

# The hydraulic transportation of thickened sludges

PT Slatter

Department of Civil Engineering, Cape Technikon, PO Box 652, Cape Town, South Africa

## Abstract

Industries which pump sludges are under continuous pressure to decrease water content, and increase concentration. Environmentally superior disposal techniques are demanding that such sludges have high mechanical strength properties. This results in a sludge with an increasing viscous character. At high concentration, the viscous forces – which are usually highly non-Newtonian and yield stress in nature – become dominant, and flows inevitably become laminar.

The objective of this paper is to demonstrate the effect and evaluate the impact that increasing non-Newtonian viscous stresses – particularly yield stress – have on the pipelining problem.

An industrially relevant sludge pipe flow study is presented, demonstrating and quantifying the relationship between sludge rheology and flow regime. It is argued that laminar flow will result in settleable solids accumulating on the pipe invert, leading to pipe blockage. Although some practical remedies have been proposed, this problem requires urgent and focussed research.

**Keywords:** rheology, pipe flow, blockage, laminar, transition, turbulent, yield stress

## Notation

Symbol Unit	Description	
D	internal pipe diameter	m
f	Fanning friction factor	
F	constitutive rheological relation function	Pa
He	Hedström number	
K	fluid consistency index, plastic viscosity	Pa.s
r	radius at a point in the pipe	m
R	internal radius of the pipe	m
Re	Reynolds number	
u	point velocity	m/s
V	average velocity	m/s
$\rho$	fluid density	kg/m <sup>3</sup>
$\tau$	shear stress	Pa
$\tau_y$	yield stress	Pa

## Subscripts

0	at the pipe wall
c	critical (at the laminar/turbulent transition)

## Introduction

Industries which pump sludges are under continuous pressure to decrease water content, and increase concentration. Environmentally superior disposal techniques are demanding that such sludges have high strength mechanical properties. This results in a sludge with an increasing viscous character. At high concentration, the viscous forces – which are usually highly non-Newtonian and yield stress in nature – become dominant, and flows inevitably become laminar (Slatter, 2002).

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This paper was originally presented at the 2004 Water Institute of South Africa (WISA) Biennial Conference, Cape Town, South Africa, 2-6 May 2004.  
☎+2721 460-3055; fax: +2721 460-3710 ; e-mail: [pslatter@ctech.ac.za](mailto:pslatter@ctech.ac.za)

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## Theory and literature

### Rheological characterisation

The rheological characterisation of non-Newtonian sludges has received much attention in the literature and the development of this discipline is ongoing. Although more complex rheological models are available, this paper has deliberately targeted a more pragmatic approach, and the simplest model which can accommodate a yield stress has been used. To this end, the Bingham plastic rheological model (Grovier and Aziz, 1972) has been found useful by many researchers to approximate the viscous flow behaviour of non-Newtonian sludges (Xu et al., 1993; Spinosa and Lotita, 2001; Slatter, 2001). The constitutive equation for pipe flow is given by:

$$\tau = \tau_y + K \left[ -\frac{du}{dr} \right] \quad [1]$$

### Laminar flow

For laminar pipe flow, the general constitutive rheological relationship F can be cast in the form

$$\tau = F \left( -\frac{du}{dr} \right) \text{ and } -\frac{du}{dr} = F^{-1}(\tau). \quad [2]$$

Also, for pipe flow, the shear stress  $\tau(r)$  varies linearly over the pipe cross-section

$$\tau(r) = \frac{\tau_0}{R} r. \quad [3]$$

The velocity profile  $u(r)$  is obtained by integration of the constitutive rheological relationship, i.e.: