Shortcommunication

Improvement on prediction for water seepage through low porosity granular media

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Abstract

A new set of equations is presented for the prediction of water seepage through granular porous media of low porosity. The results are based on the analytical modelling of form drag within a porous medium and improve on previous results which incorporate interstitial flow development within pore sections. The new formulation correlates extremely well with experimental observation as is evident from its very good correlation with the empirically based Ergun equation over the entire laminar Reynolds number range. It therefore presents a considerable improvement on results reported recently in this journal according to which the higher Reynolds number friction was severely underpredicted.

Nomenclature

d	microscopic characteristic length
d_s	cube side length
6	diameter of spherical particles (grain diameter)
F	microscopic shear factor
8	gravity force per unit mass
K	Darcy hydrodynamic permeability, e/F when $Re_{qD} \ll Re_c$,
p_f	fluid averaged pressure
	magnitude of q
q	specific discharge
Re_c	critical Reynolds number, Re_{qD} at critical point
Re_{qd}	Reynolds number, *pqd/\x*
Re_{qD}	particle Reynolds number, *pqD/u*
t	time
e	porosity (void fraction)
	fluid dynamic viscosity
P	fluid mass density

Introduction

In a recent publication (Du Plessis and Roos, 1993) the following equation was derived for the flow through granular porous media:

$$\rho\frac{\partial \boldsymbol{q}}{\partial t} + \rho \boldsymbol{q}\cdot\nabla(\boldsymbol{q}/\varepsilon) + \varepsilon\nabla p_f - \varepsilon\rho \boldsymbol{g} - \mu\nabla^2 \boldsymbol{q} + \mu F \boldsymbol{q} = \boldsymbol{0} \qquad (1)$$

In this equation all variables are known macroscopically (i.e. on average) except for the factor *F,* into which is embedded the specifics of the fluid-solid interaction. The particular form of *F* depends on the porous medium type and was shown by Du Plessis and Masliyah (1991), to be a function of the microstructure *(E,d),* the fluid properties (p,|0.) and the specific discharge *(q).*

In case of granular media, such as sand or sandstone, the granules in the porous medium are modelled by solid cubes of side lengths d_s, randomly packed into a bed with an average separation of *d* between centres of mass of neighbouring particles.

The modelling of intrapore flow phenomena over a wide range of Reynolds numbers is effected by the matching of two asymptotic solutions. In the lower extreme of very slow movement, i.e. for intrapore Reynolds numbers less than unity, fully developed flow is assumed in all pore sections, yielding an analogy to the well-known Blake-Carman-Kozeny equation. This corresponds closely with the results presented previously by Du Plessis and Roos (1993). In the present work their requirement for different velocities in transverse pores is relaxed to equal velocities in all pore sections, leading to a slight change in the coefficient from 41 to 36.

At the other extreme of relatively high intrapore velocities, i.e. for Reynolds numbers much larger than 100, the predominant effect on pressure loss is now assumed to be local recirculation within each pore section (Du Plessis, 1992). This assumption may be considered as internal form drag and succeeds in the quantitative prediction of the Forchheimer equation (Bird et al., 1960). This is in contrast to the former modelling according to which inertial effects were brought about by intrapore flow development and which underpredicted the drag at higher Reynolds number flows. Simple addition of the asymptotic expressions thus yields the following improvement on the analogous Eq.(13) of Du Plessis and Roos, 1993:

$$Fd^2 = \frac{36\,(1-\varepsilon)^{2/3}}{\left[1-(1-\varepsilon)^{1/3}\right]\left[1-(1-\varepsilon)^{2/3}\right]} + \frac{(1-\varepsilon)^{2/3}}{\left[1-(1-\varepsilon)^{2/3}\right]^2}\,Re_{qd} \qquad (2)$$

Since the characteristic length *d* is not easily measurable it is more appropriate to recast the equation into a form with the microscopic length variable more closely related to the grain diameter *D.* Assumption of equal specific surfaces (surface area per unit solid mass) for the granules and the corresponding model cubes (Bird et al., 1960), leads to direct equivalence between d_s and *D.* Eq. (2) may thus be put into the following form:

$$FD^2 = \frac{36\,(1-\varepsilon)^{4/3}}{\left[1-(1-\varepsilon)^{1/3}\right]\left[1-(1-\varepsilon)^{2/3}\right]} + \frac{(1-\varepsilon)}{\left[1-(1-\varepsilon)^{2/3}\right]^2}\,Re_{qD} \qquad (3)$$

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