A note on a light-temperature dependent model for algal blooms in the Vaal River

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Abstract

A mathematical model describing the behaviour of the total planktonic algal biomass in a river has been developed and tested on existing data from the Vaal River (South Africa). It was shown that qualitative as well as quantitive features of the development of winter algal blooms can be described by taking a few fundamental environmental variables such as light, temperature and suspended solids concentration into account.

Introduction

The Vaal River is one of the principal sources of water for the central part of South Africa. This resource is used for industrial and domestic purposes in different, and often conflicting, ways (Pieterse, 1986). For example, the use of the Vaal River as a pathway for the disposal of industrial and domestic wastes is increasing the concentrations of dissolved phosphorus and nitrogen in the water which leads to the eutrophication of the river and results in the development of algal blooms . Such increases in the algal population density may have many adverse effects on the quality of the water, and results in acute problems for water authorities responsible for the treatment and distribution of potable water to mines, cities and towns. Some problems related to the occurrence of algal growth in rivers are (Walmsley and Butty, 1983; Pieterse, 1986): Increased purification costs of water for potable purposes, tastes and odours produced by algae in water intended for potable purposes, toxins produced by certain algae that can result in losses of livestock and that may also affect humans or that may interfere with irrigation.

Thus, the prediction of the development of winter algal blooms is of importance in water resource management. The mathematical model introduced in this communication is intended to assist the relevant authorities by providing them with a tool which can both lead to a better understanding of aspects of algal population growth, and to enable short- and long-term chlorophyll-a concentration predictions.

The model

The model is based on the following three fundamental assumptions:

- The water is eutrophic, i.e. we assume there are more than enough nutrients for algal growth.
- At most *N* representatives of different algal groups such as e.g. diatoms and green algae, can be responsible for the development of an algal bloom.
- The growth and death of algae are dependent mainly on the available light as well as the average temperature of the water and these driving variables are time dependent.

In addition, we assume that other possible factors influencing algal growth are steady state for the duration of the bloom. Horizontal homogeneity of the water is also assumed in the immediate vicinity of the site at which the model is applied.

Within the boundaries of these assumptions, the scheme illustrated in Fig. 1 may be used as a basis for the algal growth model.

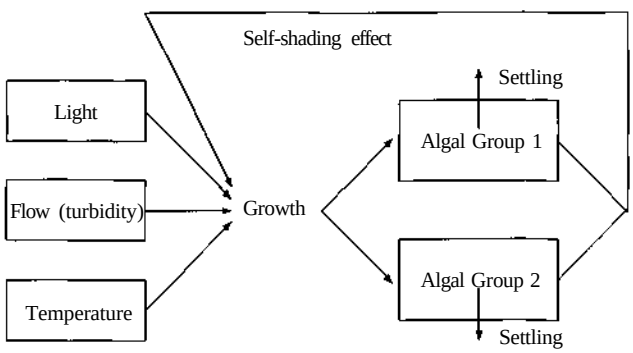

Figure 1
Vaal River N-algae growth model: A schematic representation

The transcription of the scheme in Fig. 1 in terms of mathematical relations leads to a system of 2*N* coupled nonlinear differential equations of the form:

$$\begin{cases} \dot{x}_{1i} = -(k_{Di} + S_i)x_{1i} \\ \qquad\quad k_{Gi}(\underline{E}, x_{1i}, x_{2i}, \underline{K}_j : j = 1, N; t)x_{1i} \\ \dot{x}_{2i} = k_{Di}x_{1i} - S_r x_{2i} \end{cases} \qquad i = 1,N$$

where x_{1i}, x_{2i}', is the concentration of living and dead algae belonging to group "i" respectively, and an upper dot "." stands for the derivative respect to time, *t.* The vectors $\underline{E}$ and $\underline{Kj}$ contain information on environmental parameters and information specific to a given algal species "j", respectively. The components of vectors *E* and *K,* are given in Table 1a and b.

More of the explicit form taken by the differential equations and details on the dynamics of the system of equations can be obtained from the authors.

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