

A tensor approach to the estimation of hydraulic conductivities in Table Mountain Group aquifers of South Africa

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Abstract

Based on the field measurements of the physical properties of fractured rocks, the anisotropic properties of hydraulic conductivity (HC) of the fractured rock aquifer can be assessed and presented using a tensor approach called hydraulic conductivity tensor. Three types of HC values, namely point value, axial value and flow direction one, are derived for their possible applications. The HC values computed from the data measured on the weathered or disturbed zones of rock outcrops tend to give the upper limit values. To simulate realistic variations of the hydraulic property in a fractured rock aquifer, two correction coefficients, i.e. the fracture roughness and combined stress conditions, are adapted to calibrate the tensor model application. The application results in the Table Mountain Group (TMG) aquifers show that the relationship between the HC value and fracture burial depths follows an exponential form with the power hyperbola.

Keywords: hydraulic conductivity tensor, roughness, combined stress, hydraulic aperture, Table Mountain Group (TMG)

Introduction

Darcy's law is always used to estimate the groundwater flow in both porous and fractured media, depending upon realistic estimates of aquifer hydraulic conductivities (viz. k) and hydraulic gradients (viz. J) at the scale of problem. In the case of fractured rock aquifers, presentation and determination of the hydraulic conductivity prove to be challenges. With respect to a fracture set with a mean aperture of b and a parallel face distance of d , the following classic expression is adopted for flow through the set of conduits (Talobre, 1957; Jaeger, 1972):

$$q = \frac{gb^3}{12\mu d} \cdot J \quad (1)$$

where:

ρ is the density of fluid

μ is dynamic viscosity of fluid, which is 10^{-6} m²/s for water at 20°C

g the acceleration of gravity

J hydraulic gradient

Eq. (1) represents an idealised type of flow behaviour that has been intensively studied, both experimentally and numerically, by many researchers. The term $gb^3/12\mu d$ in Eq. (1) is usually referred to as the hydraulic conductivity (HC) K for the set of fractures involved. For the determination of K value, many theories and methods have been developed. A series of results for one of the intrinsic properties of fractured rock aquifers have been obtained to various extents for more than 30 years. As a summary, there are thus far three approaches to the estimation of hydraulic conductivity of fractured rock aquifers, namely:

- HC tensor approach based on statistic or stochastic methods of *in situ* fracture geometry and physical measurements
- Fracture property field and laboratory tests for the parameter K evaluation

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Received 24 November 2005; accepted in revised form 11 April 2006.

- Inverse analysis on continuous or discontinuous problems dependent on numerical models and parametric calibrations.

The estimation of K values using either pumping or packer test is based on the assumption that the groundwater is flowing through a geological continuum. It is often an expensive exercise to estimate and predict the regional aquifer properties (viz. K and J , etc.) from local-scale hydraulic tests. Also the large variation of HC , both along borehole sections and in between holes, usually makes it difficult to determine the representivity of the parameters in terms of groundwater assessments. Even where a representative elementary volume (REV) can be defined, it may not be appropriate to directly apply the local test results to a regional aquifer. In porous media the REV can be very small, whereas in fractured media the REV may be very large or even does not exist in some cases (Kulatilake and Panda, 2000; Wang et al., 2002).

The statistic methods for calculating HC tensor were developed in 1980s (Hsieh and Neuman, 1985; Hsieh et al., 1985; Oda, 1985; Tian, 1988). The results from these methods can successfully indicate 3-D principle HC values and directions by means of coordinate rotation of the incorporation of input data that derived from the surface measurements. The basic assumptions of the tensor approach are:

- Groundwater flow is exclusively governed by fractures
- The fractures through a rock matrix are well-connected
- Flows between fracture sets do not interfere, or no deflection flow occurs.

For the ideal flow pattern with M sets of fractures involved in a study area, the hydraulic conductivity of the fracture sets is expressed in the form of matrix which reflects a sort of flow superposition:

$$\begin{cases} \bar{K} = \sum_{i=1}^M \frac{gb_i^3}{12\mu d_i} \cdot K_{jl}^i, & j, l = x, y, z \\ K_{jl}^i = [I - \bar{n}^T \cdot \bar{n}] \end{cases} \quad (2)$$