aspects of crop production within economic models of optimal water resource use. The models, methods and procedures that were developed as part of this research open up the opportunity to conduct further research. Firstly, through the application and further development of the integrated hydro-economic framework to evaluate alternative policies to re-allocate water at catchment scale and secondly, the application of the solution procedures developed in this research to evaluate water resource use using bio-economic modeling.

The following recommendations pertain to the application of the integrated hydro-economic modelling framework:

- The feasibility of upscaling the procedures developed for a specific case study farm to represent the water use of the irrigation sector in a catchment should be further investigated.
- A water accounting and auditing framework needs to be developed to give effect to water markets and capacity sharing.
- Extending the optimization algorithm to include operational variables will enable the optimization of the operating rules in the catchment.
- The application of the hydro-economic framework allows for the evaluation of farm-level variability such as heterogeneity in soils, cash reserves, farm structure, etc. on the hydro-economic feasibility of alternative water reallocation scenarios.

The following recommendations pertain to the improved modelling of bio-economic systems through the use of the optimisation methods developed in this research:

- The ability of the optimization solution procedure to optimize discontinuous systems allows for the optimization of irrigation strategies that could be applied in real life compared to the optimal irrigation schedules optimized in this research. Optimizing irrigation strategies will improve the creditability of the results.

- The solution procedure could be linked to any biophysical model that is used to simulate the impact of exogenous changes to a system. Modeling the economic impact of climate change could for example be improved through a better representation of the biophysical component of the economic model.
- The efficiency with which the solution procedure determines the near optimal solutions should be improved through the utilization of parallel processing and further algorithmic improvements.

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APPENDIX A: GAMS CODE: HYBRID GA-LP SOLUTION PROCEDURE

```
*==========================================================================================================================
* IMPORT OPTIMISATION MODEL
*==========================================================================================================================

$include "LR_OPTIMISATION_GA.gms"

*==========================================================================================================================
* GA DODE: DECLARATION OF SETS
*==========================================================================================================================
SETS
p population of individuals
  /pop1*pop20/
g genes
  /g1*g11/
v maximum number of variable identifiers
  /v1*v365/
vv(awc,cu,crop,v) subset indicating membership to number of variables to optimise

con contestants in tournament selection
  /con1*con2/
par selected parents based on tournament selection
  /par1,par2/
iter GA iteration counters
  /iter1*iter10000/;

*assign alias' to sets
*---------------------
alias(dog,dog1);
alias(p,p1);
alias(v,v1);
alias(par,par1);
alias(iter,iter_1);

*assign membership to subset vv
*------------------------------
vv(awc,cu,crop,v)$(ord(v)le count_ir(crop))=yes;
display vv;
```

```
*==========================================================================================================================
* GA DODE: DECLARATION OF PARAMETERS AND SCALARS
*==========================================================================================================================
*PARAMETERS FOR ENCODING, DECODING AND RANKING POPULATION
*---------------------------------------------------------
PARAMETERS
population(awc,cu,crop,v,t,p,g)   binary population of genes (01)
bin_exponent(g)                   exponent used to encode binary string to uniform(01)
decode(awc,cu,crop,v,t,p)         uniform(01) values decoded from binary population of genes
sdecode(awc,cu,crop,v,t,p)        uniform (01) values decoded from ranked binary population of genes
z(p)                              fitness value of the population
sZ(p)                             fitness value of the ranked population
rank(p)                           rank value (based on Z) of population
spopulation(awc,cu,crop,v,t,p,g)  ranked (based on Z) binary population of genes (01)

*PARAMETERS FOR SELECTING PARENTS WITH TOURNAMENT SELECTION
*-----------------------------------------------------------
tour_sel(awc,cu,crop,v,t,par,con) randomly selected contestants in the tournament
tour_selz(awc,cu,crop,v,t,par,p)  fitness values of selected contestants
tour_rank(awc,cu,crop,v,t,par,p)  rank value (based on Z) of contestants
tour_win(awc,cu,crop,v,t,par,p)   fitness value (Z) of winners of tournament
tour_pop(awc,cu,crop,v,t,par,p,g) winners: binary population of genes (01)
rank_of_con1(awc,cu,crop,v,t,par,con,p) rank of contestants
rank_of_con(awc,cu,crop,v,t,par,p) rank of contestant
rank_of_win(awc,cu,crop,v,t,par)   rank of winner
ord_of_win(awc,cu,crop,v,t,par,p)  ordinal value of winners rank
ord_of_win1(awc,cu,crop,v,t,par,p) ordinal value of winners rank alternative
a(awc,cu,crop,v,t)                 test parameter a
b(awc,cu,crop,v,t)                 test parameter b
select(awc,cu,crop,v,t,par)

*PARAMETERS FOR PRODUCING OFFSPRING THROUGH MUTATION AND CROSS OVER
*-------------------------------------------------------------------
mutate                             mutation probability
cross                              crossover probability
child(awc,cu,crop,v,t,par,g)       child: binary population of genes (01)
decode_child(awc,cu,crop,v,t,p)    child: uniform(01) values decoded from binary population of genes
```

```
*PARAMETERS FOR SELECTING INDIVIDUALS TO BE REPLACED WITH OFFSPRING
*-------------------------------------------------------------------
rand(par)                         ordinal value of randomly selected individual to be replaced
replace(par)                      selected individuals in population to be replaced

*PARAMETERS FOR CONTROLING THE MAIN GA LOOP
*-------------------------------------------
tau_max                           maximum time allowed for optimisation
iter_max                          maximum iterations allowed optimisation
t1                                start time
t2                                elapsed time

*PARAMETERS TO ENCODE IRRIGATION SCHEDULE FROM GA POPULATION
*----------------------------------------------------------
ga_irri(awc,cu,crop,v,t,p)        ga generated irrigation schedule (mm)
iga_irri(awc,cu,crop,v,t,p)       ga generated irrigation schedule - initial population (mm)
prob_0                            probability that irrigation event will be zero irrigation (fraction)

*PARAMETERS FOR OUTPUT GENERATION
*---------------------------------
iniZ(p)                           fitness of initial population
iterZ(iter,p)                     fitness of each iteration
iDecode(iter,awc,cu,crop,v,t,p)   updated decode of each iteration
iterPopulation(iter,awc,cu,crop,v,t,p,g) updated binary population of genes (01) for each iteration
ispopulation(iter,awc,cu,crop,v,t,p,g)  ranked (based on Z) binary population of genes (01)
isdecode(iter,awc,cu,crop,v,t,p)   updated uniform (01) values decoded from ranked binary population of genes for each iteration
iz(iter,p)                         fitness value of population for each iteration
irank(iter,p)                      rank of population for each iteration
isZ(iter,p)                        sorted fitness value of population for each iteration

*PARAMETERS FOR WATER BUDGET SIMULATION
*--------------------------------------
IRRI_sim(awc,cu,crop,dog,t,p)      irrigation quantities used in the simulation (mm)
RWC_sim(r,wb,awc,cu,crop,dog,t)    simulated root water content (mm)
BRWC_sim(r,wb,awc,cu,crop,dog,t)   simulated below root water content (mm)
BR_sim(r,wb,awc,cu,crop,dog,t)     simulated percolation below the root zone (mm)
TR_sim(r,wb,awc,cu,crop,dog,t)     contribution of water due to root growth (mm)
ETa_sim(r,wb,awc,cu,crop,dog,t)    simulated actual evapotranspiration (mm)

PARAMETERS TO CALCULATE CROP YIELD
*---------------------------------
YIELD_sim(r,wb,awc,cu,crop,t,p)    simulated crop yield (ton per ha);
```

```
*==============================================================================================================================
* INITIALISE PARAMETERS TO CONTROL OPTIMISATION AND CALCULATE TIME ELAPSED
*==============================================================================================================================
prob_0   = 0.5;       // probability of zero irrigation
mutate   = 0.1;       // mutation rate
cross    = 0.2;       // cross over rate
tau_max  = 60*60*24;  // maximum time in seconds
iter_max = 100000;    // maximum iterations

t1 = jnow;       // current time1
t2=Jnow;         // curent time2
t2=t2-t1;        // time difference
t2=t2*24*3600;   // seconds elapsed

Option limcol=0, limrow=0, solprint=off, optcr=0;

*==============================================================================================================================
*INITIALISE BINARY POPULATION OF CANDIDATE SOLUTIONS AND DETERMINE FITNESS
*==============================================================================================================================
*        INITIALISE INITIAL POPULATION OF IRRIGATION SCHEDULES
*------------------------------------------------------------------------------------------------------------------------------
*Generate genes of population
*----------------------------

*A)  Randomly generated
*----------------------
population(awc,cu,crop,v,t,p,g)$vv(awc,cu,crop,v) = round(uniform(0,1));

*B)  read from input file
*------------------------
*$GDXin iterpopulation1
*$load iterpopulation
*$GDXin
*assign values to population
*population(awc,cu,crop,v,t,p,g)=iterpopulation("iter9999",awc,cu,crop,v,t,p,g);

*Encode the binary string
*------------------------
bin_exponent(g)=2**(ord(g)-1);

*Decode to a value between 0 and 1
*---------------------------------
```

```
Decode(awc,cu,crop,v,t,p)$vv(awc,cu,crop,v)
                        =sum(g,population(awc,cu,crop,v,t,p,g)*bin_exponent(g))/(power(2,card(g))-1);

*generate irrigation schedule for each child
*--------------------------------------------
ga_irri(awc,cu,crop,v,t,p)$(vv(awc,cu,crop,v))
                         =(min_ir_day+((decode(awc,cu,crop,v,t,p)-prob_0)/(1-prob_0))
                          *(max_ir_day*length_ir -min_ir_day))
                          $(decode(awc,cu,crop,v,t,p)> prob_0);
                          // under condition that there is a chance for zero irrigation;

*-------------------------------------------------------------------------------------------------------------------------------
*        CALCULATE FITNESS OF INITIAL POPULATION
*-------------------------------------------------------------------------------------------------------------------------------
*loop over population
*--------------------
loop(p,
   //for each GA irrigation schedule, link to water application in water budget
      //---------------------------------------------------------------------
       loop(v$sum((awc,cu,crop),vv(awc,cu,crop,v)),
              IRRI_sim(awc,cu,crop,dog,t,p)$(ord(v)=crop_ir_day_cum(crop,dog))= ga_irri(awc,cu,crop,v,t,p);
            );
      //include water budget simulation model
      //-----------------------------------
$include Lock_WB_SIMULATE_GA_multi.gms

      //populate parameters with simulated yields and water applications for optimisation model
      //-------------------------------------------------------------------------------------
      ga_yield(r,wb,awc,cu,crop,t)= YIELD_sim(r,wb,awc,cu,crop,t,p);
      ga_aw(awc,cu,crop,dog,t) = IRRI_sim(awc,cu,crop,dog,t,p);

      //calculate optimisation model specific input parameters
      //-----------------------------------------------------
$include Calc_DLP_from_WB_SIM.gms

     //solve the linear short-run model water optimisation model
     //-------------------------------------------------------
     solve SR_POPULATION2 using MIP maximizing gm;

     //populate parameter with fitness value
      z(p)=gm.l;
     // end of population loop
```

```
*============================================================================================================================
* EVOLVE THE POPULATION - START OF MAIN GA LOOP
*============================================================================================================================
*start ga loop
*-------------
loop(iter$((t2 le tau_max) and (ord(iter) le iter_max)),

*initialise parameters to zero
*-----------------------------
ga_yield(r,wb,awc,cu,crop,t)= 0;
ga_aw(awc,cu,crop,dog,t) = 0;

*----------------------------------------------------------------------------------------------------------------------------
*        RANK CANDIDATE SOLUTIONS INCLUDING GENES
*----------------------------------------------------------------------------------------------------------------------------

*determine rank of the population based on fitness values
*--------------------------------------------------------
rank(p)=0;
rank(p)=sum(p1$(Z(p1) ge Z(p)), 1);

*sort population based on fitness
*--------------------------------
sZ(p)=0;
loop((p),
       loop(p1$(ord(p)=rank(p1)),
               sZ(p) = Z(p1)
            )

*extract genes based on sorted rank of population
*------------------------------------------------
spopulation(awc,cu,crop,v,t,p,g)$vv(awc,cu,crop,v)=0;  //initialise population to 0
loop(p,
       loop(p1$(ord(p)=rank(p1)),
               spopulation(awc,cu,crop,v,t,p,g)$vv(awc,cu,crop,v) = population(awc,cu,crop,v,t,p1,g)
            )
    );

*Decode ranked population to a value between 0 and 1
*---------------------------------------------------
sdecode(awc,cu,crop,v,t,p)$vv(awc,cu,crop,v)
                        =sum(g,spopulation(awc,cu,crop,v,t,p,g)*bin_exponent(g))/(power(2,card(g))-1);
```

```
*----------------------------------------------------------------------------------------------------------------------------
*        TOURNAMENT SELECTION: SELECT PARENTS TO PRODUCE OFFSPRING
*----------------------------------------------------------------------------------------------------------------------------

*select contestants in tournament
*--------------------------------
tour_sel(awc,cu,crop,v,t,par,con)$vv(awc,cu,crop,v)=0;
tour_sel(awc,cu,crop,v,t,par,con)$vv(awc,cu,crop,v)=uniformInt(1,card(p)); // all values
*display tour_sel;

*extract fitness value for contestants in tournament
*---------------------------------------------------
tour_selZ(awc,cu,crop,v,t,par,p)$vv(awc,cu,crop,v)=0;
loop((awc,cu,crop,v,t,par,p)$vv(awc,cu,crop,v),
                           loop(con$(ord(p)=tour_sel(awc,cu,crop,v,t,par,con)),
                                    tour_selZ(awc,cu,crop,v,t,par,p)= z(p)
                                );
    );

*rank contestants in tournament based on fitness
*-----------------------------------------------
rank_of_con1(awc,cu,crop,v,t,par,con,p)$vv(awc,cu,crop,v)=0;
rank_of_con1(awc,cu,crop,v,t,par,con,p)$(vv(awc,cu,crop,v) and ord(p)=tour_sel(awc,cu,crop,v,t,par,con))
                                      = rank(p);

rank_of_con(awc,cu,crop,v,t,par,p)$vv(awc,cu,crop,v)=0;
rank_of_con(awc,cu,crop,v,t,par,p)$(vv(awc,cu,crop,v) and sum(con$rank_of_con1(awc,cu,crop,v,t,par,con,p),1))
= sum(con,rank_of_con1(awc,cu,crop,v,t,par,con,p))/ sum(con$rank_of_con1(awc,cu,crop,v,t,par,con,p),1)  ;

*determine winners of the tournament
*-----------------------------------
rank_of_win(awc,cu,crop,v,t,par)$vv(awc,cu,crop,v)=0;
rank_of_win(awc,cu,crop,v,t,par)$vv(awc,cu,crop,v)
                        =smin(p$rank_of_con(awc,cu,crop,v,t,par,p),rank_of_con(awc,cu,crop,v,t,par,p));
ord_of_win(awc,cu,crop,v,t,par,p)$vv(awc,cu,crop,v)=0;
ord_of_win(awc,cu,crop,v,t,par,p)$(vv(awc,cu,crop,v) and rank_of_win(awc,cu,crop,v,t,par)=rank(p))
                        =rank_of_con(awc,cu,crop,v,t,par,p);
*choose highest ord in case of duplicates
*---------------------------------------
select(awc,cu,crop,v,t,par)$vv(awc,cu,crop,v)=0;
loop(p,
select(awc,cu,crop,v,t,par)$(vv(awc,cu,crop,v) and ord_of_win(awc,cu,crop,v,t,par,p)> 0)=ord(p);
```

```
a(awc,cu,crop,v,t)$vv(awc,cu,crop,v)=select(awc,cu,crop,v,t,"par1");
b(awc,cu,crop,v,t)$vv(awc,cu,crop,v)=select(awc,cu,crop,v,t,"par2");
*display a,b;
ord_of_win1(awc,cu,crop,v,t,par,p)$vv(awc,cu,crop,v)=0;
ord_of_win1(awc,cu,crop,v,t,"par1",p)$(vv(awc,cu,crop,v) and ord(p)=a(awc,cu,crop,v,t))
                                    =ord_of_win(awc,cu,crop,v,t,"par1",p);
ord_of_win1(awc,cu,crop,v,t,"par2",p)$(vv(awc,cu,crop,v) and ord(p)=b(awc,cu,crop,v,t))
                                    =ord_of_win(awc,cu,crop,v,t,"par2",p);

*extract genes of the winners of the tournament
*-----------------------------------------------
tour_pop(awc,cu,crop,v,t,par,p,g)$vv(awc,cu,crop,v)=0;
tour_pop(awc,cu,crop,v,t,par,p,g)$(vv(awc,cu,crop,v) and ord_of_win1(awc,cu,crop,v,t,par,p))
                                = population(awc,cu,crop,v,t,p,g);
tour_pop(awc,cu,crop,v,t,par,p,g)$(vv(awc,cu,crop,v) and tour_pop(awc,cu,crop,v,t,par,p,g))
                                = population(awc,cu,crop,v,t,p,g)/population(awc,cu,crop,v,t,p,g);

*-------------------------------------------------------------------------------------------------------------------------------
*        PRODUCE OFFSPRING THROUGH CROSS OVER AND MUTATION
*-------------------------------------------------------------------------------------------------------------------------------

* delete previous data
*--------------------
child(awc,cu,crop,v,t,par,g)=0;

* if cross over occur
*-------------------
if(uniform(0,1) le cross,

     // If cross but not mutate - all genes cross
     //-----------------------------------------
     child(awc,cu,crop,v,t,"par1",g)$vv(awc,cu,crop,v)
                                   = sum(p,tour_pop(awc,cu,crop,v,t,"par2",p,g));
     child(awc,cu,crop,v,t,"par2",g)$vv(awc,cu,crop,v)
                                   = sum(p,tour_pop(awc,cu,crop,v,t,"par1",p,g));

     //if cross and mutate - change 1 to 0 and 0 to 1
     //---------------------------------------------
     child(awc,cu,crop,v,t,"par1",g)$(vv(awc,cu,crop,v) and uniform(0,1)< mutate)
                                   = 1-sum(p,tour_pop(awc,cu,crop,v,t,"par2",p,g));
     child(awc,cu,crop,v,t,"par2",g)$(vv(awc,cu,crop,v) and uniform(0,1)< mutate)
                                   = 1-sum(p,tour_pop(awc,cu,crop,v,t,"par1",p,g));
```

```
*cross do not occur
*-------------------
else
     //if do not cross or mutate - stay the same
     //-----------------------------------------
     child(awc,cu,crop,v,t,par,g)$vv(awc,cu,crop,v)
                                = sum(p,tour_pop(awc,cu,crop,v,t,par,p,g));
*     display child ;
     //if do not cross but mutate - change 1 to 0 and 0 to 1
     //-----------------------------------------------------
     child(awc,cu,crop,v,t,par,g)$(vv(awc,cu,crop,v) and uniform(0,1)< mutate)
                                = 1-sum(p,tour_pop(awc,cu,crop,v,t,par,p,g));
  );

*-------------------------------------------------------------------------------------------------------------------------------
*        UPDATE POPULATION WITH OFFSPRING: GENES
*-------------------------------------------------------------------------------------------------------------------------------
*randomly generate two integer numbers
*--------------------------------------
replace(par)=0;
rand("par1")=uniformInt(1,ceil(card(p)/2));
rand("par2")=uniformInt(1,ceil(card(p)/2)-1);

*eliminate duplicates by adding 1
*---------------------------------
rand("par2")$(rand("par2") = rand("par1")) =rand("par2")+1;

*Assign to bottom half of population
*-----------------------------------
loop(p,
       replace(par)$(card(p)+1-rank(p)=rand(par))= ord(p) ;
    );

*replace selected individuals with the geneS of the offspring
*------------------------------------------------------------
population(awc,cu,crop,v,t,p,g)$(vv(awc,cu,crop,v) and ord(p)=replace("par1"))
                              =child(awc,cu,crop,v,t,"par1",g) ;
population(awc,cu,crop,v,t,p,g)$(vv(awc,cu,crop,v) and ord(p)=replace("par2"))
                              =child(awc,cu,crop,v,t,"par2",g) ;
*Display child,population;
iterPopulation(iter,awc,cu,crop,v,t,p,g)=population(awc,cu,crop,v,t,p,g);
```

```
*----------------------------------------------------------------------------------------------------------------------------
*        CALCULATE FITNESS OF THE OFFSPRING
*----------------------------------------------------------------------------------------------------------------------------
*Decode population to a value between 0 and 1 for each child
*------------------------------------------------------------

display decode;
*decode(awc,cu,crop,v,t,p)$(vv(awc,cu,crop,v))=0; // initialise decode to zero
decode(awc,cu,crop,v,t,p)$(vv(awc,cu,crop,v) and ord(p)=replace("par1"))
=sum((iter_1,g)$(ord(iter_1)=ord(iter)),iterpopulation(iter_1,awc,cu,crop,v,t,p,g)*bin_exponent(g))/(power(2,card(g))-1);
decode(awc,cu,crop,v,t,p)$(vv(awc,cu,crop,v) and ord(p)=replace("par2"))
=sum((iter_1,g)$(ord(iter_1)=ord(iter)),iterpopulation(iter_1,awc,cu,crop,v,t,p,g)*bin_exponent(g))/(power(2,card(g))-1);

*generate irrigation schedule for each child
*--------------------------------------------
*ga_irri(awc,cu,crop,v,t,p)=0;    // maak seker !!!!
ga_irri(awc,cu,crop,v,t,p)$vv(awc,cu,crop,v)
                         =(min_ir_day+((decode(awc,cu,crop,v,t,p)-prob_0)/(1-prob_0))
                          *(max_ir_day*length_ir -min_ir_day))
                          $(decode(awc,cu,crop,v,t,p)> prob_0);
                          // under condition that there is a chance for zero irrigation;

* track population
*------------------
iDecode(iter,awc,cu,crop,v,t,p)= decode(awc,cu,crop,v,t,p);
*display iDecode;

*loop over offspring and calculate fitness
*-----------------------------------------
loop(p$(ord(p)=replace("par1") or ord(p)=replace("par2")),
       //for each GA irrigation schedule, link to water application in water budget
       //----------------------------------------------------------------------------
       loop(v$sum((awc,cu,crop),vv(awc,cu,crop,v)),
              IRRI_sim(awc,cu,crop,dog,t,p)$(ord(v)=crop_ir_day_cum(crop,dog))= ga_irri(awc,cu,crop,v,t,p);
            );
       //include water budget simulation model
       //-------------------------------------
$include Lock_WB_SIMULATE_GA_multi.gms

      //populate parameters with simulated yields and water applications for optimisation model
      //---------------------------------------------------------------------------------------
```

```
      ga_yield(r,wb,awc,cu,crop,t)= YIELD_sim(r,wb,awc,cu,crop,t,p);
      ga_aw(awc,cu,crop,dog,t) = IRRI_sim(awc,cu,crop,dog,t,p);

      //calculate optimisation model specific input parameters
      //--------------------------------------------------------
$include Calc_DLP_from_WB_SIM.gms

     //solve the linear short-run model water optimisation model
     //----------------------------------------------------------
     solve SR_POPULATION2 using MIP maximizing gm;

     //populate parameter with fitness value
     z(p)=gm.l;
     iterZ(iter,p)=gm.l;

     // end of population loop
     //-----------------------
     );
*----------------------------------------------------------------------------------------------------------------------------------
*  DETERMINE ELAPSE TIME IN SECONDS
*----------------------------------------------------------------------------------------------------------------------------------
   t2 = Jnow ;           //current time
   t2 = t2 - t1 ;        //time difference
   t2 = 24 * 3600 * t2 ; //convert to seconds

*----------------------------------------------------------------------------------------------------------------------------------
*        RANK NEW POPULATION OF CANDIDATE SOLUTIONS INCLUDING GENES
*----------------------------------------------------------------------------------------------------------------------------------

iz(iter,p)=z(p);
iha_pop(iter,pivot,awc,cu,crop,t,p)=ha_pop(pivot,awc,cu,crop,t,p);

*determine rank of the population based on fitness values
*---------------------------------------------------------
irank(iter,p)=0;
irank(iter,p)=sum(p1$(Z(p1) ge Z(p)), 1);

*sort population based on fitness
*--------------------------------
isZ(iter,p)=0;
loop((p),
       loop(p1$(ord(p)=irank(iter,p1)),
```

```
               isZ(iter,p) = iZ(iter,p1) ;
            )
    );

*-----------------------------------------------------------------------------------------------------------------------------
*        WRITE OUTPUT AFTER CERTAIN AMOUNT OF ITERATIONS IN CASE OF SYSTEM FAILURE
*-----------------------------------------------------------------------------------------------------------------------------

loop(iter_1$((ord(iter_1)*25=ord(iter))),
    execute_unload  'isz.gdx'  isz ;
    execute_unload  'idecode.gdx'  idecode ;
    execute_unload  'iterPopulation.gdx'  iterPopulation ;
    execute_unload  'irank.gdx'  irank ;
    );

* End of GA iteration loop – do next iteration
*---------------------------------------------
);
*=============================================================================================================================
```

APPENDIX B:
GAMS CODE:
FAO 56 WATER BUDGET SIMULATION MODEL

```
*=================================================================================================================
*          FAO 56 WATER BUDGET SIMULATION MODEL
*          SOURCE: Allen, R.G., Pereira L.S., Raes D. and Smith M. (1998). 'Crop evapotranspiration. Guidelines for computing
*          crop water requirements'. Irrigation and Drainage Paper No 56. FAO. Rome.
*=================================================================================================================
*          PARAMETER DECLERATION
*          NOTE: Parameters for this module are defined in the GA module
*=================================================================================================================
*          WATER BUDGET SIMULATION MODEL CALCULATIONS
*-----------------------------------------------------------------------------------------------------------------
*__________INITIAL CONDITIONS______________________________________________________________________________________

RWC_sim(r,wb,awc,cu,crop,dog,t)
          $(ord(dog) = plantday(crop))
          = 0.5*rwcap(awc,crop,dog); // i_rwc(awc);

BRWC_sim(r,wb,awc,cu,crop,dog,t)
          $(ord(dog) = plantday(crop))
          = 0.5*(rd_max(crop) - rd(crop,dog))*soil(awc);  // i_brwc(awc);

TR_sim(r,wb,awc,cu,crop,dog,t)
          $(ord(dog) = plantday(crop))
          = 0;
ETa_sim(r,wb,awc,cu,crop,dog,t)
          $(ord(dog) = plantday(crop))
          = min(
                etm(r,crop,dog,t)
                ,
                etm(r,crop,dog,t)*(RWC_sim(r,wb,awc,cu,crop,dog,t)/(tam(awc,crop,dog)-ram(r,awc,crop,dog,t)))
               );

*========== START DAILY CALCULATION LOOP ==========================================================================

loop((awc,cu,crop,dog)
     $((ord(dog) ne plantday(crop))and sum(stage,kcdays(crop,dog,stage))),

*__________CALCULATE: WATER MOVING TO THE ROOT ZONE AS ROOTS ARE GROWING IN STAGE 2 (TR)__________________________

TR_sim(r,wb,awc,cu,crop,dog,t)
          $(ord(dog) ne plantday(crop))
          = ((rd(crop,dog)-rd(crop,dog-1))/(rd_max(crop)-rd(crop,dog-1))*BRWC_sim(r,wb,awc,cu,crop,dog-1,t))
            $((rd_max(crop)- rd(crop,dog-1)) > 0);
```

```
*__________CALCULATE: ROOT WATER CONTENT (RWC) __________________________________________________________________________________

RWC_sim(r,wb,awc,cu,crop,dog,t)
           $(ord(dog) ne plantday(crop))
           = min(
                  rwcap(awc,crop,dog)  // same as tam
                 ,
                  WC_sim(r,wb,awc,cu,crop,dog-1,t)-ETa_sim(r,wb,awc,cu,crop,dog-1,t)+ Weather(r,dog-1,t,"rain")
                  + IRRI_sim(awc,cu,crop,dog-1,t,p)*cycle(dog-1)*cu_scale(wb,cu) + TR_sim(r,wb,awc,cu,crop,dog,t)
                 );

*__________CALCULATE: WATER MOVING BELOW THE ROOT ZONE (BR)______________________________________________________________________

BR_sim(r,wb,awc,cu,crop,dog,t)
           $(ord(dog) ne plantday(crop))
           = max(
                 0
                 ,
                  RWC_sim(r,wb,awc,cu,crop,dog-1,t)-ETa_sim(r,wb,awc,cu,crop,dog-1,t)+ Weather(r,dog-1,t,"rain")
                  + IRRI_sim(awc,cu,crop,dog-1,t,p)*cycle(dog-1)*cu_scale(wb,cu) + TR_sim(r,wb,awc,cu,crop,dog,t)
                  - rwcap(awc,crop,dog)
                 );

*__________CALCULATE: WATER CONTENT BELOW ROOT ZONE (BRWC)_______________________________________________________________________

BRWC_sim(r,wb,awc,cu,crop,dog,t)
          $((ord(dog) ne plantday(crop))and sum(stage,kcdays(crop,dog,stage)))
          = min(
                (rd_max(crop) - rd(crop,dog))*soil(awc)
                ,
                BRWC_sim(r,wb,awc,cu,crop,dog-1,t) + BR_sim(r,wb,awc,cu,crop,dog,t) - TR_sim(r,wb,awc,cu,crop,dog,t)
               );
```

```
*__________CALCULATE: EVAPOTRANSPIRATION (ETa)____________________________________________________________________________

ETa_sim(r,wb,awc,cu,crop,dog,t)
           $((ord(dog) ne plantday(crop))and sum(stage,kcdays(crop,dog,stage)))
           = min(
                 etm(r,crop,dog,t)
                 ,
                 etm(r,crop,dog,t)*(RWC_sim(r,wb,awc,cu,crop,dog,t)/(tam(awc,crop,dog)-ram(r,awc,crop,dog,t)))
                );

     ); // END OF WATER BUDGET SIMULATION - DO NEXT DAY


*=================================================================================================================================
*          YIELD CALCULATION
*---------------------------------------------------------------------------------------------------------------------------------

YIELD_sim(r,wb,awc,cu,crop,t,p)
           =  PROD(stage,
                         (1-ky(crop,stage)*(1-(sum(dog
                                                $(kydays(crop,dog,stage)),
                                                ETa_sim(r,wb,awc,cu,crop,dog,t)
                                                 )
                                                 /
                                                 sum(dog
                                                     $(kydays(crop,dog,stage)),
                                                     etm(r,crop,dog,t)
                                                     )
                                             )
                          )
                                          )
                   )*yield_pot(crop)*yield_index(r,crop,t) ;

*=================================================================================================================================
```

APPENDIX C: CONSOLIDATED STUDENT CAPACITY BUILDING AND KNOWLEDGE DISSEMINATION REPORT

STUDENT CAPACITY BUILDING

Name: V Letseko
Degree: MSc Agric (Agricultural Economics)
Status of study: Completion date: November 2019
Title: An economic analysis of increasing irrigation water productivity.

Name: JJ Bezuidenhout
Degree: MSc Agric (Agricultural Economics)
Status of study: Completion date: November 2019
Registered title: An economic evaluation of alternative methods to allocate water between multiple crops using MIKE Basin Hydro

Name: M Dayimane
Degree: BSc Agric (Agricultural Economics)
Status of study: Graduated 2018
Responsibilities: Mr Dayimane acted as research assistant and was responsible for data gathering and general assistance of the team members. Specifically he got training to use SAPWAT and the short-run optimisation model which will be the base model for his M.Sc.

Name: C Steyn
Degree: BSc Agric (Agricultural Economics)
Status of study: Graduated 2018
Responsibilities: Me Steyn acted as research assistant and was responsible for data gathering and general assistance of the team members. With a strong background in computer programming se also assisted in developing some of the code to conduct Genetic algorithmic calculations.

KNOWLEDGE DISSEMINATION

Published papers

Grové B (2019) Improved water allocation under limited water supplies using integrated soil-moisture balance calculations an nonlinear programming. *Water Resour Manage* (2019) 33: 423-437. https://doi.org/10.1007/s11269-018-2110-6.

Grové B and MC Du Plessis (2019) Optimising intra-seasonal water allocation: Comparison between mixed integer nonlinear programming and differential evolution. Water SA 45(1):48-54. https://doi.org/10.4314/wsa.v45i1.06.

Working papers

Letseko V and B Grové An economic analysis of increasing irrigation water productivity. In preparation for Water.

Bezuidenhout JJ and B Grové Farm-level water allocation and irrigation scheduling optimisation under limited water availability. In preparation for Agricultural Water Management.

Grové B and M Frezghi Modeling the long-run hydro-economic impact of water curtailments using an integrated genetic algorithm dynamic nonlinear programming approach. In preparation for Water Resources Management.

Farmers day presentation

Grové B Ekonomiese waterverbruik. Groeipunt Agronomiese inligtingsdag. Wessels boerdery, Lichtenburg, 12 Februarie 2019.

Symposium presentation

Steyn, C and Grové, B. (2018). A crop water use simulation-optimization approach to allocate limited water supplies at farm level. Paper presented at the 8th South African National Commission on Irrigation and Drainage (SANCID) Symposium. White River, Mpumulanga, 14-15 November, 2018.

APPENDIX D: DATA ARCHIVING

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