A time-dependent Green's function-based model for stream-unconfined aquifer flows

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Abstract

A numerical formulation that is based on the Green element method (GEM), which incorporates a time-dependent Green's function, is used to solve transient two-dimensional flows of stream-unconfined aquifer interaction. The Green's function comes from the fundamental solution to the linear diffusion differential operator in two spatial dimensions. In classical boundary element applications, this Green's function has found use primarily in linear heat transfer and flow problems; its use here for the nonlinear stream-unconfined aquifer flow problem represents the computational flexibility that is achieved with a Green element sense of implementing the singular integral theory. The nonlinear discretised element equations obtained from numerical calculations are linearised by the Picard and Newton-Raphson methods, while the global coefficient matrix, which is banded and sparse, is readily amenable to matrix solution routines. Using four numerical examples, the accuracy of the current formulation is assessed as against an earlier one that incorporates the Logarithmic fundamental solution. It is observed that comparable accuracy is achieved between both formulations, indicating that the current formulation is a viable numerical solution strategy for the stream-aquifer flow problem.

Keywords: stream-aquifer interaction; Green element method; linearisation algorithms; numerical solutions

Introduction

Stream-aquifer interaction flows have gained considerable interest among hydrologists for quite a while because of their applications in hydrograph analysis. Of interest is estimating the baseflow from the recession hydrograph of a stream or river that could be the only water source on which surrounding communities depend for agricultural and domestic uses (Butterworth et al. 1999). This is particularly important in arid and semi-arid environments where rainfall is low and erratic, and droughts are common. Modelling of flow when there is interaction between an unconfined aquifer and a stream has generally followed two approaches. The first is based on the mathematical description of the flow with the nonlinear free-surface (water table) boundary condition being explicitly applied (Dillon and Liggett, 1983; Cabrera and Marino, 1976a; Ibrahim and Brutsaert, 1965). In this approach a vertical slice of the aquifer is considered, and the seepage face that exist at the aquifer-stream interface is captured. The drawback to this approach is that the regional distribution of flow in the aquifer and the variability of baseflow along the reach of the stream are not accounted for. This approach is suitable in instances when it can be justified that flow variability along the streamwise direction is negligibly small. The second approach is based on depth-averaging of the governing flow differential equation so that the free-surface condition is implicitly imbedded in the resulting differential equation, and the seepage face is assumed to be non-existent (Hornberger et al., 1970). By this approach, the flow is now essentially horizontal, the vertical velocity component of flow is assumed to be negligible, and the hydraulic head is assumed to be depth-invariant. These assumptions are widely known in the groundwater literature as the Dupuit-Forchheimmer assumptions and the approach is the hydraulic one. Because the ratio of the depth to lateral dimensions of most aquifers is extremely small, this assumption is valid for most parts of the aquifer but breaks down within the vicinity where flow occurs in and out of the aquifer. One advantage of this approach is that the regional distribution of base flow can be accounted for. The alternative to either of these approaches would be to adopt the hydrodynamic viewpoint whereby the governing differential equation is solved in three dimensions with the non-linear free-surface explicitly imposed. Apart from the huge computational requirements of this approach, data on hydrogeological parameters of most aquifers are not gathered to such details as to make them available for use in this model. In the best of circumstances, hydrogeological parameters from field investigations are obtained in a depth-averaged manner. It is for these reasons that the hydraulic approach is followed in this paper.

Solution strategies to stream-unconfined aquifer flows have also taken a number of directions. Experimental investigations that are designed on the Hele-Shaw apparatus have commonly been used (Ibrahim and Brutsaert, 1965; Rochester and Kriz, 1968). Though quite limited in terms of versatility of application, experimental models have provided useful insight into the phenomenon of stream-aquifer interaction flows and have served to validate other solution strategies.

The other solution strategies are analytical and numerical. To date, there is no analytic solution to the vertical slice approach where the non-linear free-surface condition is explicitly prescribed. The only widely reported exact solution to the hydraulic approximation of flow is that proposed by Boussinesq (1904) who used similarity considerations to derive the solution of flow from an aquifer into a ditch. The other analytical solutions reported in the literature were achieved after some form of simplification or linearisation was applied to the non-linear differential equation (Desai, 1973; Lockington, 1997). All these solutions apply to one spatial flow direction, with rather idealised boundary and/or initial

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