

Permeability prediction for water seepage through low porosity granular porous media

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Abstract

Momentum transport characteristic expressions for saturated water seepage through granular porous media are simplified by approximations for low and very low porosity cases. These simplifications are incorporated in momentum transport equations and result in relatively simple expressions for the hydrodynamic permeability of low porosity granular media. The porosity is allowed to differ spatially, but geometrical isotropy is demanded on an average basis. The results are restricted to laminar flow within the pores, but inertial effects due to hydrodynamic flow development within each pore section are accounted for. Results are quantitatively compared to that of the general granular porous medium model and also to experimentally correlated results.

Nomenclature

A_p	cross-sectional pore area
c	as subscript, central value
d	microscopic characteristic length
d_p	mean diameter of solid particles (=d)
d_c	cube side width
F	microscopic shear factor
f	fluid phase
g	magnitude of g
g	gravitational body force per unit mass
K	Darcy hydrodynamic permeability when $Re < Re_c$
p	pressure
P_f	$(p) f$
q	magnitude of q
q	specific discharge, (y)
Re	pore flow Reynolds number, $2pv(d-d) u$
Re_{qd}	RUC Reynolds number, pqd/u
Re	particle Reynolds number, pqd/u
S	surface area
S_{fs}	fluid-solid interface
t	time
V_f	fluid filled "void" volume within RUC
V_s	solid volume within RUC
V_o	total volume of RUC
v	fluid velocity within V_f
v_f	mean pore velocity within V_p $(v) f$
vp	mean pore velocity within pore section
e	porosity (void fraction), V_e/V_o
u	fluid dynamic viscosity
v	normal vector on S_{fs} , pointing into V_f
p	fluid mass density
t	tortuosity, de/d
	volumetric phase average of generic variable θ ,
	$\frac{1}{V} \int_V \theta dV$
$\langle \diamond \rangle_j$	volumetric intrinsic phase average of generic variable
	$\phi, \frac{1}{V} \int_V \phi dV$
	deviation, \dots

Introduction

In recent years a unified approach towards the modelling of saturated flow through porous media has been developed (Bachmat and Bear, 1986; Du Plessis and Masliyah, 1988) and applied with considerable success to various flow phenomena (Du Plessis and Masliyah, 1987; Du Plessis, 1989). Volumetric averaging of the fluid transport equations, together with explicit assumptions regarding the average geometrical properties of the void passages within a porous medium, leads to powerful transport equations capable of resembling flow conditions within a porous medium. Corresponding permeability coefficients were expressed explicitly in terms of microstructure parameters through introduction of simple rectangular representation of the mean characteristics of the microstructure.

Water filtration, percolation, ground-water flow phenomena and numerous other industrial processes such as flow through synthetic membranes frequently concern Newton fluid flow through granular porous media of very low porosity, i.e. also very low hydrodynamic permeability. The momentum transport equation, developed by Du Plessis and Masliyah (1991), is applicable for all porosity values from zero through unity. In case of low porosity granular porous media some of the expressions in their equations can be simplified extensively. This simplification and the resulting equations will be the prime objective of this paper. Such equations may be very beneficial to numerical simulation of porous flows when the entire computational domain is filled with low porosity media. Although confined to low porosities, the porosity may differ spatially.

The mathematical analysis of any isotropic porous medium requires the qualification of 3 independent parameters. In this paper the porosity, the physical dimension of the granules and the structure (granular, in this case, vs. sponge-like) will therefore be assumed known. The results to be obtained may equally well be expressed in terms of other parameters which prove to be measurable, e.g. permeability, pore length, pore area.

Modelling of the granular microstructure

The analysis of this study is based upon a granular porous medium which is rigid, stationary and locally isotropic with respect to average geometrical properties. Both variation in porosity and characteristic microscopic length are assumed to be continuous variable functions of position.

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