Modelling of a dimensionless synthetic unit hydrograph

Gaddi Ngitane-Katashaya* and Robert Cowen2

1 Dept Earth Sciences, University of Venda, P/Bag X5050, Thohoyandou 0950, South Africa
2 Department of Mathematics, University of Botswana, P/Bag 0022, Gaborone, Botswana

Abstract

When use of a synthetic unit hydrograph on ungauged and non-comparable catchments is required, a further complication is how to choose a suitable time-step to use. The computed time-step is rarely an integral multiplier or divider of the rainfall time-step thus necessitating reanalysing the rainfall chart, at the new time-step. It would be easier if the time-step of the unit hydrograph could be expressed as a continuous function.

In February 1995 there was a flash flood in mid-Eastern Botswana which resulted in all gauges being washed away. The only available data were from upland recording rain gauges from which the reconstruction of the flood hydrograph had to be done. Use of a dimensionless synthetic unit hydrograph (DSUH) was indicated. This is the first part of a two-part paper which involves analyticising (transforming a numerical/graphical function into a continuous one i.e. rendering it analytical) the dimensionless unit hydrograph i.e. developing a piecewise/zonal continuous function of time vs. the runoff.

A DSUH was piecewise modelled using usual statistical methodology and special mathematics software. In spite of having had to use several and basically different functions from piece/zone to piece/zone, the final goodness of fit was exceptional. Demonstrating the application results is shown in the unit hydrograph-based mathematical model which was developed to solve for the above flood. This will be presented in a follow-up paper, which will dwell more on the hydrology of the exercise.

Introduction

Using a unit hydrograph (UH) to compose a catchment runoff hydrograph has always relied on working from the catchment’s typical hydrographs produced from rainfall of known intensity and duration, one of the principles on which unit hydrographs are based (Sherman, 1932; Linsley et al., 1982/1988). From this one can work out instantaneous UHs and UHs of various durations to suit rainfall of various durations. Although it is known that modelling has taken over from use of UH whose basic assumptions are gross, for economical and urgency reasons where accuracy is not so crucial, one may quickly get an approximate result which might serve the purpose.

Normally, one resorts to synthetic unit hydrographs (SUHs) if there are no observed discharge hydrographs. This requires that the catchment characteristics (Snyder, 1938; Linsley et al., 1982/1988) be obtained or determined which are then used to adapt the SUH to suit a particular catchment. The catchment/basin lag-time and other catchment characteristics are required to customise the UHs to that particular catchment (Taylor et al., 1952; Mockus, 1957; Wilson, 1983). Extensive use of several catchment characterisation parameters can produce a reasonable and largely reliable runoff hydrograph suitable for design purposes (FSR, 1975/85).

The time-step of rainfall in a rainfall-runoff hydrograph is important as it is the parameter which determines the time-step of the hydrograph and the runoff input. Irrespective of the duration of the rainfall, the time-step goes a long way in determining the apparent intensity of the rainfall as experienced by the catchment, compared to the real intensity. Only accurate representation of the rainfall intensity relative to the catchment can give a true reflection of the resulting discharge. It is imperative that the time-step used in rainfall, the UH, and eventually the constituted discharge, are the same in size and reflect the true relativeness of the rainfall, UH and runoff. Starting with an SUH, this is usually the cause of doubt and difficulty, as will be explained later.

This paper, therefore, is about analyticising a UH thus rendering it easy to adjust its time-step to conform with that of rainfall thus eliminating one source of uncertainty in UH-rainfall-runoff interaction.

The flood of Mahalapye-Palapye catchments

In February 1995 there was a severe flood in the Mahalapye and Palapye catchments of Botswana (200 km NE of Gaborone and 100 km SE of Francistown - the Lotsane and Mahalapye Rivers are south-flowing tributaries of the Limpopo River). The river-gauging stations were washed away as were some houses near the rivers. The only data available were the rain-gauge records and of flood marks on trees and bushes in the floodways.

The condition of the catchment was such that the floods came within a day of the start of the rainfall and since the catchments were reasonably steep, with little infiltration capacity, the rivers were back to nil discharge within a few days afterwards. A flash flood situation was thus indicated. This showed that the solution would be obtained by using a unit hydrograph, specifically an SUH as there were no prior records of similar circumstances.

The necessity for modelling the flood was to obtain a flood hydrograph thus being able to reconstruct and delineate the flood zone, flood duration, and thus work out assistance and compensation programmes. Mathematical modelling, deterministic simulation-based (for more accuracy and representativity), would have taken longer and would have been more expensive, contrary to the basic objective of the exercise.

The problem: Use of a dimensionless synthetic unit hydrograph

The commonly used dimensionless synthetic unit hydrograph (DSUH) is the US Soil Conservation Service (USSCS) type (Linsley et al., 1982/1988). It is usually presented as a graph thus
leaving the user to determine its dimensions from a graphical representation. This allows the user to set the time-step for sampling to one which will suit the rainfall data and catchment characteristics at hand. Bearing in mind that the dimensional time-steps of the UH are determined using the catchment characteristics and rainfall intensity (in some formulations) the UH characteristics are determined after the rainfall, thus making it impossible to determine the time-step in advance for a one-off event as the present case.

If one is dealing with an instantaneous unit hydrograph (IUH), it is possible to determine the desired time-step mathematically. It is desirable that the extraction of DSUH ordinates be done at a relative time-step which, when multiplied by \( T_p \) gives a time-step equal to that of rainfall. The time-step thus determined is usually in non-integral multiples/dividers of the time to peak \( T_p \). It is thus imperative that dimensionless time-step similar to that of rainfall. Thus a need to be able to manipulate the time-step is presented.

The solution: Stepless time increment scale

It was realised that in order to achieve the requirements, one had to have a DSUH which is continuous (analytical), thus able to assume any time-step that might be imposed on it. The solution thus lay in the ability of the DSUH to assume the analytical state i.e. \( U\) = \( F(X) \), where \( U\) (the DSUH ordinate) is a function of the time-step \( X \). In this case \( n \) represents the fact that one may need to use several functions in order to represent the SUH curve properly.

With this analytical SUH, and ordinates \( U\), one only needs to specify at what time(\( X \)) one requires \( U\). One would get a \( U\)(\( X \)) which is the dimensionless SUH ordinate at relative time \( X \). Thus if the rainfall time-step is 1 h then the relative time-step (Xut) will be 1 h divided by \( T_p \) and the \( U\) corresponding to \( X \) units of time would be equal to 1.0 (at Xut is equal to 1.0 i.e. at the peak real time (Tp)). The corresponding CUH ordinates and time-steps would be \( UH=U\) Qpk and 4q = Xut Tp (Wilson, 1983).

Subsequent presentation will dwell exclusively on the procedure for analysing the DSUH as the main objective of this paper, the flood simulation will be in a subsequent paper. The resulting hydrograph closely resembles \( f(x) = x.exp(x) \) Walpole and Myers (1990) which indicates that one should try and model the data by functions whose terms are products of powers of \( x \) and exponentials. The data were divided into the following four regions:

- **Region A**: initial rise of the hydrograph
- **Region B**: peak region (from mid-rise, peak, to upper-recession - an inverted bell shape).
- **Region C**: mid-recession to mid-lower recession
- **Region D**: beyond mid-lower recession.

Subsequent use of the software Mathematica (Wolfram Research Inc., 1993) was to determine the values of the constants when the functions had been determined.

**Modelling Region A**

We proposed that a model of the form \( F(x) \) [as in Eq. (1)] should be fitted to the data points (a cubic polynomial with a little exponential growth added).

\[
F(x) = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4
\]  

where:

- \( \alpha_1 \) and \( k \) are constants to be determined

Let the data be represented by \( \{(X_i, Y_i): 1 \leq i \leq 101 \} \). As solving simultaneous equations involving transcendental functions is extremely difficult, the constant \( k \) (multiplier of exponent \( x \)) was estimated using computer simulation to be \( k = 1.57 \). The constants \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) were then estimated by the method of least squares which we briefly describe:

Choosing the data pairs \( \{(X_n, Y_n) \} \) in zone: \( 0 \leq i \leq 6 \) we define:

\[
J(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \Sigma(Y-f(X))^2
\]

From:

\[
\frac{\delta J}{\delta \alpha_1} = 0, \quad \frac{\delta J}{\delta \alpha_2} = 0, \quad \frac{\delta J}{\delta \alpha_3} = 0, \quad \frac{\delta J}{\delta \alpha_4} = 0, \quad \text{we obtain:}
\]

\[
\begin{align*}
\sum XY &= \sum Xf(X) \\
\sum X^2 Y &= \sum X^2 f(X) \\
\sum X^3 Y &= \sum X^3 f(X)
\end{align*}
\]

which give (Mathematica, 1993):

- \( \alpha_1 = 2.0876 \) \( \alpha_2 = -1.93324 \) \( \alpha_3 = 4.23726 \) \( \alpha_4 = 3.69121 \)

**Modelling Region B**

We attempted to model this region by a polynomial and computer simulation showed that at least a quintic (a 5th power polynomial) was necessary. Hence we proposed the model \( g(x) \) such that:

\[
g(x) = \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5
\]

By considering the data points \( \{(X, Y) \} \) in zone: \( 15 \leq i \leq 32 \) and using the method of least squares we obtained (Mathematica, 1993):

- \( \beta_1 = -1.91541 \) \( \beta_2 = 9.70405 \) \( \beta_3 = -9.9143 \)
- \( \beta_4 = 3.42073 \) \( \beta_5 = -0.289851 \)

**Modelling Region C**

We proposed an exponential model of the form \( h(x) \) such that:

\[
h(x) = \alpha e^{\beta x}
\]

where:

- \( x \) is a time variable, and \( \alpha \) and \( \beta \) are constants.
Pairs of data were chosen and inserted into the equation. This was then solved to obtain the constants. We found that one obtained the closest fit when the equation was solved for \( \{X_i, Y_i\} \) in zone: \( 46 < i < 74 \) which gave (Mathematica, 1993):

\[ \alpha = 6.603689528 \quad \beta = -1.554251744 \]

### Modelling Region D

We proposed a negative exponential model \( \{\text{function } K(x)\} \) of the form:

\[ k(x) = \alpha e^{\beta x} \quad (4) \]

where \( x, \alpha, \beta \) have the same functional meanings as before and used the same method as for region B. Using \( \{\{X_i, Y_i\} \text{ in zone: } 74 < i < 101\} \) we obtained the following (Mathematica, 1993):

\[ \alpha = 116.9078954 \quad \beta = -2.252735315 \]

### The overall model

By comparing the value of the obtained functions to the actual data we were able to deduce their respective domain. Thus, this gave us the following model:

\[ \phi = \{f(x): x \in [0.0, < 0.7]\} \text{ valid in } X_t \text{ range (zone) } 0.0 \text{ to } < 0.7 \]

\[ = \{g(x): x \in [0.7, < 1.6]\} \text{ valid in } X_t \text{ range (zone) } 0.7 \text{ to } < 1.6 \]

\[ = \{h(x): x \in [1.6, < 4.15]\} \text{ valid in } X_t \text{ range (zone) } 1.6 \text{ to } < 4.15 \]

\[ = \{k(x): x \in [4.15, 5.0]\} \text{ valid in } X_t \text{ range (zone) } 4.15 \text{ to } 5.0 \]

### Transition zones

As is the normal practice in engineering, the jump discontinuities of the model (from one zone to another) are amended in such a way that the values for the model in the regions \( [0.65, 0.75], [1.55, 1.65], \) and \( [4.1, 4.2] \) which are referred to as “shared zones” in the program, are determined by taking fractions of function values which lie on either side of the jump (This aspect was programmed in such a way that unequal fractions of the functions, in the shared zone, could be used to minimise the error within the zone).

### Validation of the model

Let:

\[ Y_n = \text{the data}; \quad y = \text{mean value of the data}; \]

\[ Z_n = \text{estimate from model}; \quad z = \text{mean value of estimate from model}; \]

The coefficient of correlation, \( r \), is defined by

\[ r = \frac{\Sigma(Y_n - y)(Z_n - z)}{\sqrt{\Sigma(Y_n - y)^2 \Sigma(Z_n - z)^2}} \quad (5) \]

where: \(-1.0 \leq r \leq 1.0\)

The coefficient of correlation can be interpreted in the following way:

<table>
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<th>Range of correlation coefficient</th>
<th>0.0 to 0.2</th>
<th>0.2 to 0.4</th>
<th>0.4 to 0.7</th>
<th>0.7 to 0.9</th>
<th>0.9 to 1.0</th>
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<td>weak model</td>
<td>moderate model</td>
<td>strong model</td>
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</table>

In our case \( r = 0.99915 \), which is considered to be an almost perfect model.

The standard error of estimate, SEE is defined in Eq. (6) which, for our results, is equal to 0.00131.

\[ \text{SEE} = \{1 - r^2\} / \{N - 2\} \quad (6) \]

Thus our 99.7% confidence interval is \( r \pm \text{SEE} = [0.995985, 1.003845] \). Due to the large number of data points, it was not necessary to use Fishers Z-transformation (Mathematica, 1993). This confidence interval can be interpreted in the following way:

If another 101 data points were randomly chosen and this was done 1000 times, we would expect that for 997 times the new value of the coefficient of correlation would exceed 0.99598. See the Appendices.

Appendix 1 gives the numerical results obtained after applying typical values to the unit hydrograph function. Note that at this stage our objective was to fit the developed function to the known DUH ordinates.

Appendix 2 gives the subroutine EXPOLYFT of the function which was used to test its performance and as contained in the simulation model.

### Conclusions

- An analytical/functional unit hydrograph facilitates the continuous adjustment of time to peak with changing rainfall onset times and subsequently rainfall durations. This conforms more to the assumptions on which use of a UH is based.
- A function of a unit hydrograph can be obtained using various curve fitting techniques thus making a unit hydrograph a continuous function of time and rainfall amount. This facilitates proper setting of the hydrograph time-step according to the conditions prevailing when it was raining.
- The complete hydrological simulation program or only the EXPOLYFT subroutine can be made available by e-mail, on request.

### Acknowledgement

We would like to thank Dr E Lungu (University of Botswana) for his assistance on computer modelling of the mathematics involved and Mr B Jay, a government engineer, who supplied the data and other necessary information.

### References


Appendix 1

Results of unit hydrograph data fitted to the model (performance of expolift)

Old values of X vs old and estimated values of Y, showing errors after fitting to functions FX,GX,HX,XX which describe the unit hydrograph.

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Yo = YOLD; Yom = yo.mean; Ye = y. estm; Yem = y.estmean; [(10F+9G means in transition zone 10% of functn F value added to 90% of funct G value give the best Ye value)]

Yom(mean) = 0.259465  Yem(mean) = 0.259456  TOTVAR(TotVar) S[(Yo-Yom)^2] = 10.25252
ESTMER(EstmErr) S[(Yo-Ye)^2] = 0.001288  TOTERR S[(Yo-Yom)*(Ye-Yem)] = 10.24110
Subroutine EXPOLYFT

C Subroutine EXPOLYFT evaluates the UH ordinates
C using ExponPolynomial based on USSCS UnitHydrograph

SUBROUTINE EXPOLYFT(XT,YOLD,UHDT,FUNC)
INTEGER FNC
REAL KX
CHARACTER FUNC*9
DATA FNC,F5,F1 /1,0.5,0.1/
DATA F2,F3,F4 /0.2,0.3,0.4/
DATA F5,F6,F7 /0.5,0.6,0.7/
DATA F8,F9,FX /0.8,0.9,0.0/
DATA GX,HX,KX /0.0,0.0,0.0/

C--Setting XT range & go to appropriate equations
8000 IF(XT.ge.0.0.and.XT.lt.0.65) THEN
  FNC= 1
  GOTO 8010
ENDIF
IF(XT.ge.0.65.and.XT.le.0.75) THEN
  FNC = 12
  GOTO 8010
ENDIF
IF(XT.gt.0.75.and.XT.lt.1.55) THEN
  FNC= 2
  GOTO 8020
ENDIF
IF(XT.ge.1.55.and.XT.le.1.65) THEN
  NC= 23
  GOTO 8020
ENDIF
IF(XT.gt.1.65.and.XT.lt.4.10) THEN
  FNC= 3
  GOTO 8030
ENDIF
IF(XT.ge.4.10.and.XT.le.4.20) THEN
  FNC= 34
  GOTO 8030
ENDIF
IF(XT.ge.4.20) THEN
  FNC = 4
  GOTO 8040
ENDIF

C==Function FX models initial rise of hydrograph
C--Range XT:ge 0 to le 0.65 FX; share: >0.65-<0.75 UH
8010 CONTINUE
FX1 = 2.08760*XT
FX2 = 1.93324*XT*EXP(1.57*XT)
FX3 = 4.23726*XT**2
FX4 = 3.69121*XT**3
FX = FX1 + FX2 + FX3 + FX4
IF(FNC.eq.12) GOTO 8020
GOTO 8100

C==Function GX models mid-rise-peak-mid-recession
C--Range XT:ge 0.75 to 1.55 UH; share: >1.55-< 1.75 FX
8010 CONTINUE
GX1 = 1.915410*XT
GX2 = 9.700540*XT**2.0
GX3 = 9.914300*XT**3.0
GX4 = 3.420730*XT**4.0
GX5 = 0.289581*XT**5.0
GX = - GX1 + GX2 - GX3 + GX4 - GX5
IF(FNC.eq.12) GOTO 8110
GOTO 8020

Appendix 2

C==Function HX models midrec->lower-mid-recession
C--Range XT:ge 1.65->le 4.1 HX; share: >1.55-< 1.65
8030 CONTINUE
HX = 6.603689528*EXP(-1.554251744*XT)
IF(FNC.eq.23) GOTO 8123
IF(FNC.eq.34) GOTO 8040
GOTO 8130

C==Function KX models the lower recession (tail)
C--Range XT:ge 4.20 HX; share: >4.10-< 4.20 with HX
8040 CONTINUE
KX = 116.9078954*EXP(-2.252735315*XT)
IF(FNC.eq.34) GOTO 8134
GOTO 8140

C The following routine tests combinations of func
C lages in SHARED ZONES which give UHT with the
C least error
C---Function FX only
8110 IF(FNC.eq.1) THEN
  UHDT = FX
  FUNC = '[100FX]'!
  GOTO 8200
ENDIF

C SHZones/UnSh>Shared/transition zone; Unshared
C FValUnShZ---->Function value in unshared zone
C ErDuModSh---->Error due to mode of share/assign
C Comp50%ShZ--->Compute at 50% funcs in Sh-zone
C Z%FVCompUHT-->In Sh-zone % of FVals to make UHT
C ErDu%Sh------>Error due to  percent  of sharing
C
C==Functions FX and GX========================
8112 IF(FNC.eq.12) THEN
  SH1 = FX                   ! FValUnShZ
  U1  = ABS((SH1-YOLD)/YOLD) ! ErDuModSh
  SH2 = GX                   ! FValUnShZ
  U2  = ABS((SH2-YOLD)/YOLD) ! ErDuModSh
  SH12= F5*FX+(1.0-F5)*GX    ! Comp50%ShZ
  U12 = ABS((SH12-YOLD)/YOLD)! ErDuModSh
  SHL1= F1*FX+(1.0-F1)*GX    ! Z%FVCompUHT
  UL1 = ABS((SHL1-YOLD)/YOLD)! ErDu%Sh
  SHL2= F2*FX+(1.0-F2)*GX    ! Z%FVCompUHT
  UL2 = ABS((SHL2-YOLD)/YOLD)! ErDu%Sh
  SHL3= F3*FX+(1.0-F3)*GX    ! Z%FVCompUHT
  UL3 = ABS((SHL3-YOLD)/YOLD)! ErDu%Sh
  SHL4= F4*FX+(1.0-F4)*GX    ! Z%FVCompUHT
  UL4 = ABS((SHL4-YOLD)/YOLD)! ErDu%Sh
  SHL5= F5*FX+(1.0-F5)*GX    ! Z%FVCompUHT
  UL5 = ABS((SHL5-YOLD)/YOLD)! ErDu%Sh
  SHL6= F6*FX+(1.0-F6)*GX    ! Z%FVCompUHT
  UL6 = ABS((SHL6-YOLD)/YOLD)! ErDu%Sh
  SHL7= F7*FX+(1.0-F7)*GX    ! Z%FVCompUHT
  UL7 = ABS((SHL7-YOLD)/YOLD)! ErDu%Sh
  SHL8= F8*FX+(1.0-F8)*GX    ! Z%FVCompUHT
  UL8 = ABS((SHL8-YOLD)/YOLD)! ErDu%Sh
  SHL9= F9*FX+(1.0-F9)*GX    ! Z%FVCompUHT
  UL9 = ABS((SHL9-YOLD)/YOLD)! ErDu%Sh

  UM1 = MIN(U1,U2,U12,UL1,UL2,UL3,UL4)
  UMIN= MIN(UM1,UL6,UL7,UL8,UL9)

  IF(UMIN.eq.U1) THEN
    UHDT = SH1
    FUNC = '[100F+0G]'!
    GOTO 8200
  ENDIF

IF (UMIN.eq.U12) THEN
  UHDT = SH12
  FUNC = ' [50F+50G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.U2) THEN
  UHDT = SH2
  FUNC = ' [0F+100G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.U11) THEN
  UHDT = SH11
  FUNC = ' [10F+90G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.U12) THEN
  UHDT = SH2
  FUNC = ' [20F+80G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.U13) THEN
  UHDT = SH13
  FUNC = ' [30F+70G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.U14) THEN
  UHDT = SH14
  FUNC = ' [40F+60G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.UH6) THEN
  UHDT = SHH6
  FUNC = ' [60F+40G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.UH7) THEN
  UHDT = SHH7
  FUNC = ' [70F+30G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.UH8) THEN
  UHDT = SHH8
  FUNC = ' [80F+20G]'
  GOTO 8200
ENDIF
IF (UMIN.eq.UH9) THEN
  UHDT = SHH9
  FUNC = ' [90F+10G]'
  GOTO 8200
ENDIF
FNC = 2
GOTO 8200
ENDIF
C--Function GX only
8120 IF (FNC.eq.2) THEN
  UHDT = GX
  FUNC = ' [100GX]'
  GOTO 8200
ENDIF
C--Functions GX and HX
8123 IF (FNC.eq.23) THEN
  SH2 = GX ! FValUnShZ
  U2 = ABS((SH2-YOLD)/YOLD) ! ErDu%Sh
  SH3 = HX ! FValUnShZ
  U3 = ABS((SH3-YOLD)/YOLD) ! ErDuModSh
  SH23 = F1*GX+(1.0-F1)*HX ! Comp50%Sh2
  U23 = ABS((SH23-YOLD)/YOLD) ! ErDuModSh
  SHL1 = F1*GX+(1.0-F1)*HX ! Z%FVCompUHT
  UL1 = ABS((SHL1-YOLD)/YOLD) ! ErDu%Sh
  SHL2 = F2*GX+(1.0-F2)*HX ! Z%FVCompUHT
  UL2 = ABS((SHL2-YOLD)/YOLD) ! ErDu%Sh
  SHL3 = F3*GX+(1.0-F3)*HX ! Z%FVCompUHT
  UL3 = ABS((SHL3-YOLD)/YOLD) ! ErDu%Sh
  SHL4 = F4*GX+(1.0-F4)*HX ! Z%FVCompUHT
  UL4 = ABS((SHL4-YOLD)/YOLD) ! ErDu%Sh
  SHH6 = F6*GX+(1.0-F6)*HX ! Comp50%Sh6
  UH6 = ABS((SHH6-YOLD)/YOLD) ! ErDuModSh
  SHH7 = F7*GX+(1.0-F7)*HX ! Comp50%Sh7
  UH7 = ABS((SHH7-YOLD)/YOLD) ! ErDuModSh
  SHH8 = F8*GX+(1.0-F8)*HX ! Comp50%Sh8
  UH8 = ABS((SHH8-YOLD)/YOLD) ! ErDuModSh
  SHH9 = F9*GX+(1.0-F9)*HX ! Comp50%Sh9
  UH9 = ABS((SHH9-YOLD)/YOLD) ! ErDuModSh
  UM1 = MIN(U2, U3, U23, UL1, UL2, UL3, UL4)
  UMIN = MIN(UM1, UH6, UH7, UH8, UH9)
ENDIF
IF (UMIN.eq.U2) THEN
  UHDT = SH2
  FUNC = ' [100G+0H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.U23) THEN
  UHDT = SH23
  FUNC = ' [50G+50H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.U3) THEN
  UHDT = SH3
  FUNC = ' [0G+100H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.UL1) THEN
  UHDT = SHL1
  FUNC = ' [10G+90H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.UL2) THEN
  UHDT = SHL2
  FUNC = ' [20G+80H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.UL3) THEN
  UHDT = SHL3
  FUNC = ' [30G+70H]'!
  GOTO 8200
ENDIF
IF (UMIN.eq.UL4) THEN
  UHDT = SHL4
  FUNC = ' [40G+60H]'!
  GOTO 8200
ENDIF
UM1 = MIN(U2, U3, U23, UL1, UL2, UL3, UL4)
UMIN = MIN(UM1, UH6, UH7, UH8, UH9)
IF (UMIN.eq.UH9) THEN
    UHDT = SHH9
    FUNC = '[(90G+10H)]'
    GOTO 8200
ENDIF
FNC = 3
GOTO 8130
ENDIF
C--Function HX only
8130 IF (FNC.eq.3) THEN
    UHDT = HX
    FUNC = '[(100HX)]'
    GOTO 8200
ENDIF
C--Functions HX and KX
8134 IF (FNC.eq.34) THEN
    SH3 = HX ! FValUnShZ
    U3 = ABS((SH3-YOLD)/YOLD) ! ErDuModSh
    SH4 = KX ! FValUnShZ
    U4 = ABS((SH4-YOLD)/YOLD) ! ErDuModSh
    SH34 = F1*HX+(1.0-F1)*KX ! Comp50%ShZ
    U34 = ABS((SH34-YOLD)/YOLD)! ErDuModSh
    SHL1 = F1*HX+(1.0-F1)*KX ! Z%FVCompUHT
    U3 = ABS((SHL1-YOLD)/YOLD)! ErDuModSh
    SHL2 = F2*HX+(1.0-F2)*KX ! Z%FVCompUHT
    U2 = ABS((SHL2-YOLD)/YOLD)! ErDuModSh
    SHL3 = F3*HX+(1.0-F3)*KX ! Comp50%ShZ
    U3 = ABS((SHL3-YOLD)/YOLD)! ErDuModSh
    SHL4 = F4*HX+(1.0-F4)*KX ! Z%FVCompUHT
    U4 = ABS((SHL4-YOLD)/YOLD)! ErDuModSh
    U34 = ABS((SH34-YOLD)/YOLD)! ErDuModSh
    IF (UMIN.eq.U3) THEN
        UHDT = SH3
        FUNC = '[100H+0K]'
        GOTO 8200
    ENDIF
    IF (UMIN.eq.U34) THEN
        UHDT = SH34
        FUNC = '[50H+50K]'
        GOTO 8200
    ENDIF
C--Function KX only
8140 IF (FNC.eq.4) THEN
    UHDT = KX
    FUNC = '[100KX]'
    GOTO 8200
ENDIF
C==============================================
8200 CONTINUE
8201 RETURN
END