An efficient optimisation method in groundwater resource management

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Abstract

Uncertainty in input parameters to groundwater flow problems has been recognised as an impediment to designing efficient groundwater management strategies. The most popular approach to tackling this problem has been through the Monte Carlo approach. However, this approach is generally too expensive in terms of computer time because of the number of scenarios required to ensure reliable statistics. Furthermore, solutions obtained through this approach are not necessarily robust. In this paper, it is shown how groundwater management problems, where input parameters are uncertain can be reformulated as second-order cone optimisation (SOCO) problems, which are efficiently solved by recently developed interior-point methods. Results for a real-world case application of a groundwater aquifer found in Kenya are presented.

Keywords: Groundwater management, uncertainty, second-order cone, Laikipia, Kenya

Introduction

Theories of groundwater flow are well developed and over the last three decades, several applications have been made to model groundwater flow systems. A large number of these works have been based on deterministic flow simulation, and over the last decade a considerable volume of literature has been written and theories developed in the field of stochastic subsurface hydrology. In reality, many real-world aquifers are characterised by a few measurement points, which are used to derive the aquifer characteristics. Traditionally, the few measurement points available have been used for zonation purposes after which the management problem is solved deterministically. However, recognising the fact that the material forming aquifers varies enormously spatially, it is not immediately clear how optimal management strategies designed deterministically perform in an environment of uncertainty. Having noted that the parameters of the earth material which dictate the water-flow conditions vary a great deal spatially, one may ask the questions: Does it make sense to model groundwater systems deterministically using the sparse data available? How do solutions based on such an approach perform in a real-world scenario?

Geohydrologists, have for some time now, used the Monte Carlo approach in an attempt to desensitise the optimal solutions, thus including some robustness within the optimisation problem. However, the Monte Carlo approach is generally CPU-intensive (because of the large number of scenarios which have to be considered to arrive at a relatively insensitive optimal solution), hence its main drawback. Recently, some researchers have applied the multi-stage optimisation approach (in particular a two-stage optimisation approach) with some promising results (Ndambuki et al., 2000a; Wagner et al., 1992; Mulvey et al., 1995; Mark et al., 1999).

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In this paper, we transform our stochastic groundwater quantity management problem into a second-order cone optimisation problem, which is then solved by some powerful interior-point method. We first present a general introduction to second-order cone optimisation (SOCO) problems followed by the formulation of a second-order cone groundwater quantity management problem. Subsequently, we present results for a real world case aquifer found in Laikipia District, Kenya.

Consider a linear optimisation problem (LOP) of the following form:

$$\text{minimize } c^T x \tag{1}$$

subject to:

$$a_i^T x \le b_i,\ i = 1,\ldots,m \tag{2}$$

$$x \ge 0 \tag{3}$$

where:

c, $a_i \in R^n$; $b_i \in R$ are the problem parameters; while x are the optimisation variables.

Assuming that all the problem parameters except a_i are accurately known and that a_i is uncertain but lying in ellipsoids e_i defined as:

$$a_i \in \varepsilon_i = \left\{ \overline{a_i} + \mathbf{P}_i u_i \;\middle|\; \|u_i\| \le 1 \right\} \tag{4}$$

where:

$\mathbf{P} = \mathbf{P}^T$ are n x n perturbation matrices;
a_i overstrike are the nominal values and the norm of u_i ensure convexity.

Then a robust solution of the optimisation problem given by Eqs. (1) to (3) is as follows:

$$\text{minimize } c^T x \tag{5}$$

subject to:

$$a_i^T x \le b_i,\ \forall a_i \in \varepsilon_i,\ i = 1,\ldots,m \tag{6}$$

$$x \ge 0 \tag{7}$$