

The entropy theory as a tool for modelling and decision-making in environmental and water resources

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Abstract

Since the development of the entropy theory in the late 1940s and of the principle of maximum entropy (POME) in the late 1950s, there has been a proliferation of applications of the entropy theory in a wide spectrum of areas, including environmental and water resources. The real impetus to entropy-based modelling in water resources was provided in 1970s. A great variety of entropy-based applications in environmental and water resources have since been reported, and new applications continue to unfold. Most of these applications have, however, been in the realm of modelling and a relatively few applications have been reported on decision-making. This paper revisits the entropy theory and discusses its usefulness in environmental and water resources, and is concluded with comments on its implications in developing countries.

Introduction

Environmental and water resource systems are inherently spatial and complex, and our understanding of these systems is less than complete. Many of the systems are either fully stochastic, or part-stochastic and part-deterministic. Their stochastic nature can be attributed to randomness in one or more of the following components that constitute them: system structure (geometry); system dynamics; forcing functions (sources and sinks); and initial and boundary conditions. As a result, a stochastic description of these systems is needed, and the entropy theory enables development of such a description.

Engineering decisions concerning environmental and water resource systems are frequently made with less than adequate information. Such decisions may often be based on experience, professional judgment, thumb rules, crude analyses, safety factors, or probabilistic methods. Usually, decision-making under uncertainty tends to be relatively conservative. Quite often, sufficient data are not available to describe the random behavior of such systems. Although probabilistic methods allow for a more explicit and quantitative accounting of uncertainty, their major difficulty occurs due to the availability of limited or incomplete data. Small sample sizes and limited information render estimation of probability distributions of system variables with conventional methods quite difficult. This problem can be alleviated by use of the entropy theory which enables determination of the least-biased probability distributions with limited knowledge and data. Where the shortage of data is widely rampant as is normally the case in developing countries, the entropy theory is particularly appealing. The objective of this paper is to revisit the entropy theory and underscore its usefulness for both modelling and decision-making in environmental and water resources.

Entropy theory

The entropy theory is comprised of three main parts: Shannon entropy, principle of maximum entropy, and principle of minimum

cross entropy. Before discussing these parts, it will be instructive to briefly discuss the meaning of entropy.

Meaning of entropy

The zeroth law of thermodynamics is related to the concept of temperature T , the first law of thermodynamics is related to the concept of internal energy U , and the second law of thermodynamics is related to the thermodynamic variable, called entropy, S , which is defined for a system as:

$$dS = \frac{dQ}{T}, \oint dS = 0 \quad (1)$$

where:

- dS is the change in entropy
- dQ is the change in heat
- T is the temperature.

\oint indicates that the integral is evaluated for a complete traversal of the system response cycle. In Eq. (1), temperature is a state variable.

Heat is disordered energy. Energy can exist without disorder. The general principle is that energy becomes heat as soon as it is disordered. Conversely, disorder can exist without energy, and disorder becomes heat as soon as it is energised. Thus, to specify heat two numbers are needed: one to measure the quantity of heat, and the other to measure the quantity of disorder. The quantity of heat energy is measured in terms of calories and the quantity of disorder is measured in terms of entropy.

If there is a connection between disorder and entropy, then disorder, like entropy, must increase in natural processes. This is indeed the case, i.e. there is a tendency for a natural process to proceed toward a state of greater disorder. To illustrate, consider the confluence of two rivers having different sediment concentrations C_1 and C_2 . Downstream of the confluence, the downstream reach attains an intermediate concentration C . The river system has been more disordered in this natural process because we have lost our ability to classify sediment concentration. The statement that discharge in the river corresponds to concentration C is weaker than the statement that discharge in River A corresponds to sediment concentration C_1 and discharge in River B corresponds to sediment

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