A modified boundary integral solution of recharging and dewatering of an unconfined homogeneous aquifer

Onyejekwe OO[1]*, Karama AB[1] and Kuwornoo DK[2]
[1]*Department of Civil Engineering, University of Durban-Westville, Durban 4000, South Africa*
[2]*Department of Chemical Engineering, University of Durban-Westville, Durban 4000, South Africa*

Abstract

A numerical procedure is presented to deal with recharging and dewatering of unconfined homogeneous aquifer. The new procedure, referred to as the Green element method (GEM), relies on the singular integral theory of the boundary element method (BEM) and permits the solution integral replication of the governing non-linear partial differential equation by domain discretisation. Hence, the advantages of both BEM and the finite element method (FEM) are utilised. Numerical results from test problems obtained by GEM are compared with those from the literature, and for all cases, the results were found to be very encouraging.

Introduction

World-wide, the demand for water has greatly increased while the available amount of water is limited. As a result, reclamation and reuse of water resources has received a great amount of attention in recent times. This has necessitated the need to explore new methods of exploiting water, especially water in aquifers. One of the earlier studies on unconfined aquifers was made by Marino and Yeh (1972). They considered a recharge well with unsteady radial flow in an unconfined homogeneous and isotropic aquifer together with a source term and solved the governing partial differential equation by a method involving transformation and Lagrange interpolation. Their results were very close to the analytical solution of the problem under consideration. Marino (1973) developed analytical expressions that describe the water fluctuations in semi-pervious stream-unconfined aquifer systems. He considered the water level in the stream to be lowered suddenly below its initial elevation and then suddenly raised above its initial elevation while the storage capacity of the stream bed remained insignificant. The only setback to his work was that the expression derived are applicable only when the rise or decline of the water table does not exceed 50% of the initial depth of saturation. Further work on unconfined aquifer done by Marino (1975) on water table fluctuations beneath a circular uniformly recharging area resulted in a numerical solution based on the Douglas-Jones Predictor-corrector method. His solution gave encouraging results when compared with those available in literature. Rao and Sarma (1984) developed an analytical solution for determining a groundwater profile resulting from localised recharge to a finite unconfined aquifer with mixed boundary conditions. In their study, they used the extended finite Fourier transforms and the method of images to arrive at the analytical solution. Their analytical results were validated by experimental results. Latinopoulos (1981) presented an analytical solution for groundwater flow in an unconfined aquifer under seasonal recharge. He showed that to have a uniform recharge rate over the whole period does not enhance an accurate analytical solution. Another contribution was made by Lockington (1997) in determining the water table in an unconfined aquifer bounded by a stream. He considered the aquifer dewatering and recharging. In his work, he presented analytical solutions of the Boussinesq equation that describes the recharging and dewatering process in an unconfined aquifer.

The scarcity of published work in groundwater literature concerning the use of the boundary element method (BEM) to solve the non-linear Boussinesq equation that describes the flow of moisture in an unconfined aquifer is mainly due to the numerical difficulties encountered in applying this method to resolve non-linearity. In the work reported herein, we adopt a novel numerical procedure (Onyejekwe 1995; Taigbenu and Onyejekwe, 1997) based on the boundary integral theory to resolve non-linearity in an efficient and straight-forward way. This is the key motivation for this work.

* To whom all correspondence should be addressed.
☎ (031) 204-4571; fax (031) 204-4291; e-mail okey@pixie.udw.ac.za
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Problem formulation

The non-linear Boussinesq equation that describes the flow of water in an unconfined aquifer obtained under Dupuit-Forchheimer assumptions is given by :

$$\frac{K}{S}\frac{\partial}{\partial x}\left(\phi\frac{\partial\phi}{\partial x}\right) = \frac{\partial\phi}{\partial t} + \frac{R(x,t)}{S} \qquad (1)$$

where:

- $\phi\ (x,t)$ is the height of water table above impervious layer
- x and t are space and time co-ordinates respectively
- K is the hydraulic conductivity
- S is the specific yield
- R(x,t) is the source-sink term.

For Eq. (1) to be well posed, appropriate initial and boundary conditions should be specified. The initial condition for Eq. (1) that describes the distribution of the scalar variable at initial time for a computational domain, Ω, is given by:

$$\phi(x,t_o) = \phi_o \quad \text{on } \Omega \qquad (2)$$

The Dirichlet data that can be specified on a portion of the