

# A review of mathematical programming models of irrigation water values

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## Abstract

By introducing the user-pays principle into the irrigation water pricing debate, the 1998 National Water Act created a demand for models to measure willingness-to-pay for irrigation water. Water values are traditionally simulated with mathematical programming models. Models differ in their treatment of crops, irrigation options and water constraints, and other firm-level characteristics but they all use shadow prices as an indication of water value. The 17 models reviewed here, report average annual water values of between \$0.0042·m<sup>-3</sup> and \$0.1899·m<sup>-3</sup>. Crops modelled influence water values, but there is no apparent relationship between objective function specification and average value. Nor does the number of irrigation options seem to influence water value either. The policy implication is that while similar models for the same region produce consistent estimates, each region requires its own model that has to be updated regularly.

**Keywords:** Q15, Q12, water value, irrigation, mathematical programming

## Introduction

By raising the issue of full cost-recovery, the 1998 National Water Act made it clear that up to that point irrigation water was considerably subsidised in South Africa, and that price increases would be necessary to achieve optimal allocation. The Department of Water Affairs and Forestry (DWAF) responded immediately by doubling water rates in many cases, but it is unclear whether full cost-recovery would be feasible in the short term. While clear consensus exists about the conditions for allocative efficiency, as outlined by Sampath (1992), the international debate on the feasibility of full cost-recovery is far from over. The conclusion of this debate is an empirical issue; when the net present value of the total benefit of irrigation water is smaller than the net present value of total costs, full cost-recovery is impossible. When it is greater, full cost-recovery is feasible. One thus needs an accurate estimate of the demand for irrigation water to resolve the empirical issue. This creates a demand for models to measure willingness-to-pay for irrigation water. While isolated alternatives exist, mathematical programming is traditionally the tool of choice to simulate irrigation water values (Gibbons, 1986). This review discusses the theoretical framework on which this class of model relies and then describes 17 models reported over the past 20 years before concluding on the potential usefulness of mathematical programming models to resolve the empirical cost-recovery debate.

## Theoretical framework

Ricardian rents provide the theoretical framework for residual imputation with mathematical programming models. Under per-

fect competition, when the firm is a price-taker in factor and product markets, payments to all variable factors of production are exactly equal to total revenue, making economic profits zero. Here the firm is a farm. The theory is simple. Economic profits, if any exist, are returns to fixed factors of production, potentially including irrigation water.

By this logic static and dynamic linear and non-linear programming models have been used not only to simulate total benefits of irrigation water, but also to derive demand functions for it. Models are set up to maximise profit subject to, amongst others, water constraints, which are then parametrically tightened to derive a schedule of shadow prices. A linear programming shadow price is the marginal contribution to the objective function (profit) attributable to an additional unit of the binding constraint (Beneke and Winterboer, 1973). The shadow price of water directly estimates the marginal value product of water; the demand for the resource is simply the quantity constraint as a function of the shadow price and total value is the integral of this function. To be an accurate estimate of water value, water must be the only fixed factor in the model and the input-output coefficients must be correct.

The vast majority of irrigation water value models use residual imputation. Even Howe (1985), who based his demand curve for water on the gross margin of individual crops, uses the idea that residual profits indicate the value of water. There are three alternatives to residual imputation. The first estimates a crop-water production function from field trials and then scales this physical production function by the price of the product (Colby, 1989; Penzhorn and Marais, 1998). The second approach is to estimate a demand function directly from water price data. Griffin and Perry (1985) presented an econometric model using panel data of irrigation prices in Texas. The third approach is to use Hedonic pricing methods to measure the contribution of water value to farm prices. Torell et al. (1990) estimated water in the Ogallala Aquifer to be worth between \$0.0009·m<sup>-3</sup>·a<sup>-1</sup> and \$0.0077·m<sup>-3</sup>·a<sup>-1</sup>. Faux and Perry (1999) estimated the water value in Malheur County, Oregon, to be between \$0.0073·m<sup>-3</sup>·a<sup>-1</sup> and \$0.0357·m<sup>-3</sup>·a<sup>-1</sup>.

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