

Analytical solutions for the recovery tests after constant-discharge tests in confined aquifers

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ABSTRACT

A new analytical solution for residual drawdown during the recovery period after a constant rate pumping test is described. A comparison between the proposed solution, existing solutions and experimental data from field observation are presented. The proposed analytical solution is in perfect agreement with the experimental data for $\alpha = 0.01$, in contrast to the Cooper-Jacob solution. A new analytical solution for the determination of the skin factor without any restriction on the variables t and t' is derived. An analytical solution for the drawdown response in a confined aquifer that is pumped step-wise or intermittently at a different discharge rate is suggested. On the basis of the suggested solution, a new analytical solution for the analysis of residual drawdown data after a pumping test with step-wise or intermittently changing discharge rates is provided.

Keywords: recovery equations, residual drawdown, skin effect, variable discharges

INTRODUCTION

As soon as the pump is closed up after a test, the water levels in the well and the piezometers will start to rise. This rise in water level is referred to as residual drawdown, s' . In groundwater studies, this is expressed as the difference between the initial water level prior to the start of pumping and the water level measured at a time t' after the termination of pumping. In a hydraulic test, it is very important to measure the residual drawdown during the recovery period. Recovery test measurements allow the transmissivity of the aquifer under investigation to be determined more accurately. The residual drawdown field data are more reliable than the pumping test data because recovery occurs at a constant rate, whereas in practise, a constant discharge during pumping is often difficult to achieve (Kruseman and Ridder, 1994). For any well-flow equations, an adequate recovery equation can be mathematically represented (Kruseman and Ridder, 1994).

The concept of a skin effect is derived from petroleum engineering, which uses the concept to account for the head losses in the vicinity of the well (Ramey, 1982). If the effective radius of the well r_{ew} is larger than the real radius of the borehole r_w in groundwater studies, this concept is referred to as a positive skin effect. If it is smaller, the well is usually poorly developed or its screen is clogged and this is referred to as a negative skin effect (De Marsily, 1986). In groundwater studies, the theory behind this concept is that the aquifer is assumed to be homogeneous up to the wall of the borehole, while all the head losses are assumed to be concentrated in a thin, resistant skin against the wall of the borehole. In groundwater hydraulics, the skin effect is defined as a difference between the total drawdown observed in a well and loss component, assuming that the non-linear well losses are negligible (Kruseman and Ridder, 1994; Atangana, 2014; Atangana and Vermeulen, 2014; Cloot and Botha, 2006; Barker, 1986; Bear, 1972; Boonstra and Kselik, 2002).

In this paper a new analytical solution to the Theis recovery equation for the confined aquifer and the skin effect will be provided. It is very important to point out that the flow equation does not always give aquifer parameters accurately, because the data collected during the pumping test are obtained when the aquifer is under stress. However, the data collected after the pump is shut down are natural. Therefore, rather than estimating the aquifer parameters with the drawdown, which is the solution of the flow equation, one will instead consider the estimation with the recovery solution, which gives us the data while the aquifer is not under stress.

Confined aquifers and Theis's recovery method

In 1935, Theis (Theis, 1935) was the first to develop an equation for unsteady state flow, which introduced the time factor and the storativity as:

$$\frac{S}{T} D_t(s(r, t)) = D_{rr}^2(s(r, t)) + \frac{1}{r} D_r(s(r, t)) \quad (1)$$

and provided a solution to this equation as:

$$s(r, t) = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy, \quad u = \frac{Sr^2}{4Tt} \quad (2)$$

where:

T is the transmissivity

S the storativity

Q the constant discharge rate

On the basis of Eq. (2) Theis developed his recovery method for confined aquifers. The Theis recovery method is widely used for the analysis of recovery tests.

Analytical solutions

After a constant rate pumping test, the residual drawdown during the recovery period according to Theis is given by:

$$s(r, t) = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-y}}{y} dy - \frac{Q}{4\pi T} \int_{u'}^\infty \frac{e^{-y}}{y} dy, \quad u = \frac{Sr^2}{4Tt} \text{ and } u' = \frac{S'r^2}{4Tt'} \quad (3)$$

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