

A generalised solution for step-drawdown tests including flow dimension and elasticity

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Abstract

Step-drawdown and multi-rate tests present convenient tools for the estimation of the long-term yield of boreholes. However, the analytical methods commonly employed for the analysis of such tests are all based on the assumption that the drawdown in a borehole is a linear function of the discharge rate. Numerous constant rate tests, of which a few are discussed in this paper, has shown that this is not necessarily the case with boreholes drilled in the Karoo formations of South Africa. The drawdowns in these boreholes are not only influenced by the peculiar geometry of the aquifers, but also the non-linear deformation of the aquifers during the pumping of a borehole. The two new non-linear models for the analysis of step-drawdown and multi-rate tests introduced here, tries to account for these factors; in particular the deformation of the aquifer, flow dimension and dewatering of discrete fractures. Although the model proposed for multi-rate tests is still based on constant time steps, the one for step-drawdown tests allows the user to use arbitrary time steps, when performing the test in the field.

Non-linearities in drawdown curves should always be treated with caution, especially when used to assign sustainable yields for boreholes. However, the example of a step-drawdown test performed at the Campus Test Site of the University of the Free State, shows that non-linearities can be addressed with an appropriate model.

Introduction

Step-drawdown tests were introduced by Jacob (1947) to study the influence that the discharge rate, Q , has on the drawdown, $s(r, t)$, of the water level in a borehole. His conclusion, based on a number of drawdown tests, was that the observed drawdown consists of two components—one linear in Q and the other one non-linear. He also showed that the linear component can be divided into what he called 'the linear aquifer loss coefficient', which he denoted by the symbol $B_1(r_w, t)$, and a 'linear well loss coefficient, B_2 , caused by the loss of energy in the borehole itself. The former of these components can be viewed as the drawdown one would observe if water could be withdrawn from an aquifer without the loss of energy represented by the term B_2 . In other words, $B_1(r_w, t)$ can be interpreted as the theoretical solution of the groundwater flow equation for the actual, physical aquifer. It is, consequently, impossible (at least at this moment) to distinguish between the two linear losses in practice. Jacob, therefore, combined the two terms into the linear loss coefficient, defined by the equation:

$$B(r_e, t) = B_1(r_w, t) + B_2$$

where r_e is known as the effective radius of a borehole, with physical radius r_w . Jacob defined r_e as the radial distance from the vertical axis of the borehole to a point where the water level in the aquifer equals the water level in the borehole. This interpretation led him to describe the observed drawdown in a pumped borehole, s_w , with the equation:

$$s_w = B(r_e, t)Q + CQ^2 \quad (1)$$

where the term CQ^2 , represents the non-linear losses.

The main effect of the non-linear losses is to drive the water level in the borehole down, without contributing to Q . This could not only affect the operational costs of a borehole adversely, but could also cause irreparable damage to the borehole, pump and even the aquifer. It is very important that one should never operate a borehole in such a way that the non-linear energy losses become dominant. However, it may sometimes be necessary to sacrifice energy for the borehole to perform optimally. Since this was the main motivation for Jacob to introduce step-drawdown tests, it is not surprising to find that Eq. (1) can be very useful in this regard.

It is common practice to assume that the coefficient C in Eq. (1) is constant and attribute the existence of the term CQ^2 to turbulent flow, caused by the pump in and near the borehole (Helweg, 1994). However, there are indications that the drawdown is not only a function of Q , but also the geometry of the aquifer and that this may contribute to the non-linear term in Eq. (1) and cause the parameters B_2 and C to be time-dependent. Helweg suggests that Eq. (1) be replaced by the equation:

$$s = [A + B' \log(t)]Q + [C' \log(t)]Q^2 \quad (2)$$

which he claims is more general than Eq. (1). This is certainly true in the sense that Eq. (2) allows the coefficients to be time-dependent. However, to achieve this he assumed that the theoretical drawdown, $B_1(r_w, t)$, could be represented by the Cooper-Jacob approximation of the Theis solution for an infinite uniform aquifer. Since this assumption is not necessary in Eq. (1), the possibility exists that Eq. (1) may describe the drawdowns of boreholes in heterogeneous aquifers better than Eq. (2), if the time is kept constant.

Another consequence of Helweg's assumption is that the flow towards the borehole must be radial, which need not be the case. This seems to be particularly the case with the shallow aquifers in the geological formations associated with the Karoo Supergroup in South Africa. These formations, which underlie approximately 50% of the country, consist mainly of sandstones, mudstones,

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Received 22 July 1999; accepted in revised form 23 February 2001.